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Theory of Storage and Option Pricing: Analyzing Determinants of Implied Skewness and Implied Kurtosis

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Theory of Storage and Option Pricing:

Analyzing Determinants of Implied Skewness and Implied Kurtosis

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Theory of Storage and Option Pricing: Analyzing Determinants of Implied Skewness and Implied Kurtosis

Practitioner's Abstract: *Options on agricultural futures are popular financial instruments used for agricultural price risk management and to speculate on future price movements. Poor performance of Black's classical option pricing model has stimulated many researchers to introduce pricing models that are more consistent with observed option premiums. However, most models are motivated solely from the standpoint of the time series properties of futures prices and need for improvements in forecasting and hedging performance. In this paper we propose a novel arbitrage pricing model motivated from the economic theory of optimal storage. We introduce a pricing model for options on futures based on a Generalized Lambda Distribution (GLD) that allows greater flexibility in higher moments of the expected terminal distribution of futures price. We show how to use high-frequency data to estimate implied skewness and kurtosis parameters. We propose an economic explanation for variations in skewness based on the theory of storage. We use times and sales data for corn futures and options on futures for the period 1995-2009. After controlling for changes in planned acreage, we find a statistically significant negative relationships between ending stocks-to-use and implied skewness and kurtosis, as predicted by the theory of storage.*

Keywords: arbitrage pricing model, options on futures, generalized lambda distribution, theory of storage, skewness, kurtosis

1. Introduction

Options written on commodity futures have been investigated from several aspects in the commodity economics literature. For example, Lence (1994), Vercammen (1995), Lien and Wong (2002), and Adam-Müller and Panaretou (2009) considered the role of options in optimal hedging. Use of options in agricultural policy was examined by Gardner (1977), Glauber and Miranda (1989), and Buschena (2008). The effects of news on options prices had been investigated by Fortenbery and Sumner (1990), Isengildina-Massa et al. (2008) and Thomsen (2009). The informational content of options prices has been looked into by Fackler and King (1990), Sherrick, Garcia and Tirupattur (1996), and Egelkraut et al. (2007). Some of the most interesting work done in this area considers modifications to the standard Black-Scholes formula that accounts for non-normality (skewness, leptokurtosis) of price innovations, heteroskedasticity, and specifics of commodity spot prices (i.e. mean-reversion). Examples include Kang and Brorsen (1995), and Ji and Brorsen (2000).

In this article we revisit the well-known fact that the classical Black's (1976) model is inconsistent with observed option premiums. Previous studies like Fackler and King (1990) and Sherrick, Garcia and Tirupattur (1996) address this puzzle by identifying properties of futures prices that deviate from assumptions of the Black's model, i.e. leptokurtic and skewed distributions of the logarithm of terminal futures prices and stochastic volatility. A common feature of past studies is the grounding of their arguments in the time-series properties of stochastic processes for futures prices and the distributional properties of terminal futures prices. In other words, their arguments are primarily statistical. In contrast to previous

studies, we offer an economic explanation for the observed statistical characteristics. We focus on presenting an alternative pricing model that is not focused on improving the forecasts of options premiums compared to Black's or other models, but linking premium deviations to fundamental characteristics associated with the theory of storage.

The article is organized as follows. In the next section we examine in detail the implications of Black's classical option pricing model and contrast it with predictions from the theory of storage as outlined by Williams and Wright (1991). We put forward a testable hypothesis grounded in the described economic analysis and propose a novel arbitrage pricing model based on the Generalized Lambda Distribution (GLD) that we then use to devise a method for testing our hypothesis. The estimation process requires multiple data steps that we describe in detail in the third section. The fourth section describes the estimation procedure and presents results. A set of conclusions completes the paper.

2. Theory

2.1. Foundations of arbitrage pricing theory for options on futures

Black (1976) was the first to offer an arbitrage pricing model for options on futures contracts. Despite numerous extensions and modifications proposed in the literature, and the inability of the model to explain observed option premiums, traders still use this model in practice. This is likely due to its simplicity and ability to forecast option premiums after appropriate "tweaks" are put in place. Black proposes that futures prices follow a stochastic process as described below:

$$dF = \sigma F dz \tag{2.1}$$

where F stands for futures price, σ for volatility, and dz is an increment of Brownian motion.

The implication is that futures prices are unbiased expectations of terminal futures prices (ideally equal to the spot price at expiration), and the stochastic process followed by futures prices is geometric a Brownian motion.

The option premium V is equal to the present value of the expected option payoff under a risk-neutral distribution for terminal prices. For example, for a call option with strike K, volatility σ, risk-free interest rate r and time left to maturity T:

$$V\left(K, F_0, T, \sigma, r\right) = e^{-rT} \int_0^{\infty} Max\left(F_T - K, 0\right) f\left(F_T; F_0, \sigma, r, T\right) dF_T \tag{2.2}$$

Because delta hedging options on futures does not require a hedger to pay the full value of the futures contract due to margin trading, a risk-neutral terminal distribution for futures prices is equivalent to a risk-neutral terminal distribution for a stock that pays a dividend yield equal to the risk-free interest rate:

$$\ln F_T \sim N\left(\ln F_0 - \frac{1}{2}\sigma^2, \sigma^2 T\right) \tag{2.3}$$

Thus, Black's model postulates that distribution of terminal futures prices, conditional on information known at time zero, is lognormal with the first four moments fully determined by the current futures price and volatility parameter σ. In particular, the first four moments of the risk-neutral terminal distribution are equal to:

$$\tilde{\mu} = F_0 \quad \tilde{\sigma}^2 = F_0^{\,2}\left(e^{\sigma^2 t} - 1\right) \quad SKEW = (e^{\sigma^2 t} + 2)\sqrt{e^{\sigma^2 t} - 1} \quad KURT = e^{4\sigma^2 t} + 2e^{3\sigma^2 t} + 3e^{2\sigma^2 t} \quad (2.4)$$

For example, if a futures price is \$2.50, volatility is 30%, and there are 160 days left to maturity, the standard deviation of the terminal distribution would be \$0.50, skewness would be 0.60 and kurtosis would be 3.64. Therefore, the standard Black's model implies that the expected distribution of terminal prices would be positively skewed, and leptokurtic. When complaints are raised that Black's model imposes normality restrictions, it is the logarithm of the terminal price that the critique refers to.

The standard way to check if Black's model is an appropriate pricing strategy is to exploit the fact that for given futures price, strike price, risk-free interest rate, and time to maturity, the model postulates a one-to-one relationship between the volatility coefficient and the option premium. Thus, the pricing function can be inverted to infer the volatility coefficient from an observed option premium. Such coefficients are referred to as *implied volatility* and the principal testable implication of Black's model is that implied volatility does not depend on how deep in-the-money or out-of-money an option is. If the logarithm of terminal price is not normally distributed, then Black's model is not appropriate, and implied volatility (IV) will vary with option moneyness – a flagrant violation of model's assumptions. Black's model gives us a pricing formula for European options on futures. Prices of American options on futures that are assumed to follow the same stochastic process as in Black's model must also account for the possibility of early exercise. For that reason, their prices cannot be obtained through a closed-form formula, but must be estimated through numerical methods such as the Cox, Ross and Rubinstein (CRR) (1979) binomial trees. While CRR binomial trees preserve the basic restrictions of Black's model (i.e. terminal distribution of log-prices is normal), Rubinstein (1994, 1998) shows how that can be relaxed to allow for non-normal skewness and kurtosis. To illustrate the effect of skewness and kurtosis on Black's implied volatility we used Edgeworth binomial trees (Rubinstein, 1998). This allows for pricing options that exhibit skewed and leptokurtic distributions of terminal log-prices. As can be seen in Figure 1, zero skewness and no excess kurtosis (S=0, K=3) corresponds to a flat IV curve, i.e. CRR implied volatility estimated from options premiums is the same no matter what strike is used to infer it. A leptokurtic distribution will cause so called "smiles", i.e. options with strikes further away from the current futures price will produce higher implied volatility coefficients. Positive skewness creates an upward sloping curve, and negative skewness a downward sloping IV curve. Typically, CRR implied volatility curves for storable agricultural commodities (i.e. corn, soybeans, wheat) are upward sloping. As an example consider the December 2006 corn contract. The futures price on June 26, 2006 was \$2.49/bu. As seen in Figure 2, the implied volatility curve associated with calculating IV using various December option strikes is strongly upward sloping, with the implied volatility coefficients for the highest strike options close to 15 percentage points higher than the implied volatility for options with lower strikes.

Three approaches can be used to address this issue:

1) Start from the end: relax the assumptions concerning risk-neutral terminal distributions of underlying futures prices, i.e. allow for non-lognormal skewness and kurtosis. As long as delta hedging is possible at all times (i.e. markets are complete), it is still possible to calculate option premiums as the present value of expected option payoffs. Examples of this approach include Jarrow and Ruud (1982), Sherrick et al. (1996), and Rubinstein (1998). While the formulas that derive option premiums as discounted expected payoffs assume that options are European, one can still price American options using implied binomial trees calibrated to the terminal distribution of choice (Rubinstein, 1994).

2) Start from the beginning: start by asking what kind of stochastic process is consistent with a non-normal terminal distribution? By introducing appropriate stochastic volatility and/or jumps, one might be able to fit the data just as well as by the approach above. Examples of this approach are Kang and Brorsen (1995), and Ji and Brorsen (2009).

3) "Tweak it so it works good enough" approach: if one is willing to sacrifice mathematical elegance, the coherence of second approach, and insights that might emerge from the first approach, and if the only objective is the ability to forecast day-ahead option premiums one can simply tweak Black's model. An example of such approach would be to model volatility coefficient as a quadratic function of the strike. Even though it makes no theoretical sense (this is like saying that options with different strikes live in different universes), this approach will work good enough for many traders. A seminal article that evaluates the hedging effectiveness of such approach is Dumas, Fleming and Whaley (1996).

In this article we take the first approach, and modify the Black's model by modifying the terminal distribution of futures price. Instead of a lognormal, we propose a generalized lambda distribution (GLD) developed by Ramberg and Schmeiser (1974) and introduced to options pricing by Corrado (2002). An alternative would be to use Edgeworth binomial trees, but preliminary analysis showed that such an approach may not be adequate for situations where skewness and kurtosis are rather high. In addition, Edgeworth trees work with the skewness of terminal log-prices, while we prefer to have implied parameters for the skewness of terminal futures prices directly, not their logarithms. In addition, the GLD pricing model allows for a higher degree of flexibility in terms of skewness and kurtosis, i.e., its' parameters are easy to imply from observed options prices and it is straightforward to develop a closed-form solution for pricing options. While these are all favorable characteristics, it is in fact the ability to gain additional economic insight that truly justifies yet another option pricing model. GLD allows us to get an explicit estimate of skewness and kurtosis of the terminal distributions, and we can use that to test predictions of the theory of storage that have thus far remained conjectures.

2.2. Theory of storage and time-series properties of commodity spot and futures prices

Deaton and Laroque (1992) used a rational expectations competitive storage model to explain nonlinearities in the time series of commodity prices: i.e. skewness, rare but dramatic substantial increases in prices, and a high degree of autocorrelation in prices from one harvest season to the next. The basic conclusion of their work is that inability to carry negative inventories introduces a non-linearity in prices that manifests itself in the above characteristics.

This is an example of theory being employed in an attempt to replicate patterns of observed price data. In a similar fashion, but subtly different, Williams and Wright (1991) postulate that the features of expected price distributions at harvest time are conditional on information concerning the current (pre-harvest) price and available stocks. According to them, when observed at annual or quarterly frequency, spot prices exhibit positive autocorrelation, and an autoregressive structure emerges because storage allows unusually high or low excess demand to be spread out over several years. Furthermore, the variance of price changes depends on the level of inventory. When stocks are high, and spot price is low, the abundance of stored stocks serves as a buffer to price changes, and variance is low. When stocks are low, and thus spot price is high, stockpiles are near empty and unable to buffer price changes. Finally, the third moment of the price change distribution also varies with inventories. Since storage can always reduce the downward price pressure of a windfall harvest, but cannot do as much for a really bad harvest, large price increases are more common than large decreases. The magnitude of this cushioning effect of storage depends on the size of the stocks. In conclusion, one should expect commodity prices to be mean-stationary, heteroskedastic and with conditional skewness, where both the second and third moments depend on the size of the inventories.

Testing the theory proceeds with this argument: if we can replicate the price pattern using a particular set of rationality assumptions, then we cannot refute the claim that people indeed behave in such manner. That is the road taken by Deaton and Laroque (1992) and Miranda and Rui (1995). However, since in the spot price series we only see realizations of prices, not the conditional expectations of them, we cannot use spot price data to directly test what the market *expected* to happen. To the best of our knowledge, predictions from storage theory focused on the scale and shape of expected distributions of new harvest spot prices has remained untested. In this paper we use options data to infer the conditional expectations of terminal futures prices, and therefore test the following predictions of theory of storage:

- The lower inventories are, the higher will be the kurtosis of the conditional harvest futures price distribution

- The lower inventories are, the more positive will be the skewness of the conditional harvest futures price distribution

We use an options pricing formula based on the generalized lambda distribution to estimate skewness and kurtosis of expected (conditional) harvest futures price distributions. Implied parameters from the model are then used to test the hypotheses above.

2.3. Option pricing formula using generalized lambda distribution

The generalized lambda distribution (GLD) was invented by Ramberg and Schmeiser (1974), Ramberg et al. (1979) described further its' properties, and it was introduced to options pricing by Corrado (2002). Corrado derived a formula for pricing options on non-dividend paying stocks. Here we review the properties of GLD and adopt his formula to options on futures.

GLD is most easily described by a percentile function[1] (i.e. inverse cumulative density function):

$$F(p) = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2} \tag{2.5}$$

For example, to say that for $p = 0.90, F(p) = 4.5$ means that the market expects with a 90% probability that the terminal futures price will be lower than or equal to \$4.50/bu.

GLD has four parameters: λ_1 controls location, λ_2 determines variance, and λ_3 and λ_4 jointly determine skewness and kurtosis. In particular the mean and variance are calculated as follows:

$$\begin{aligned} \mu &= \lambda_1 + A/\lambda_2 \\ \sigma^2 &= \left(B - A^2\right)/\lambda_2^2 \end{aligned} \tag{2.6}$$

with $A = \frac{1}{1+\lambda_3} - \frac{1}{1+\lambda_4}$ and $B = \frac{1}{1+2\lambda_3} + \frac{1}{1+2\lambda_4} - 2\beta(1+\lambda_3, 1+2\lambda_4)$,where $\beta(\ )$ stands for complete beta function. We see that the λ_3 and λ_4 parameters influence both location and variance, however λ_1 influences only the first moment, and λ_2 influences only the first two moments, i.e. skewness and kurtosis do not depend on λ_1 and λ_2.

The skewness and kurtosis formulas are:

$$\begin{aligned} \alpha_3 &= \frac{\mu_3}{\sigma^3} = \frac{C - 3AB + 2A^3}{\lambda_2^2 \sigma^3} \\ \alpha_4 &= \frac{\mu_4}{\sigma^4} = \frac{D - 4AC + 6A^2B - 3A^4}{\lambda_2^4} \end{aligned} \tag{2.7}$$

with $C = \frac{1}{1+3\lambda_3} - \frac{1}{1+3\lambda_3} - 3\beta(1+2\lambda_3, 1+\lambda_4) + 3\beta(1+\lambda_3, 1+2\lambda_4)$

and $D = \frac{1}{1+4\lambda_3} + \frac{1}{1+4\lambda_3} - 4\beta(1+3\lambda_3, 1+\lambda_4) - 4\beta(1+\lambda_3, 1+3\lambda_4) + 6\beta(1+2\lambda_3, 1+2\lambda_4)$

[1] F here stands for futures price, not for cumulative density function.

Standardized GLD has a zero mean and unit variance, and has a percentile function of the form:

$$F(p)=\frac{1}{\lambda_2(\lambda_3,\lambda_4)}\left(p^{\lambda_3}-(1-p)^{\lambda_4}+\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right) \tag{2.8}$$

Where $\lambda_2(\lambda_3,\lambda_4)=sign(\lambda_3)\times\sqrt{B-A^2}$

From here, we can move more easily to an options pricing setting. We wish to make GLD an approximate generalization of the log-normal distribution so we keep the mean and the variance the same as in (2.4), while allowing skewness and kurtosis to be separately determined by the λ_3 and λ_4 parameters. Therefore, the percentile function relevant for option pricing will be

$$F(p)=F_0\left(1+\frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(p^{\lambda_3}-(1-p)^{\lambda_4}+\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right)\right) \tag{2.9}$$

Note that this is equivalent to (2.5) with $\lambda_1=F_0+\frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right)$

and $\lambda_2=\frac{\lambda_2(\lambda_3,\lambda_4)}{\sqrt{e^{\sigma^2 t}-1}}$. This will guarantee that the first two moments of the terminal distribution will be $\tilde{\mu}=F_0 \quad \tilde{\sigma}^2=F_0^{\,2}\left(e^{\sigma^2 t}-1\right)$, just like in Black's model.

The pricing formula for European calls is

$$V(K,F_0,T,\sigma,r,\lambda_3,\lambda_4)=e^{-rT}\int_0^\infty Max(F_T-K,0)\,dp(F) \tag{2.10}$$

As shown by Corrado (2002), we can simplify this problem through a change-of-variable approach where $F(p)=F_T$:

$$\int_0^\infty Max(F_T-K,0)\,dp(F)=\int_K^\infty (F_T-K)\,dp(F)=\int_{p(K)}^1 \left(F(p)-K\right)dp \tag{2.11}$$

Here $p(K)$ stands for the cumulative density function, evaluated at K. While there is no closed form formula for the function, values can be easily found with numerical approaches by using the percentile function.

Integrating $F(p)$ we get

$$G_1 = \int_{p(K)}^{1} F(p)\,dp = F_0\left(p + \frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(\frac{1}{\lambda_3+1}p^{\lambda_3+1} + \frac{(1-p)^{\lambda_4+1}}{\lambda_4+1} + \frac{1}{\lambda_4+1}p - \frac{1}{\lambda_3+1}p \right)\right)\Bigg|_{p(K)}^{1}$$

$$= F_0\left(1-p(K) + \frac{\sqrt{e^{\sigma^2 t}-1}}{\lambda_2(\lambda_3,\lambda_4)}\left(\frac{p(K)-p(K)^{\lambda_3+1}}{\lambda_3+1} + \frac{1-p(K)-(1-p(K))^{\lambda_4+1}}{\lambda_4+1} \right)\right)$$

with the final European call pricing formula as:

$$V(K,F_0,T,\sigma,r,\lambda_3,\lambda_4) = F_0 e^{-rt} G_1 - e^{-rt} K G_2 \tag{2.12}$$

where G_1 is defined above and $G_2 = 1-p(K)$

In a similar way it can be shown that the price for a put is

$$V_P(K,F_0,T,\sigma,r,\lambda_3,\lambda_4) = e^{-rt}K(1-G_2) - F_0 e^{-rt}(1-G_1) \tag{2.13}$$

3. Data

Commodity futures for corn, soybeans and wheat as well as options on futures are traded on the Chicago Mercantile Exchange (formerly the Chicago Board of Trade). A dataset comprising of all recorded transactions, i.e. times and sales data for both futures and options on futures, for the period 1995 through 2009, was obtained. It includes data for both the regular and electronic trading sessions. The total number of transactions exceeds 90 million. Options data were matched with the last preceding futures transaction. LIBOR interest rates were obtained from British Bankers' Association, and represent the risk-free rate of return. Overnight, 1 and 2 weeks, and 1 through 12 months of maturity LIBOR rates for period the 1995 through 2009 were used to obtain the arbitrage-free option pricing formulas. In particular, each options transaction was assigned the weighted average of interest rates with maturities closest to the contract traded. To avoid serial correlation in residuals from estimating implied coefficients in subsequent steps, the data frequency was reduced to not less than 15 minutes between transactions for the same options contract. This resulted in 11,139 data files, each containing between 200 and 500 recorded transactions for a particular trading day for a given commodity. For each data point we separately estimated implied volatility using CRR binomial trees with 500 steps. We then, for each data point, calculated the price of a European option using Black's formula and assuming the same parameters (futures price, interest rate, time to maturity) as that recorded for the American option, while setting volatility equal to the one implied for American options. These 'artificial' European options are then used in calibration and econometric analysis.

U.S. corn, soybean, and wheat market balance sheets were produced by the RENK Agribusiness Institute at UW-Madison. Balance sheet data for stocks-to-use are used as a measure of the adequacy of ending stocks to serve as a buffer against a poor crop harvest.

In this paper we present results using only corn data. Research on the other two commodities is ongoing. Before estimating the parameters of the GLD pricing model, we can use the observations from Figure 1 to get some ideas concerning the evolution of the conditional terminal distribution of futures prices. Figure 3 shows the implied volatility curves for options on the December 2004 and December 2006 corn futures contracts. Note that about 400 days to maturity the IV curves are nearly flat. The slopes gradually increase up to July of the contract year, and then become flatter again. As we enter the harvest season, smiles start to dominate the picture. This is a typical pattern that emerges for nearly all years in our sample, with the noted exception of 2009 where symmetric smiles dominate the figure throughout. The evolution of IV curves can be best understood by examining an individual growing season for corn and the timing of market-sensitive USDA reports. At the end of March, USDA publishes the Prospective Plantings report that indicates the acreage farmers intend to plant to each of the major crops. In addition, reports on remaining stock levels are published at the end of each quarter. Finally, an acreage report is published at the end of June. This report estimates the number of acres actually planted for each crop. Note that reductions in implied volatility for at-the-money options as well as the slopes of the IV curves regularly follows the publications of stocks and acreage reports. Therefore, to test predictions of storage theory, it is appropriate to focus on information contained in options premiums for the June trading period when considerable information is available concerning crop progress and intended acreage, but a lot of uncertainty is still unresolved relative to actual production levels.

4. Research Methods and Results

Our main hypothesis is that higher moments of the conditional distribution of terminal futures prices reflect market sentiment relative to the adequacy of current inventories to mitigate any production problems in the forthcoming harvest.

For each contract, for each trading day, we separately estimate the parameters $\sigma, \lambda_3, \lambda_4$ in the GLD option pricing formula. In particular, we minimize the squared difference in option premiums calculated with the GLD formula, and prices of European options as implied by Black's model.

We first need a starting value for the implied volatility of an option with a strike price closest to the underlying futures price. The starting values for the λ_3 and λ_4 parameters were chosen to correspond to the skewness and kurtosis of the terminal stock price with the restriction that the logarithm of the terminal price is normally distributed with variance equal to $\sigma^2 t$, where σ^2 is the square of the starting value for the implied sigma parameter. We use Excel Solver to run the minimization problem, utilizing a FORTRAN compiled library (.dll file) created by Corrado (2002) that estimates GLD European Call prices. A formula for the GLD European put option was then programmed in Visual Basic for Applications.

We use estimated lambda parameters to calculate implied skewness and kurtosis. While we are working on extensions, this paper is focused on an analysis of December corn futures (this is often referred to as the harvest contract). In particular, for the purposes here, we used only implied higher moments for trading days occurring between the 10th and 20th of June each year

in the sample period. For trading days used in following analysis, GLD option prices seem to work rather well, with an average absolute pricing error about 3/8 of a cent per bushel, and a maximum pricing error usually reaching not more than 2 cents (this occurs for the least liquid and most away from the money options). While there may be issues regarding the robustness of implied parameters with respect to starting values, the implied parameters seem to be rather stable from one day to the next. For example, for Dec '05 corn, the skewness estimated between June 10 and June 20, 2005 varies between 1.09 and 1.22, with kurtosis ranging between 3.96 and 4.82. For that year, the average absolute pricing error was 5/8 of a cent per bushel, with a maximum pricing error of 2.4 cents.

For all years in the sample, the implied skewness is 1.2 to 3 times higher than it would be if the logarithm of the terminal futures price was really expected to be normal. Implied kurtosis is 1.2 to 1.6 times higher than that predicted by Black's model. We thus see that deviations from Black's model are particularly pronounced in implied skewness.

Relationship between implied skewness and kurtosis, ending stocks-to-use and planned changes in acreage is illustrated in Figure 4. We estimate simple linear regressions for the period 1995-2009 using implied skewness and kurtosis as dependent variables with a constant, ending stocks-to-use and planned change in planted acreage as reported in Prospective Plantings report as explanatory variables. Regression statistics are reported in Table 1.

A reduction in stocks-to-use by 1 percentage point increases skewness by 0.015, and kurtosis by 0.091, and these estimated coefficients are statistically significant at the 95% confidence level. Both results support the central hypothesis of the paper – deviations from Black's model arise because conditional implied higher moments of the terminal distribution are higher than implied by the lognormality restriction. Controlling for planned change in acreage planted, moments vary systematically with the adequacy of stocks to buffer a poor harvest.

5. Conclusions and further research

An options pricing model based on a generalized lambda distribution provides a useful heuristic in thinking about determinants of the shape of terminal futures price conditional distributions. Results indicate that crop inventories play a significant role in determining the expected asymmetry and density in the tails of the distribution. In particular, results reveal that implied skewness is much more persistent than implied by Black's model, and in years with low implied volatility implied skewness remains much higher than would be the case under the lognormality restriction. Further research will focus on extending this analysis to other storable crops. Of particular interest will be a comparison of soybeans and corn. Based on Brazil's role in international soybean markets, there are effectively two harvests per year. Implications for the level, persistence, and evolution of implied skewness and kurtosis needs to be examined in this larger context.

Thus far, the literature has focused on evaluating the impact of government reports on implied volatility coefficients. Our model allows us to extend this to higher moments and examine how reports (i.e., information) influence the entire distribution of prices, not just the second moment.

In absence of high frequency data, many researchers use end of day reported prices for futures and options to evaluate implied higher moments. By re-estimating this model using only end of day data we will be able to examine the amount of noise and possible direction of bias such an approach brings to estimates of implied higher moments.

Our conjecture is that positive skewness emerges due to the storability of commodities. It would be interesting to use the GLD option pricing model to examine the evolution and determinants of higher moments of non-storable commodities. Further research is needed to examine the impact of durability of production factors for commodities that are themselves not storable.

Finally, impacts of market liquidity and trader composition on levels and stability of implied higher moments is a new promising area for research. With careful design of the analysis, we may be able to find a way to separate the part of the option price that is due to implied terminal price distributions from additional premium influences incurred due to hedging pressure or lack of market liquidity.

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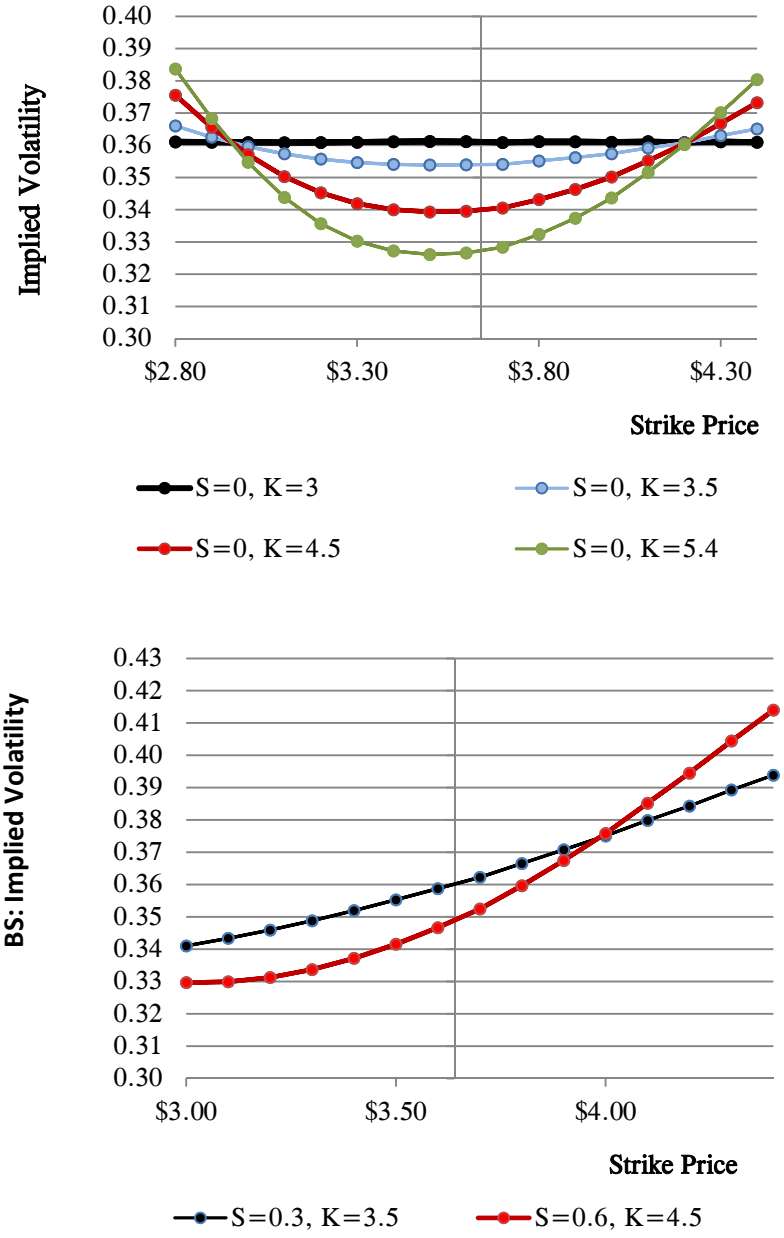
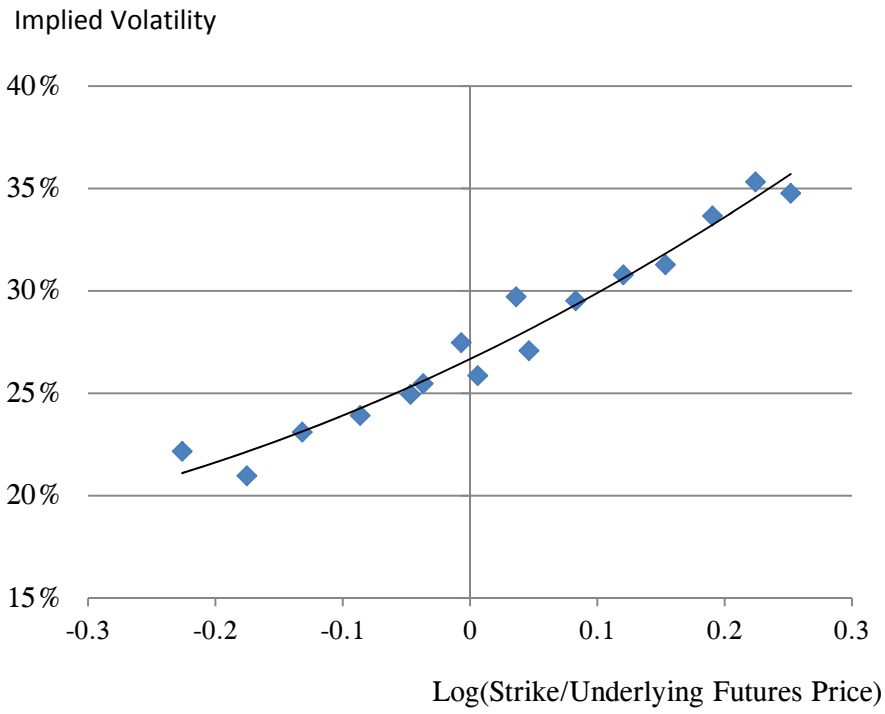
Figure 1. Effects of Excess Kurtosis and Positive Skewness on Implied Volatility



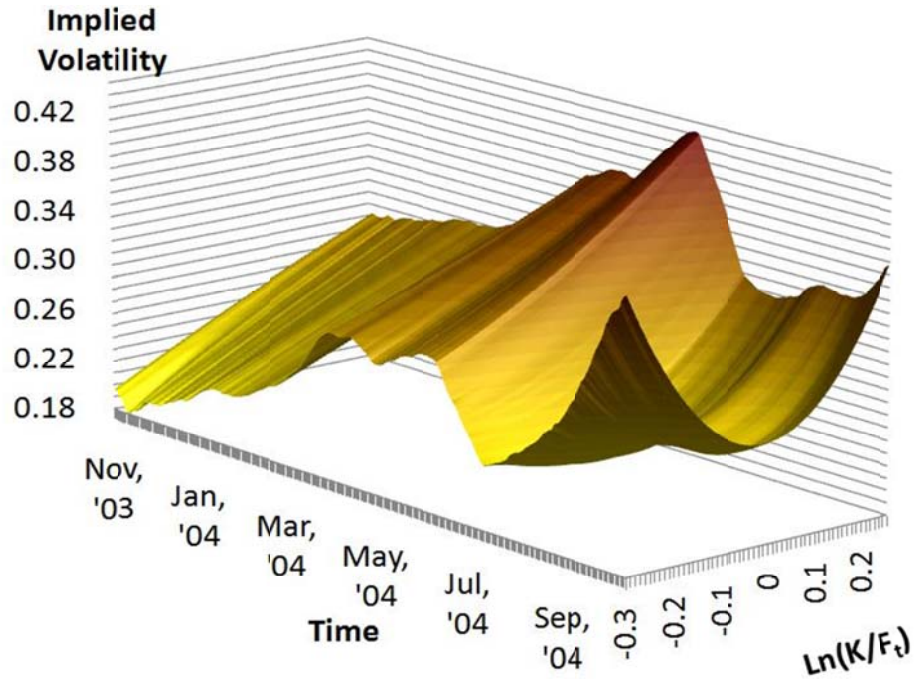
Notes: S stands for skewness, K for kurtosis of terminal futures log-prices. Option premiums are calculated via Rubinstein's Edgeworth binomial trees that allow for non-normal skewness and kurtosis, and implied volatility is inferred using Cox, Ross and Rubinstein's binomial tree which assumes normality in terminal futures prices. Black line in the above diagram with $S=0$ and $K=3$ corresponds to assumptions of Black's model, and in such scenario implied volatility curve is flat across all strikes. Excess kurtosis ($K>3$) creates convex and nearly symmetric "smiles", and positive skewness produces upward sloping implied volatility curve.

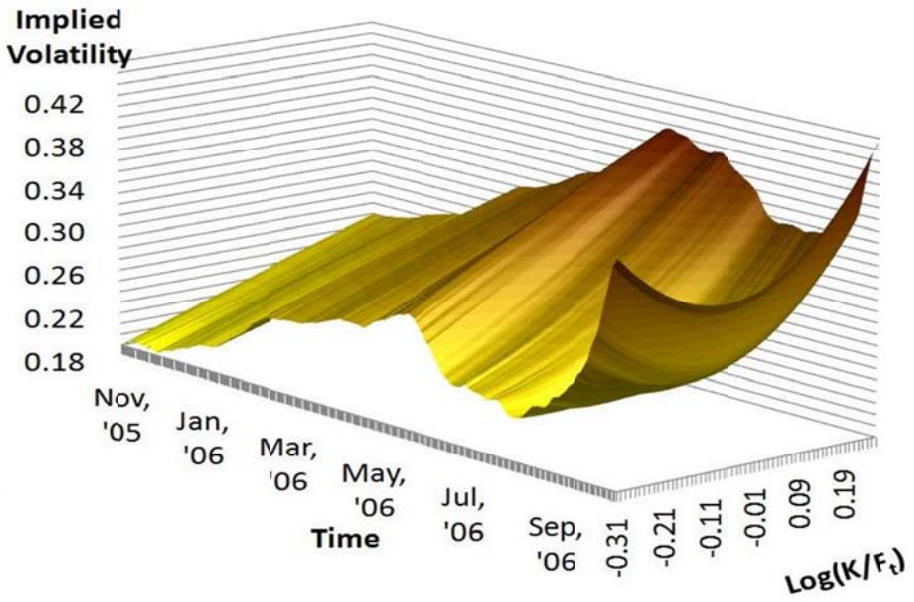
Figure 2. Typical pattern for Implied Volatility coefficients for options on agricultural futures



Notes: Implied volatility coefficients are estimated for options on Corn December 2006 futures contract, on 6/26/2006 using Cox, Ross and Rubinstein's binomial tree with 500 steps. Underlying futures price was 2.49$/bu. Dots represent implied volatility coefficients for each strike, and smooth line is fitted quadratic trend.

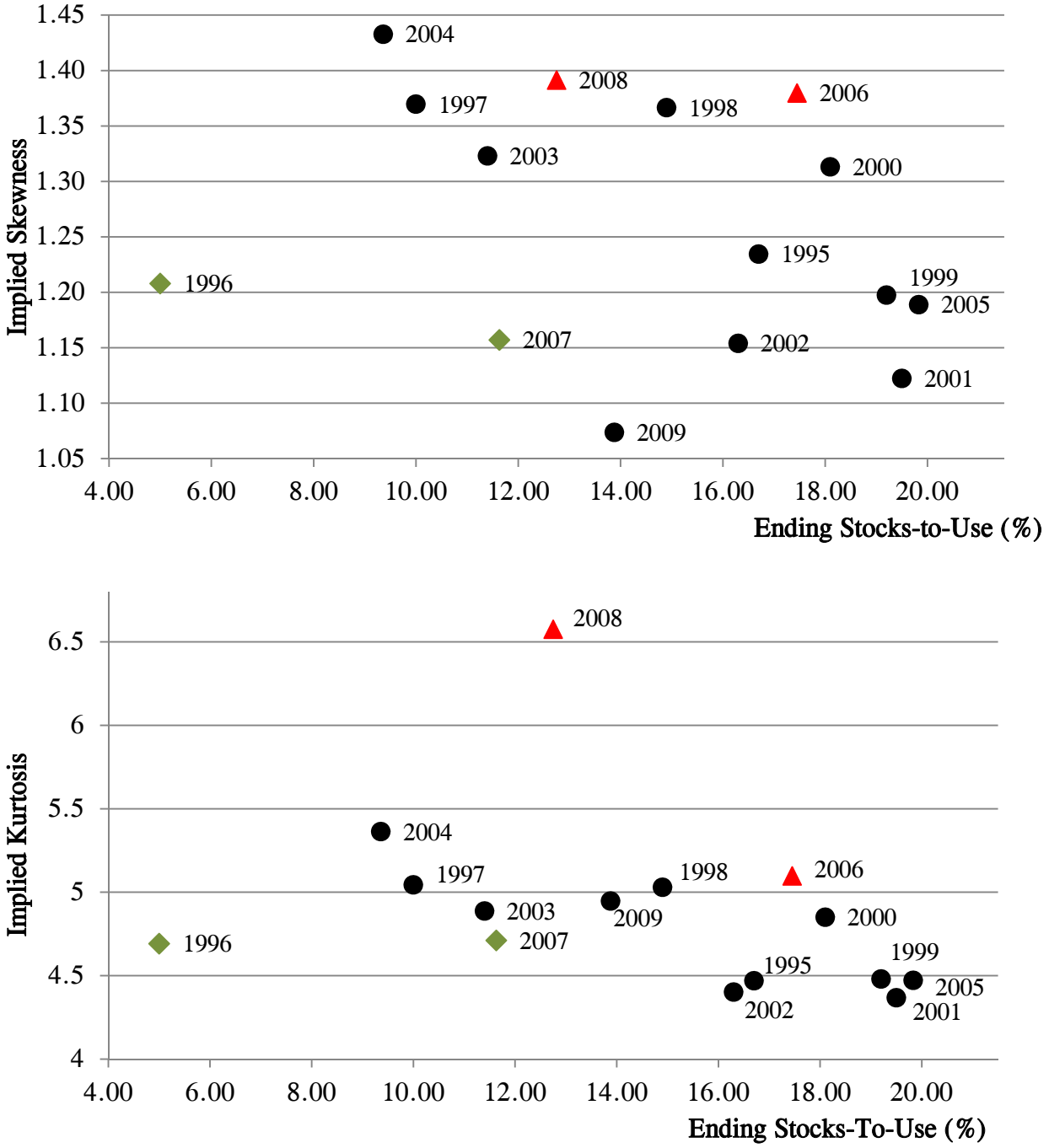
Figure 3. Evolution of implied volatility curve for options on Dec' 04 and Dec '06 corn futures.





Notes: For each day, implied volatility is estimated for each traded option using 15 minute interval data. Quadratic trend curve is fitted to produce implied volatility curve. 30 day moving average is calculated to increase smoothness of the volatility surface and make it easier to see principal characteristic of the IV curve evolution.

Figure 4. Relationship between implied skewness and kurtosis, stocks-to-use and planned change in planted acreage



Note: Years with increase in intended cultivated acreage of 5 or more percent are drawn using green rhombs. Years with decrease in intended cultivated acreage of 5 or more percent are drawn using red triangles.

Table 1. Determinants of implied skewness and kurtosis: descriptive statistics and regression results

Descriptive Statistics

Variable	Mean	Standard Deviation	Min	Max
Implied Skewness	1.26	0.11	1.07	1.43
Implied Kurtosis	4.89	0.55	4.36	6.57
Ending Stocks-to-Use (Percentage)	14.4	4.33	5	19.83
Intended Acreage Planted – Percentage Change	0.73	6.08	-8	15

Note: Implied skewness and kurtosis were calculated for December corn contracts as average for implied parameters on trading days in period 6/10-6/20 of each year. On average, 100-150 data points were used in estimating implied parameters for each trading day in the stated periods, and 9 trading days per year were used in obtaining average implied skewness and kurtosis.

Regression Results

Explanatory variables	Dependent Variable: GLD Implied Skewness	Dependent Variable: GLD Implied Kurtosis
Constant	1.49065	6.2572
	(0.1067)	(0.4529)
Ending Stocks-to-Use ratio	-0.01539	-0.0915
	(0.0071)	(0.0299)
Intended Acreage Planted –	-0.0113	-0.0653
Percentage change	(0.0050)	(0.0214)
Degrees of Freedom	12	12
Mean Root Square Error	0.089	0.374
R^2	0.35	0.50

Note: Critical t-statistic for 12 d.f. for 95% is 2.17881. All coefficients are statistically significant at 95% confidence level.