NCCC-134

APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

Volatility Persistence in Commodity Futures: Inventory and Time-to-Delivery Effects

by

Berna Karali and Walter N. Thurman

Volatility Persistence in Commodity Futures:

Inventory and Time-to-Delivery Effects

Berna Karali

and

Walter N. Thurman[*]

---

[*]Berna Karali (bkarali@uga.edu) is an Assistant Professor in the Department of Agricultural and Applied Economics at the University of Georgia and Walter N. Thurman (wally_thurman@ncsu.edu) is a Professor in the Department of Agricultural and Resource Economics at North Carolina State University.

Volatility Persistence in Commodity Futures:
Inventory and Time-to-Delivery Effects

Most financial asset returns exhibit volatility persistence. We investigate this phe-nomenon in the context of daily returns in commodity futures markets. We show that the time gap between the arrival of news to the markets and the delivery time of futures contracts is the fundamental variable in explaining volatility persistence in the lumber futures market. We also find an inverse relationship between inventory levels and lum-ber futures volatility.

Key words: volatility persistence, theory of storage, volatility, futures markets, lumber

Introduction

Volatility of financial asset returns persists. High-volatility periods are apt to be followed by high-volatility periods, and similarly for low-volatility periods. This has been observed for returns to publicly traded stocks (Engle, 2004) and for returns to futures contracts–both those written on financial assets (Li and Engle, 1998) and those written on physical commodities (Pindyck, 2001, 2004; Ng and Pirrong, 1994).

Explanations for volatility persistence have been proposed from microstructure models. The process of market price reaction to information flows is argued to result in such persistence. See, for example, Kyle (1985) and Andersen and Bollerslev (1997). But there are other reasons to expect volatility persistence in the case of futures contracts written on commodities. Aggregate physical inventories play a shock absorbing role in commodity markets, implying that when physical inventories are large, the size of commodity futures price changes is small (see Thurman, 1988; Williams and Wright, 1991; Karali and Thurman, 2008). Further, aggregate inventories evolve slowly. Thus, commodity futures volatility is characterized by phases of varying length, depending on the speed of inventory changes for the particular commodity.

A second factor influencing volatility in commodity futures is the time gap between when news arrives to the market and when the contract calls for delivery. The major futures exchanges trade several contracts for a commodity, differing only by delivery date. Near-term information shocks should have greater impact on contracts for near-term delivery than on contracts for farther-out delivery due to the smaller elasticities of supply and demand for shorter runs. The implications for volatility's dependence on time to delivery can only be studied by analyzing the multiple contracts that are simultaneously traded.

We test these two theoretical implications in the lumber futures market. In earlier work (Karali and Thurman, 2008), we study the effect of housing starts announce-

ment surprises relative to Money Market Services survey forecasts on lumber futures prices. We find statistically and economically significant announcement, inventory, and time-to-delivery effects and show that the price response to observed information flows depends on inventories and time to delivery. In this study, we extend the earlier work by studying the response of lumber futures prices to unobserved information flows. We hypothesize that, as in the case of observed information, the price response to unobserved information flows should also depend on inventories and time to delivery. Because the sign of the unobserved shock is unknown, we are naturally lead to use absolute log price changes, a commonly used measure for volatility in the literature. Specifically, we analyze daily lumber futures prices from the Chicago Mercantile Exchange (CME) from 1992 to 2005, defining volatility as the absolute value of log price changes over a day. Studying time-to-delivery effects requires using data on all contracts traded on a given day. This, in turn, requires a recognition of the correlation among price observations from the same day, which are subject to common shocks. We use a Generalized Least Squares (GLS) procedure similar to the one in Karali and Thurman (2008) to take this contemporaneous correlation into account, resulting in efficient use of futures price data and consistent standard errors of our estimates.

We find an inverse relationship between inventory levels and lumber futures volatility, as predicted by the theory of storage. As inventory levels become smaller lumber futures contracts become more volatile. The relationship is both statistically and economically significant. We also find an inverse relation between time to delivery and volatility. The closer the contract trade date is to delivery time, the higher is price volatility. We interpret this result in standard microeconomic terms. Lumber supply and demand curves become more inelastic as time to delivery nears. Thus, shocks originating from either the supply or demand side of the market have a larger price impact as time to delivery nears. Further, we find that while volatility persistence is statistically significant in the marginal distribution of lumber futures returns, much of that persistence can be explained by the dependence on time to delivery.

A Three-Period Storage Model

We use the simple, finite-horizon storage model presented by Williams and Wright (1991) to derive optimal storage rules via a social planner's perspective. We then use these storage rules in a simulation study to investigate the relationship between expected price changes and inventories, which plays an important role in our empirical analysis of futures prices.

Analytic Solution

The model has a finite horizon with three periods. The third and last period is period T; the second period is period $T - 1$; and the first period is period $T - 2$. The carry-out from the last period has no value since the world ends beyond that time. Therefore, carry-out in the last period is zero and everything available in the last period is consumed, that is, $S_T = 0$. Inverse consumption demand is assumed to be

linear in quantity consumed such that

$$P_t = \alpha + \beta q_t, \qquad \text{with} \quad \alpha > 0, \quad \beta < 0. \tag{1}$$

Supply is perfectly inelastic with constant mean $\bar{y}$ and is subject to a random additive disturbance v_t. The supply equation is given by

$$y_t = \bar{y} + v_t, \tag{2}$$

where v_t is a uniformly distributed random disturbance with mean zero and standard deviation σ. Thus, the probability density function of v_t, which is observable by everyone, is defined as

$$f(v_t) = \begin{cases} \dfrac{1}{2\sqrt{3}\sigma}, & -\sqrt{3}\sigma \leq v_t \leq \sqrt{3}\sigma \\ 0, & \text{elsewhere.} \end{cases} \tag{3}$$

In any given period, total availability in the market is the sum of production in that period and carry-in into that period, that is, $A_t \equiv y_t + S_{t-1}$. Consumption in any period satisfies

$$q_t = y_t + S_{t-1} - S_t = A_t - S_t. \tag{4}$$

Mean production level $\bar{y}$ and v_t is observed by all market participants in the beginning of period t; thus, all decisions made in period t are conditioned on the realization of v_t. Marginal physical storage cost is denoted by c and is constant over time. The one-period interest rate is r.

In order to find optimal storage in period $T - 2$, the social planner uses backward induction. The planner first solves for the optimal storage rule in period $T - 1$, and then uses it to derive the optimal storage rule in period $T - 2$. The social planner's optimal choice for storage in any period j is determined by the first order condition of the following objective function:

$$V_j(A_j) = \max_{S_j} \left\{ \int_0^{A_j - S_j} P(q)dq - cS_j + (1+r)^{-1} E_j \left[V_{j+1} \left(y_{j+1} + S_j - S_{j+1} \right) \right] \right\}$$

subject to
$$S_j \geq 0. \tag{5}$$

The first-order condition is then given by

$$\frac{\partial V_j}{\partial S_j} = -P_j - c + (1+r)^{-1} E_j[P_{j+1}], \qquad S_j > 0. \tag{6}$$

Using this first order condition, the planner's optimal choice in period $T - 1$ satisfies:

$$P_{T-1} + c = (1+r)^{-1} E_{T-1}[P_T], \qquad S_{T-1} > 0. \tag{7}$$

3

Substituting the inverse demand equation (1) into equation (7) and then using supply equation (2) in equation (4) yields the optimality condition:

$$\alpha + \beta(A_{T-1} - S_{T-1}) + c = (1+r)^{-1} E_{T-1}\left[\alpha + \beta(\bar{y} + v_T + S_{T-1})\right], \qquad S_{T-1} > 0. \quad (8)$$

The expectation on the right-hand side can be calculated using the probability density function of the random disturbance v_T, since v_T is the only random variable in equation (8). The optimal level of storage in period $T-1$ is then found by solving the following equation for S_{T-1}

$$\alpha + \beta(A_{T-1} - S_{T-1}) + c = \left(\frac{1}{1+r}\right)\left(\frac{1}{2\sqrt{3}\sigma}\right)\int_{-\sqrt{3}\sigma}^{\sqrt{3}\sigma} \left(\alpha + \beta(\bar{y} + v_T + S_{T-1})\right) dv, \quad (9)$$

and the solution is given by:

$$S_{T-1}^* = \max\left\{\frac{(1+r)A_{T-1} - \bar{y}}{(2+r)} + \left(\frac{1+r}{2+r}\right)\left(\frac{c}{\beta}\right) + \left(\frac{r}{2+r}\right)\left(\frac{\alpha}{\beta}\right), 0\right\}. \quad (10)$$

If the interest rate is set equal to zero, as done by Williams and Wright (1991) for simplicity, the optimal storage rule for period $T-1$ is the same as their equation (3.8) on p. 59.

The first-order condition for optimal storage in period $T-2$ is

$$P_{T-2} + c = (1+r)^{-1} E_{T-2}[P_{T-1}], \qquad S_{T-2} > 0. \quad (11)$$

The solution for period $T-2$ becomes complicated due to the possibility of storage in period $T-1$ and a fundamental nonlinearity. As seen from equation (10), optimal storage in period $T-1$ depends on total availability A_{T-1}, which, in turn, depends on the random disturbance to supply in that period, v_{T-1}. Depending on the random shock in period $T-1$, one will either observe no storage or a positive amount of storage. This critical value of the random shock can be found by substituting total availability in period $T-1$ into the optimal storage rule for that period, and then by setting S_{T-1}^* equal to zero. The critical value of v_{T-1} is

$$v_{T-1} = -S_{T-2} - \left(\frac{c}{\beta}\right) - \left(\frac{r}{1+r}\right)\bar{y} - \left(\frac{r}{1+r}\right)\left(\frac{\alpha}{\beta}\right). \quad (12)$$

Whenever v_{T-1} is less than the critical value, the planner will choose not to store in period $T-1$, that is, $S_{T-1} = 0$. Whenever v_{T-1} exceeds the critical value, the planner will choose to store a positive amount in period $T-1$, that is, $S_{T-1} > 0$. After substituting the inverse demand function and consumption identity into equation (11),

the optimality condition becomes

$$\alpha + \beta(A_{T-2} - S_{T-2}) + c =$$

$$\left(\frac{1}{1+r}\right)\left(\frac{1}{2\sqrt{3}\sigma}\right)\left\{\int_{-\sqrt{3}\sigma}^{-S_{T-2}-(c/\beta)-(r\bar{y}+r\alpha/\beta)/(1+r)} \left(\alpha + \beta(\bar{y} + v_{T-1} + S_{T-2})\right) dv\right.$$

$$\left.+ \int_{-S_{T-2}-(c/\beta)-(r\bar{y}+r\alpha/\beta)/(1+r)}^{\sqrt{3}\sigma} \left(\alpha + \beta(\bar{y} + v_{T-1} + S_{T-2} - S_{T-1}^*)\right) dv\right\}.$$

$$(13)$$

The right-hand side of equation (13) shows the weighted average of possible situations with and without storage in period $T-1$. Substituting equation (10) for S_{T-1}^* in equation (13), and integrating over v_{T-1}, results in a quadratic equation, the solution to which gives the optimal storage rule in period $T-2$:

$$S_{T-2}^* = \max\left\{\sqrt{3}\sigma\left(\frac{2r^2 + 7r + 7}{1+r}\right) - \left(\frac{c}{\beta}\right) - \left(\frac{r\bar{y} + r\alpha/\beta}{1+r}\right)\right.$$

$$- \sqrt{12\sigma^2\left(\frac{r^4 + 7r^3 + 19r^2 + 24r + 12}{(1+r)^2}\right) - 4\sqrt{3}\sigma\left((2+r)A_{T-2} - \left(\frac{2+r}{1+r}\right)\bar{y}\right)}$$

$$\overline{-4\sqrt{3}\sigma\left(\frac{2r^2 + 7r + 6}{1+r}\right)\left(\frac{c}{\beta}\right) - 4\sqrt{3}\sigma\left(\frac{r^2 + 2r}{1+r}\right)\left(\frac{\alpha}{\beta}\right)}$$

$$\overline{\left.-4\sqrt{3}\sigma\left(\frac{r^2 + 4r + 4}{1+r}\right)\left(\frac{r\bar{y} + r\alpha/\beta}{1+r}\right)}, 0\right\}.$$

$$(14)$$

As seen from equation (14), the optimal level of storage in the first period, $T-2$, depends on current availability A_{T-2} and model parameters. Availability in period $T-2$ equals the sum of supply in that period and carry-in from period $T-3$. Formally,

$$\begin{aligned} A_{T-2} &= y_{T-2} + S_{T-3} \\ &= \bar{y} + v_{T-2} + S_{T-3}. \end{aligned}$$

$$(15)$$

The level of carry-in into the first period will affect optimal storage in this period, which, in turn, will affect optimal storage in the second period. Therefore, one should expect varying storage and price paths depending on initial carry-in levels.

Simulation Results

With a starting value of carry-in into the first period, S_{T-3}, one can simulate the three-period model by drawing from the probability distribution of the random supply shock. The explicit storage rules derived in the previous section show how many units

will be stored in each period given availability in those periods. The amount stored in each period then will determine consumption and price in those periods. In this way, one obtains a path for each endogenous variable. When this process is repeated many times with the same initial values but with different draws of the random shock, one obtains a conditional frequency distribution for each variable in each period. For instance, for price, we obtain $P_{t,i}$ with $t = T-2, T-1, T$ and $i = 1, 2, \cdots, n$, where n is the number of iterations.

We repeat this process for several values of initial carry-in, S_{T-3}. For each value of S_{T-3}, we compute

$$\left| \frac{1}{n} \sum_{i=1}^{n} (P_{T-2,i} - P_{T-3}) \right|, \tag{16}$$

where

$$P_{T-3} = \left(\frac{1}{1+r} \right) \left(\frac{1}{n} \sum_{i=1}^{n} P_{T-2,i} \right) - c,$$

to represent $E_{T-3}[|P_{T-2} - P_{T-3}|]$. We choose model parameters as follows. Inverse consumption demand equation has intercept of $\alpha = 600$ and slope of $\beta = -5$, that is, $P_t = 600 - 5q_t$. Marginal physical storage cost, c, is \$2 per period. The interest rate, r, is zero. The mean production level, $\bar{y}$, is 100 units. The random disturbance v_t lies between -15 and +15, that is, $\sqrt{3}\sigma = 15$, where σ is the standard deviation of the random disturbance. The number of iterations, n, is 100,000.

Figure 1(a) shows the relationship between the expected absolute price change from the initial period $T-3$ to the first period $T-2$ and the level of initial carry-in. As inventory levels become larger, the expected magnitude of price movement becomes smaller. Figure 1(b), shows how the variance of price movement declines with inventories. So, the absolute price changes seem to capture the variance pattern pretty well. This simulation result demonstrates one important conclusion: the assertion of the theory of storage that $E_t[P_{t+1}]$ is decreasing in S_t, also holds for price movements, $E_t[|P_{t+1} - P_t|]$. This result is the main motivation behind our empirical analysis.

This relationship should also hold in commodity futures markets, which reveal the market's expectation of future spot price changes. Further, the price response of a futures contract to a shock should be smaller when its delivery time is farther away due to greater elasticity of supply and demand curves over longer runs. These hypotheses can be tested in the following linear model of volatility:

$$|\ln F_t - \ln F_{t-1}| = a + f(S_t) + g(TTD_t) + \varepsilon_t, \tag{17}$$

where $\ln F_t$ is the natural logarithm of the price of a futures contract on day t,[1] S_t is the physical inventory level on day t, $f(S_t)$ is a function of inventories, TTD_t is time to delivery, the number of days remaining to contract expiration on day t, between

---

[1]We take the natural logarithm of futures prices in order to eliminate the effects of inflation.

zero and 169, and $g(TTD_t)$ is a function of time to delivery. As an implication of the theory of storage and larger elasticities over longer runs, one should expect to see:

$$\partial(|\ln F_t - \ln F_{t-1}|)/\partial S_t = \partial f(S_t)/\partial S_t < 0$$
$$\partial(|\ln F_t - \ln F_{t-1}|)/\partial TTD_t = \partial g(TTD_t)/\partial TTD_t < 0.$$

Measuring Inventory and Time-to-Delivery Effects on Volatility

We use daily settlement prices of lumber futures from the Chicago Mercantile Exchange (CME), from 77 contracts between July 14, 1992 and November 15, 2005. Lumber futures contracts expire every two months and the delivery months are January, March, May, July, September, and November. On the CME, a new contract is listed on the day after the front month expires. At any point in time, a total of seven contracts are listed, each with a different delivery date up to 14 months into the future. However, we trim the data set purchased from the Commodity Research Bureau to include 170 observations—the number of trading days of the shortest-lived contract—for all contracts, resulting in at most five contracts on a given day.

We use the Lumber & Other Construction Materials inventory series (NAICS 4233) from Monthly Wholesale Trade reports published by the U.S. Census Bureau from January 1992 to December 2005. We convert the inventory series, released in current dollars, into constant 1982 dollar values using the Lumber Producer Price Index published by the Bureau of Labor Statistics. We interpolate the resulting monthly series by a cubic spline method to obtain daily inventories.[2] The daily inventory data are shown in figure 2.

To combine simultaneously traded contracts, we write the empirical version of equation (17) as:[3]

$$|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})| = \alpha + \beta S_t + \gamma_1 TTD_{it} + \gamma_2 TTD_{it}^2 + \varepsilon_{it},$$
$$i = 1, 2, \cdots, k_t, \quad t = 1, 2, \cdots, T, \quad (18)$$

where $|\%\Delta F_{it}|$ is the approximate percentage change in the volatility of daily return on lumber futures, k_t is the number of contracts traded on day t, between one and five, and T=3,365, the number of trading days in our sample. The total number of observations is $\sum_{t=1}^{T} k_t = 13,090$. Descriptive statistics of the variables are presented in table 1.

If we just estimate equation (18) via Ordinary Least Squares (OLS), we would be ignoring the large correlation among the price observations on a given day. To account for

---

[2]We also tried linear and step function interpolation methods and found little substantive change in the results.

[3]Because Karali and Thurman (2008) find evidence of a nonlinear time-to-delivery effect on lumber futures price response to housing starts announcement shocks, we choose a quadratic function of time to delivery.

this contemporaneous correlation, we define the following structure for the disturbance:

$$\boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{k_t t} \end{bmatrix}, \quad E(\boldsymbol{\varepsilon}_t) = \mathbf{0}, \quad E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_{t-1}) = \mathbf{0}, \quad V(\boldsymbol{\varepsilon}_t) = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1k_t} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2k_t} \\ \vdots & \vdots & & \vdots \\ \sigma_{1k_t} & \sigma_{2k_t} & \cdots & \sigma_{k_t}^2 \end{bmatrix}.$$

Because the number of contracts varies across trading days it is difficult to implement a GLS method that integrates this structure for the disturbance. For detailed computations, see Karali (2007) and Karali and Thurman (2008). Here, we define $V(\boldsymbol{\varepsilon}_t)$ in such a way that it assumes both covariance stationarity over time and identical covariances between contracts that have the same discrepancy in delivery month. That is, the covariance between the first and second nearby contracts (σ_{12}) is assumed to be the same as that between the second and third nearby contracts (σ_{23}) as there are two-month delivery discrepancy between these contracts. The estimates $\hat{V}(\boldsymbol{\varepsilon}_t)$ and $\hat{Corr}(\boldsymbol{\varepsilon}_t)$ from equation (18), which give an idea of the importance of the GLS method, are:

$$\hat{V}(\boldsymbol{\varepsilon}_t) = \begin{bmatrix} 0.97 & 0.73 & 0.61 & 0.50 & 0.25 \\ 0.73 & 0.97 & 0.73 & 0.61 & 0.50 \\ 0.61 & 0.73 & 0.97 & 0.73 & 0.61 \\ 0.50 & 0.61 & 0.73 & 0.97 & 0.73 \\ 0.25 & 0.50 & 0.61 & 0.73 & 0.97 \end{bmatrix}, \quad \hat{Corr}(\boldsymbol{\varepsilon}_t) = \begin{bmatrix} 1 & 0.75 & 0.63 & 0.51 & 0.26 \\ 0.75 & 1 & 0.75 & 0.63 & 0.51 \\ 0.63 & 0.75 & 1 & 0.75 & 0.63 \\ 0.51 & 0.63 & 0.75 & 1 & 0.75 \\ 0.26 & 0.51 & 0.63 & 0.75 & 1 \end{bmatrix}.$$

Results from equation (18) are presented in table 2. As seen in the table, the inventory coefficient is negative. This is the sign the theory of storage predicts for inventories. The smaller the inventories, the higher the volatility and vice versa. When inventories are large enough they prevent large fluctuations in price that would be caused by demand and supply shocks. When inventories are small, demand and supply shocks cause larger fluctuations in price.[4] The coefficient on the linear TTD term is negative and the one on the quadratic term is positive. The overall time-to-delivery effect on volatility with the GLS estimates over the range of the TTD variable is shown in figure 3(a). As the figure shows, holding everything else constant, the time-to-delivery effect is negative and increases from 0.002 to 0.008 in magnitude over the life of a contract. Figure 3(b) shows that when evaluated at the mean value of inventories, volatility of a futures contract increases from 0.9 percentage points to 1.7 percentage point from the first to the last trading day. This indicates that as a contract approaches delivery its

---

[4]One might be concerned that slowly evolving inventories are proxying for an exogenous nonlinear trend. To investigate this possibility, we added a quadratic function of time trend to our model and found it to be statistically significant. However, inventory effects become even larger and stronger in this model. Thus, deviations of inventories from their trend also affect volatility inversely, indicating that the significant inventory effect in table 2 is not simply an exogenous time trend.

volatility rises. When a contract is far from its delivery date its volatility is lower.[5]

In terms of economic significance, a change in inventories from their minimum value of \$3.1 billion to their maximum value of \$7.5 billion causes a 0.5 percentage point decrease in volatility with the OLS estimates and a 0.4 percentage point decrease in volatility with the GLS estimates. As to time-to-delivery effects, OLS and GLS estimates imply 0.9 and 0.8 percentage point increases in volatility over the life of the contract. Compared to a typical day's absolute log price change, 1.2 percentage points, we consider these changes to be significant.

Volatility Persistence

In empirical studies of financial assets, volatility persistence is a common concern. Most financial asset returns exhibit volatility persistence. Days with high volatility are followed by high volatility, and days with low volatility are followed by low volatility. To investigate this phenomenon in the lumber futures contracts, and to compare across other financial assets, table 3 presents AR(1) regression results for volatility on selected financial assets. Specifically, the regression equation is

$$|\%\Delta P_t| \equiv |100 \times (\ln P_t - \ln P_{t-1})| = a + b|\%\Delta P_{t-1}| + \varepsilon_t \qquad (19)$$

where $\ln P_t$ is the natural logarithm of the price of a financial asset on day t. As a sample of financial assets, we choose the S&P 500 index, the Dow Jones Industrial Average index, the Nasdaq Composite index, the Canadian Dollar/US Dollar exchange rate, the Japanese Yen/US Dollar exchange rate, and the returns to holding 3-month, 6-month, and 1-year treasury bills (secondary market). For lumber futures, we use data on all contracts in our sample.

Results given in table 3 show that there is statistical evidence for volatility persistence in the lumber futures market similar to, albeit at the low end of, that seen in other markets. The OLS estimates imply that a 1.5 percentage point increase in today's volatility results from a 7.7 percentage point increase—the largest movement in the sample—in yesterday's price movement. The GLS estimates imply a 1.2 percentage point increase today for a 7.7 percentage point increase yesterday.

---

[5]The relationship between trading volume and volatility has been an issue in the literature. To investigate whether our time-to-delivery effect is proxying for a volume effect, we added volume into our model for the sample period during which we had volume data (January 11, 2001-November 11, 2005). We found a positive and statistically significant volume effect. Holding everything else constant, a 100 unit increase in volume causes a 0.04 percentage point increase in volatility. The inventory effect does not change much with the addition of volume while the time-to-delivery effect decreases in magnitude from -0.91 to -0.65 over the life of a contract. Even though time to delivery and volume are correlated, the time-to-delivery effect remains significant even with volume. While the volume effect itself is statistically significant, we choose to exclude it in our model because while time to delivery is exogenous, volume is endogenous—volatility and volume are jointly determined and we wish to measure the total effect on volatility from a change in time to delivery.

To incorporate volatility persistence into our previous model with covariates, we add the lagged value of the dependent variable in equation (18) to the right-hand side of (18):

$$|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})| = \alpha + \beta S_t + \gamma_1 TTD_{it} + \gamma_2 TTD_{it}^2 + \psi|\%\Delta F_{i,t-1}| + \varepsilon_{it},$$
$$i = 1, 2, \cdots, k_t, \quad t = 1, 2, \cdots, T. \quad (20)$$

Results are presented in table 4. As before, we find negative and statistically significant coefficient estimates for the inventory and linear TTD variables. The quadratic TTD term is positive and statistically significant. Figure 4 shows the marginal time-to-delivery effect and predicted volatility evaluated at the mean values of inventories and lagged absolute price changes. We obtain a positive and statistically significant estimate for ψ with both OLS and GLS. However, when we use the GLS transformation to eliminate contemporaneous correlation among contracts, the estimates of the volatility persistence parameter become smaller. Also, note that the magnitude and significance of ψ decline dramatically compared to the case when we only include intercept and lagged volatility (table 3). The OLS estimate falls from 0.19 to 0.13, and the GLS estimate falls in half, from 0.15 to 0.07—a nine standard deviation decrease in volatility persistence. The GLS estimates imply that a 7.7 percentage point increase in yesterday's price movement will cause only a 0.5 percentage point increase in today's volatility—instead of the 1.2 percentage point increase reported before—when we correct for correlation among contracts and account for inventories and time to delivery.

Another important result is seen by a comparison between tables 2 and 4. Including the lagged dependent variable in the volatility model does not much affect the parameter estimates for inventories and time to delivery. When evaluated at the sample range of inventories, both OLS and GLS estimates in table 4 imply a 0.4 percentage point change in volatility. While the OLS estimates imply a 0.7 percentage point increase in volatility over the life of the contract, the GLS estimates imply a 0.8 percentage point increase. These results are very similar to what are found without the lagged dependent variable.

Conclusions

In lumber futures markets, volatility is inversely related to inventories. When inventories are low, futures contracts are relatively volatile. When inventories are large, their role in absorbing supply and demand shocks makes futures contracts less volatile. This confirms empirically a central prediction from the theory of storage.

We also find an inverse relationship between price volatility and time to delivery. As futures contracts approach delivery, their price fluctuations become larger. When contracts are far away from their delivery date, they are less volatile. We interpret this result as an implication of lumber supply and demand curves becoming more inelastic

as time to delivery approaches. Thus, supply and demand shocks have a larger price impact on near-term contracts than on those farther out.

As with other financial assets, there is strong statistical evidence of volatility persistence in the lumber futures market. However, much of the persistence in lumber futures can be explained by the dependence of volatility on inventories and time to delivery.

References

Andersen, T.G., and T. Bollerslev. 1997. "Intraday Periodicity and Volatility Persistence in Financial Markets." *Journal of Empirical Finance* 4:115–158.

Engle, R.F. 2004. "Risk and Volatility: Econometric Models and Financial Practice." *The American Economic Review* 94:405–420.

Karali, B. 2007. "Information and Price Response in Storable Commodity Futures Markets: An Application to Lumber Contracts." Ph.D. dissertation, North Carolina State University.

Karali, B., and W.N. Thurman. 2008. "Announcement Effects and the Theory of Storage: An Empirical Study of Lumber Futures." Unpublished manuscript, available at SSRN: http://ssrn.com/abstract=1025453.

Kyle, A.S. 1985. "Continuous Auctions and Insider Trading." *Econometrica* 53:1315–1336.

Li, L., and R.F. Engle. 1998. "Macroeconomic Announcements and Volatility of Treasury Futures." Discussion paper 98-27, University of California, San Diego.

Ng, V.K., and C. Pirrong. 1994. "Fundamentals and Volatility: Storage, Spreads, and the Dynamics of Metals Prices." *The Journal of Business* 67:203–230.

Pindyck, R.S. 2001. "Volatility and Commodity Price Dynamics." Draft working paper, M.I.T. Center for Energy and Environmental Policy Research, August 19, 2001.

—. 2004. "Volatility and Commodity Price Dynamics." *The Journal of Futures Markets* 24:1029–1047.

Thurman, W.N. 1988. "Speculative Carryover: An Empirical Examination of the U.S. Refined Copper Market." *The RAND Journal of Economics* 19:420–437.

Williams, J.C., and B.D. Wright. 1991. *Storage and Commodity Markets*. Cambridge University Press.
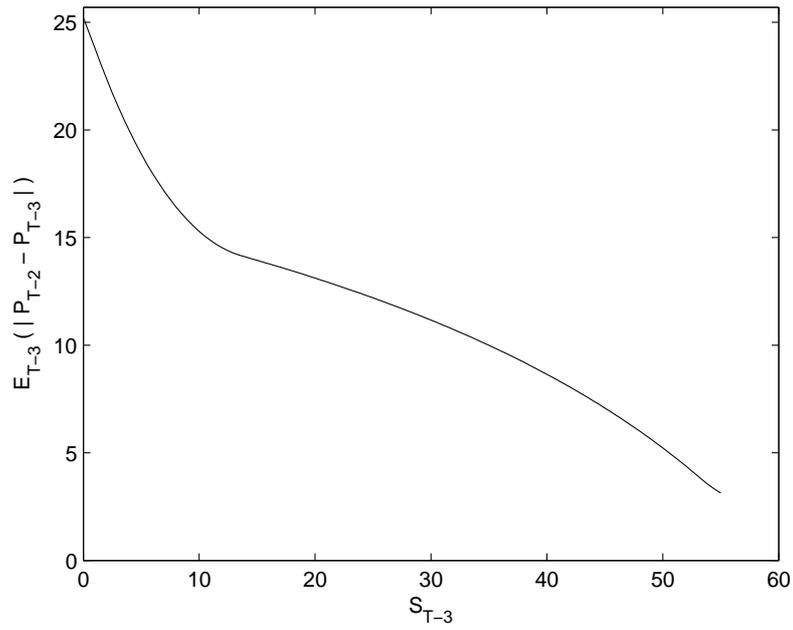
(a) Expected Absolute Price Changes
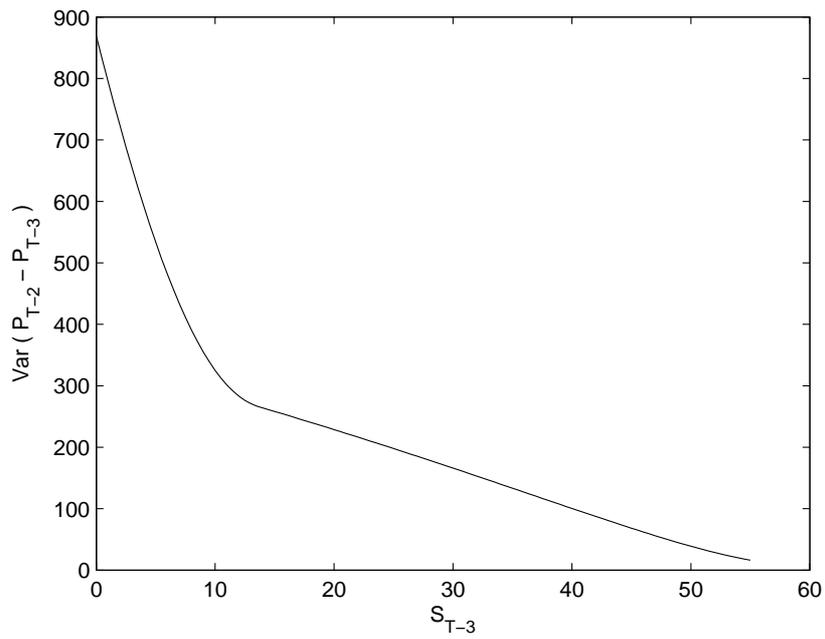
(b) Expected Variance of Price Changes

Figure 1: Expected Absolute Price Changes and Expected Variance of Price Changes at Different Levels of Inventories
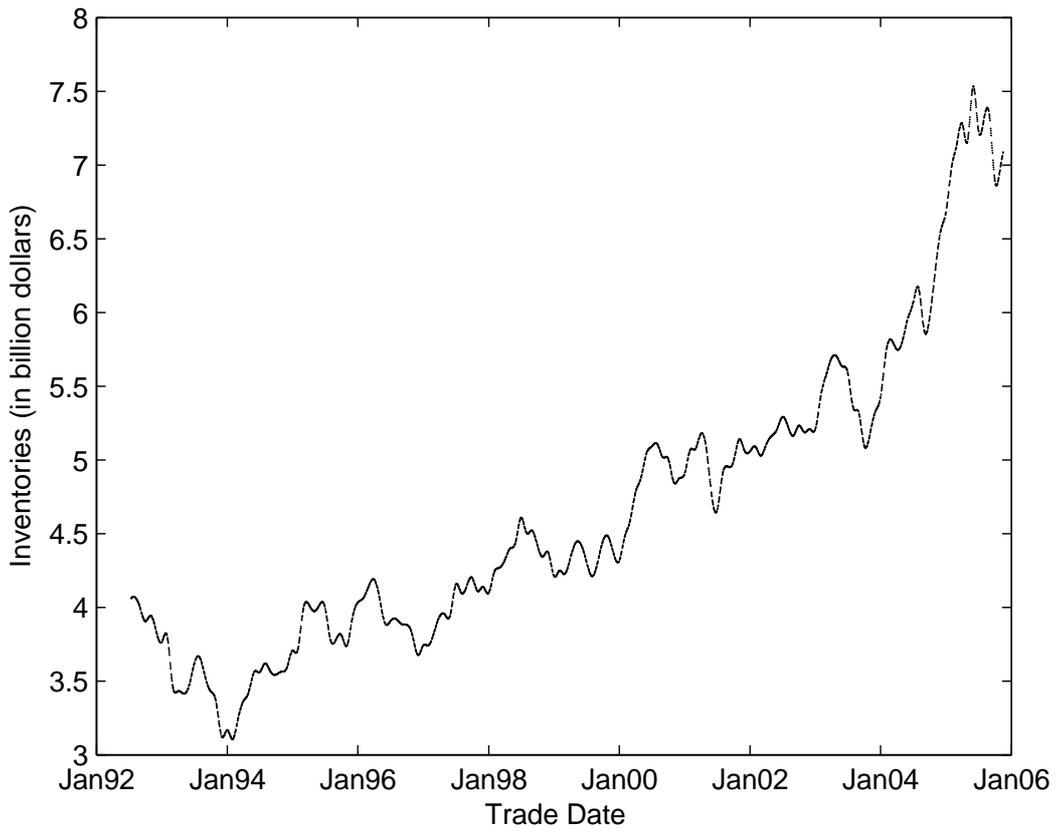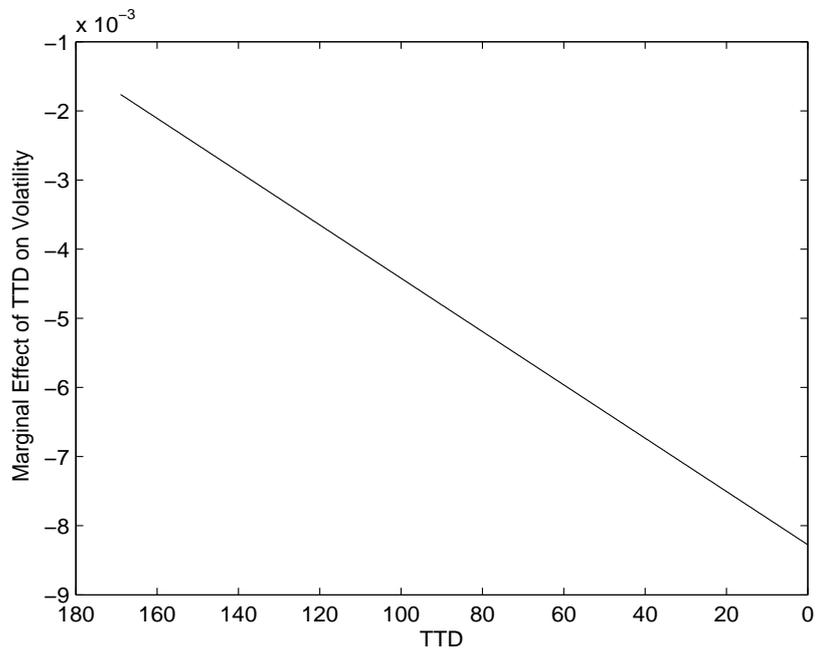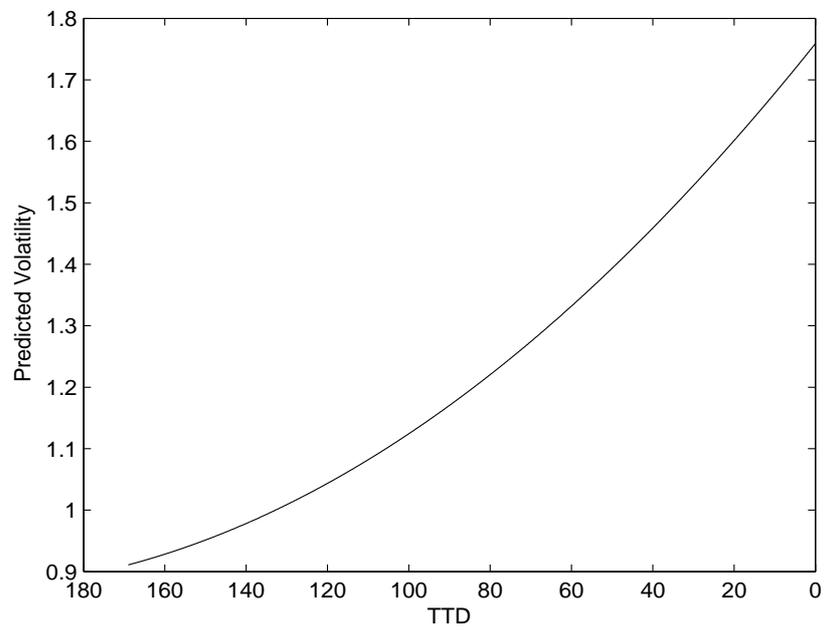
13

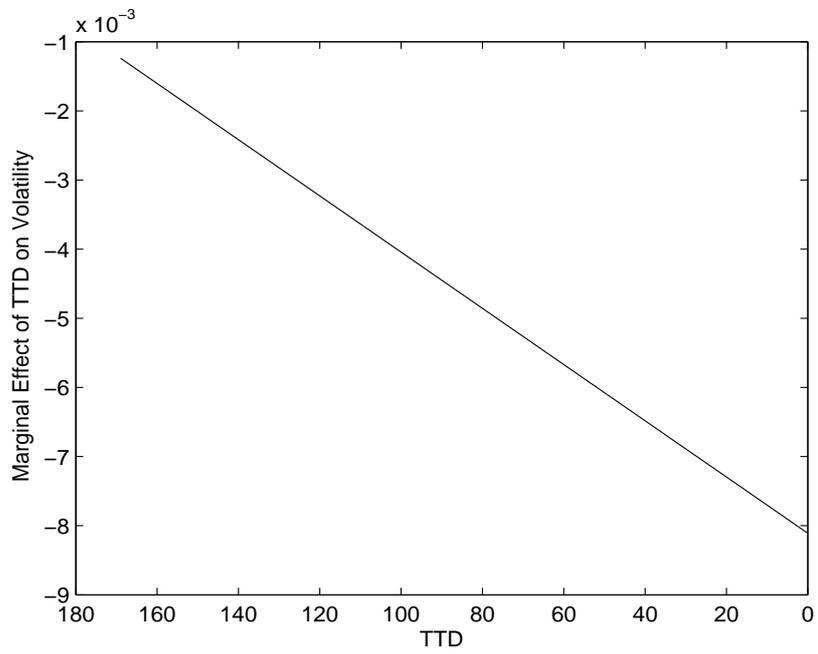Figure 2: Lumber Inventories (billions of dollars)

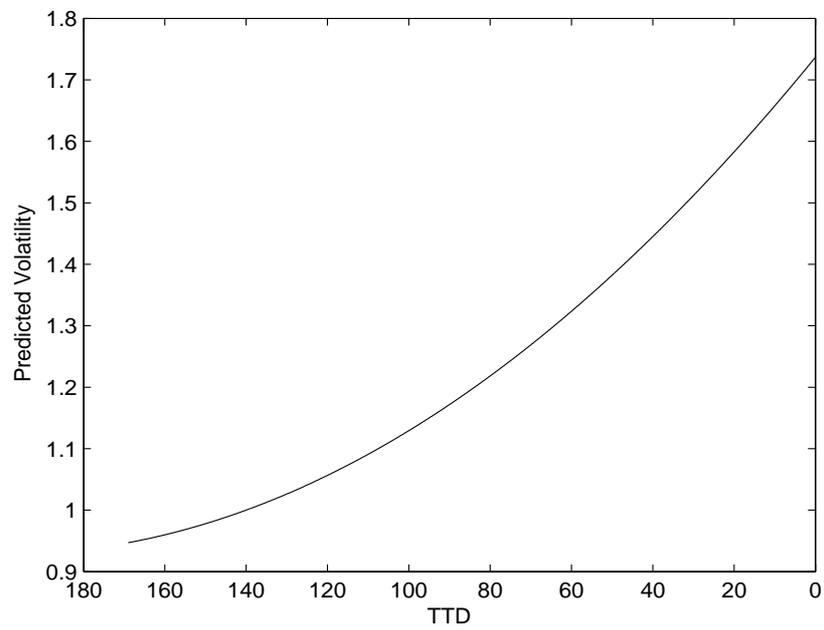(a) Marginal Effect of Time to Delivery

(b) Predicted Volatility

Figure 3: Time-to-Delivery Effect on Volatility

(a) Marginal Effect of Time to Delivery

(b) Predicted Volatility

Figure 4: Time-to-Delivery Effect on Volatility with Persistence

16

Table 1: Summary Statistics of Daily Variables

N=13,090	$\%\Delta F_{it}$	$|\%\Delta F_{it}|$	Inventories	TTD
Mean	-0.0105	1.2433	4.6154	84.50
Median	0	0.9739	4.3835	84.50
Min	-7.8560	0	3.1054	0
Max	14.1945	14.1945	7.5337	169
Std. Deviation	1.6097	1.0225	0.9541	49.08

Notes: $\%\Delta F_{it} = 100 \times (\ln F_{it} - \ln F_{i,t-1})$ and $|\%\Delta F_{it}| = |100 \times (\ln F_{it} - \ln F_{i,t-1})|$, $i = 1, 2, \cdots, k_t$, $t = 1, 2, \cdots, T$, where k_t is the number of contracts traded on day t, $T = 3,365$, the number of days in the sample, and $\ln F_{it}$ is the natural logarithm of futures price on day t, during which a total of k_t contracts were traded. Inventories are measured in billions of 1982 dollars.

Table 2: Inventory and Time-to-Delivery Effects on Volatility

	Least Squares Estimates	Generalized Least Squares Estimates
α	2.281 [0.050] (45.977)	2.195 [0.071] (30.806)
β	-0.114 [0.009] (-12.583)	-0.094 [0.015] (-6.448)
γ_1	-0.008 [0.001] (-11.474)	-0.008 [0.000] (-21.536)
γ_2	0.00002 [0.000] (4.335)	0.00002 [0.000] (9.024)

Notes: Regression results from $|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})| = \alpha + \beta S_t + \gamma_1 TTD_{it} + \gamma_2 TTD_{it}^2 + \varepsilon_{it}$. $|\%\Delta F_{it}|$ is the approximate percentage change in the volatility of daily return on lumber futures, $\ln F_{it}$ is the natural logarithm of futures price on day t, during which a total of k_t contracts were traded, S_t is the lumber inventory level on day t, and TTD_{it} is time to delivery, the number of days remaining to contract expiration on day t. Standard errors and t-values of estimates are given in the brackets and parentheses, respectively.

Table 3: Volatility Persistence in Financial Assets Returns

	S&P 500	Dow Jones	Nasdaq	CAD-USD	YEN-USD	3-m T-Bill	6-m T-Bill	1-yr T-Bill	Lumber Futures OLS	GLS
a	0.48	0.52	0.50	0.16	0.34	0.67	0.60	0.57	1.01	1.14
	[0.01]	[0.01]	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
	(65.98)	(67.99)	(42.29)	(50.33)	(51.49)	(47.81)	(50.98)	(50.96)	(72.47)	(56.67)
b	0.22	0.28	0.36	0.26	0.20	0.32	0.26	0.20	0.19	0.15
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
	(27.01)	(39.74)	(36.60)	(25.47)	(19.23)	(37.00)	(28.41)	(19.44)	(21.91)	(15.44)

Notes: Regression results from $|\%\Delta P_t| \equiv |100 \times (\ln P_t - \ln P_{t-1})| = a + b|\%\Delta P_{t-1}| + \varepsilon_t$, where $|\%\Delta P_t|$ is the approximate percentage change in the volatility of daily return on the financial asset on day t. Standard errors and t-values of estimates are given in the brackets and parentheses, respectively. Data periods for selected financial series are as follows: S&P 500, 01/03/1950-06/12/2006; Dow Jones, 01/02/1930-06/12/2006; Nasdaq, 02/05/1971-06/12/2006; CAD-USD exchange rate, 01/04/1971-06/12/2006; Yen-USD exchange rate, 01/04/1971-06/12/2006; 3-month T-bill, 01/04/1954-06/09/2006; 6-month T-bill, 12/09/1958-06/09/2006; 1-year T-bill, 02/01/1962-08/24/2001; lumber futures, 07/14/1992-11/15/2005.

Table 4: Volatility Persistence in Lumber Futures

	Least Squares Estimates	Generalized Least Squares Estimates
α	1.986 [0.053] (37.397)	2.066 [0.074] (28.007)
β	-0.098 [0.009] (-10.861)	-0.089 [0.015] (-6.105)
γ_1	-0.007 [0.001] (-10.207)	-0.008 [0.000] (-20.622)
γ_2	0.00002 [0.000] (4.023)	0.00002 [0.000] (9.378)
ψ	0.132 [0.001] (14.962)	0.068 [0.009] (7.404)

Notes: Regression results from $|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})| = \alpha + \beta S_t + \gamma_1 TTD_{it} + \gamma_2 TTD_{it}^2 + \psi |\%\Delta F_{i,t-1}| + \varepsilon_{it}$. $|\%\Delta F_{it}|$ is the approximate percentage change in the volatility of daily return on lumber futures, $\ln F_{it}$ is the natural logarithm of futures price on day t, during which a total of k_t contracts were traded, S_t is the lumber inventory level on day t, and TTD_{it} is time to delivery, the number of days remaining to contract expiration on day t. Standard errors and t-values of estimates are given in the brackets and parentheses, respectively.