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Practitioner's Abstract

In a well-functioning futures market, the futures price on the expiration date equals the price of the underlying asset on that date. An unprecedented episode of non-convergence in Chicago Board of Trade (CBOT) corn, soybeans, and wheat began in late 2005, and with the exception of some brief periods, largely persisted through 2010. Most recently, the Kansas City Board of Trade (KCBOT) wheat contract also has demonstrated convergence problems. During this unprecedented and extended episode of non-convergence, futures contracts have expired at prices up to 35 percent greater than the prevailing cash grain price. Using a rational expectations commodity storage model, we show how such non-convergence can be produced by the institutional structure of the delivery market. Specifically, we show how a wedge between the marginal cost of storing the physical commodity and the cost of carrying the delivery instrument causes non-convergence. We fit the model to corn, soybeans, and wheat and find strong support for our model.

Key words: convergence, delivery, futures, grains, storage,

Introduction

In a well-functioning futures market, the futures price on the expiration date equals the price of the underlying asset. This simple "convergence" condition is necessary for effective risk-shifting and efficient price discovery in a futures market (e.g., Telser and Higinbotham 1977, Pirrong, Haddock, and Kormendi 1993). Non-convergence therefore threatens the basic functioning of a futures market. Historically, large differences between cash and futures prices during the delivery period of futures markets have been relatively rare (Peck and Williams 1991). When such problems have emerged in the past the duration was relatively short-lived and attention directed to potential problems with market manipulation in the form of "corners" and "squeezes" (Kyle 1984, Pirrong 1993, 2004, Allen, Litov, and Mei 2006).

An unprecedented episode of non-convergence in Chicago Board of Trade (CBOT) corn, soybeans, and wheat began in late 2005, and with the exception of some brief periods, largely persisted through 2010. Most recently, the Kansas City Board of Trade (KCBOT) wheat contract also has demonstrated convergence problems (Figure 1). During this unprecedented and extended episode of non-convergence, futures contracts have expired at prices up to 35 percent greater than the prevailing cash grain price. This failure in the most basic of futures market functions would appear to create an extraordinary opportunity for grain traders to make massive arbitrage profits by acquiring inexpensive grain in the cash market and delivering it at much higher futures contract prices. However, enacting such arbitrage is not as simple as filling a train car with grain and depositing it at a grain elevator. When delivery occurs on a grain futures contract, the firm on the short side of the market delivers a delivery instrument (a warehouse receipt or shipping certificate) to the firm on the long side of the market. The firm may hold the delivery instrument for as long as it wishes, although it must pay a daily storage fee while it holds the instrument. Thus, an arbitrageur who delivers on the futures contract faces an uncertain period before realizing profit from eventual convergence.

Two recent studies examine the causes of the convergence failures. Irwin, Garcia, Good, and Kunda (2011) identify a correlation between non-convergence and the occurrence of large carrying charges in corn, soybeans, and wheat. When carrying charges (spreads between futures prices with different delivery dates) are high enough, Irwin et al. argue that an incentive is created for takers of delivery (longs) to hold the delivery instruments and "earn the carry" rather than cancelling the instrument by converting it into grain (loading out), which would bring about convergence. Aulerich, Fishe, and Harris (2011) show that the long's carry-induced incentive to hold delivery instruments can be modeled as an embedded real option. The option becomes more valuable as the volatility of cash and futures prices increases; if the option value becomes large enough the cash and futures market can disconnect.

While informative, the previous two studies do not explain the underlying economic forces that generate these large episodes of convergence failure. In this article, we develop a dynamic rational expectations commodity storage model that explains the observed convergence failures. Specifically, we show that non-convergence arises in equilibrium when the market price of physical grain storage exceeds the cost of holding delivery instruments. The storage fee on delivery instruments is set by the futures exchange and does not vary much over time. However, the price of physical grain storage varies substantially over time as the level of inventory changes. Plentiful inventories generate a high price of physical storage and small inventories cause the price of storage to become negative as the market is willing to pay a convenience yield to store the commodity (Working 1948 1949, Brennan 1958). We call the difference between the cost of carrying physical grain and the cost of carrying delivery instruments the *wedge*. We show that the magnitude of the non-convergence equals the expected present discounted value of future positive wedges.

Our model rationalizes convergence failure as an equilibrium outcome under rational expectations. However, some futures traders may not trade as though they have rational expectations, or may not understand the rational expectations solution. This possibility evokes models of the limits of arbitrage in the presence of noise trading in asset markets (e.g., DeLong et al 1990, Shleifer and Vishny 1997), especially as non-convergence has occurred during a period of dramatic growth in participation in futures markets. Commodity index trading has attracted particular attention as a possible cause of non-convergence. For example, on July 21, 2009, Thomas Coyle, the Chairman of the National Grain and Feed Association, testified to the United States Senate Permanent Subcommittee on Investigations (USS/PSI) that "…disproportionate participation of investment capital has been the significant contributing factor to a disconnect between cash wheat values and wheat futures prices." One month earlier, the USS/PSI released a 247-page report prepared by its own researchers purporting to show that commodity index trading had caused the non-convergence in wheat markets.

Consistent with DeLong et al (1990), the USS/PSI report maintains that index fund capital overpowers arbitrageurs, who may be limited by credit constraints and uncertainty over the time it would take to achieve convergence. To incorporate this possibility, we incorporate into our model the possibility of a bubble solution in which the magnitude of the non-convergence (i.e., the basis) is driven by a non-fundamental noise term.

We estimate an econometric model to test the predictions of the theoretical model for CBOT corn, soybeans, and wheat, and the KCBOT wheat markets. The dependent variable in the econometric model is a measure of the wedge between the market price of physical grain storage and the cost of holding delivery instruments. The explanatory variables include: grain inventory at deliverable locations, contract storage rates, inventories of materials and supplies divided by total sales for food products manufacturing firms (to capture convenience yield), a credit spread measure (e.g., 3-month commercial paper minus 3-month treasury bills), and the market position of commodity index traders (to represent a bubble component). The empirical evidence strongly supports our rational expectations model. Specifically, we find that the storage rate correlates negatively with the wedge, and that the wedge is greatest early in the crop year when inventory is at its largest. We also find evidence that high stocks in deliverable locations correlate strongly with the wedge as the convenience yield has dropped which contributes to a larger wedge for corn and wheat. We find no evidence of a futures bubble caused by commodity index traders. Graphical analysis highlights the important role that the difference between the futures storage fee and the price of physical grain storage played in explaining recent non-convergence.

Theoretical Model of the Delivery Market

Institutional Background

As noted by Pirrong, Kormendi, and Haddock (1993, p. 9), "The delivery terms of futures contracts specify the types and grades of deliverable goods, and denote the places and times of delivery that must be met to avoid default on an outstanding contract." These terms evolve over time to reflect changes in the commercial standards for market transactions. If the terms become misaligned with prevailing standards then the contract may no longer serve as a useful hedging instrument and its continued existence is threatened. Grain futures contracts traded at the CBOT and KCBOT specify a par delivery location and grade for each contract. Delivery at non-par locations and grades is permissible at fixed premiums or discounts.[1]

In a competitive market with costless physical delivery at one particular location and date, arbitrage would force the futures price at expiration to equal the cash price. If the futures price exceeds the cash price, the cash commodity would be bought, futures sold, and delivery made. If the cash price exceeds futures, then futures would be bought and the buyer stands for delivery. This type of arbitrage would prevent the law of one price from being violated. In such a well-functioning delivery system, only a minimal number of futures deliveries would be needed because long and short futures position holders are indifferent to offsetting their positions rather than making and taking delivery. As Hieronymus (1977, p. 340) notes, "A futures contract is a temporary substitute for an eventual cash transaction. In markets that work, delivery is rarely made and taken; futures contracts are entered into for reasons other than exchange of title."

In reality, delivery arbitrage is more complex than the simple description provided above. When a futures contract allows multiple delivery days, locations and grades, as is the case for CBOT and KCBOT grain contracts, delivery will occur at the "cheapest-to-deliver" date, location, and grade, as this will provide makers of delivery (shorts) the lowest cost alternative for sourcing the commodity to satisfy delivery obligations (Stulz 1982, Johnson 1987). The value of these delivery options to the short (timing, location, grade) in grain markets may vary over time

(Hranaiova, and Tomek 2002, Hranaiova, Jarrow, and Tomek 2005). Furthermore, both longs and shorts involved in the delivery process incur costs, which in turn determine arbitrage bounds for the convergence of cash and futures prices at delivery locations.

An important feature of the physical delivery system for CBOT and KCBOT grain futures contracts is the delivery instrument. Delivery is not satisfied directly by physical grain, but instead is satisfied by a warehouse receipt in the case of KCBOT wheat or a shipping certificate in the case of CBOT corn, soybeans, and wheat.[2] A warehouse receipt is a legal document that provides proof of ownership (title) of a certain grade and quantity of a commodity at a given storage facility; e.g., 5,000 bushels of number one hard red winter wheat in firm *x*'s warehouse in Kansas City, Kansas. Crucially, warehouse receipts used in the futures delivery process are negotiable, and thus transferable between parties. A shipping certificate is also a legal document, but rather than representing actual grain in store, it gives the holder the right but not the obligation to demand load-out of the designated commodity from a particular shipping station; e.g., 5,000 bushels of number two yellow corn loaded on a barge at firm *y*'s shipping station on the Illinois River at LaSalle, Illinois.[3] The advantage of a shipping certificate is the flexibility it offers to makers of delivery (shorts) since the grain can be sourced over time and space. Like warehouse receipts, shipping certificates are transferable. Neither warehouse receipts nor shipping certificates have expiration dates, and hence, are in theory infinitely lived instruments.

"Regular firms" play a key role in the CBOT and KCBOT delivery systems. Only firms approved by the exchange as regular for delivery are allowed to issue warehouse receipts or shipping certificates. Firms must meet certain exchange requirements to be eligible for regularity, such as a minimum net worth of $5 million, and have storage warehouses or shipping stations within the delivery territory of the futures contract. Regular firms are the source of all delivery instruments for their designated warehouses or shipping stations. If a maker of delivery is not a regular firm, he/she must buy a receipt or certificate from a regular firm, another holder of a receipt or certificate, or have taken delivery on a previous long futures position. A regular firm that is short is the only party that has the ability to make an "original" delivery with a newly issued delivery instrument. Regular firms are typically large commercial grain firms, such as Cargill, Bunge, and Archer Daniels Midland.

In CBOT and KCBOT grain futures contracts, the holder of a short position has discretion about when in the delivery month to deliver. The delivery process consists of a three-day sequence: 1) an intention day where the short declares their intention for delivery to the exchange clearinghouse, 2) a notice day where the clearinghouse notifies the oldest outstanding long position holder with an invoice for delivery, and 3) a delivery day where the seller and the buyer exchange delivery instruments and payment. The first three-day sequence can be initiated two business days before the first business day of the expiration month and the last three-day sequence can be initiated on the business day prior to the 15th calendar day of the expiration month. This results in a total delivery period of about 10 business days for each contract.

A long taking delivery of a CBOT or KCBOT grain futures contract and holding the delivery instrument incurs an exchange determined daily storage fee. When load-out of grain occurs, the long also incurs a load-out fee, as well as costs of weighing, grading, elevation, trimming, and blending the delivered grain (Pirrong, Haddock, and Kormendi 1993).[4] Finally, regular firms often argue that giving up warehouse space or tying up shipping facilities in the delivery process is costly. This means that they will only deliver on futures if they receive a premium to compensate them for the "inconvenience cost" associated with use of their facility.

Two-Period Model

Consider a storable grain commodity in a two-period world. We model three markets connected to the commodity: the spot market, the futures market, and the delivery instrument market. These markets are populated by two representative traders: the regular firm and the financial firm. The regular firm may issue delivery instruments (warehouse receipts or shipping certificates depending on the particular market) and has the capacity to store grain, whereas the financial firm may not issue delivery instruments and cannot store grain. The financial firm has capital cost r^f, which may be less than the capital cost r faced by the regular firm, reflecting possible advantages in capital markets. The two firms operate with identical information sets, have homogeneous rational expectations, and behave competitively in all markets.

The regular firm enters period 1 with an endowment of I_0 units of the commodity. In period 1, it chooses how much of the commodity to store for sale in the second period (I_1) and how much to sell in the current period ($I_0 - I_1$). It faces market net inverse demand curves $P_1 = f(I_0 - I_1)$ and $P_2 = f(I_1, \varepsilon_2)$ in periods 1 and 2, respectively. The net demand shock ε_2 is the only source of uncertainty in the model. We allow the regular firm to be risk-averse and to be willing to pay a convenience yield to store the commodity Brennan (1958). We specify the convenience yield to be a monotonically non-decreasing function of inventory, denoted by $y(I_1)$. Without loss of generality, we set the risk premium to be a constant π and we also impose a constant cost of physical storage, δ. In our formulation, the cost of physical storage includes the rental fee for warehouse space, handling and in- and out-charges, and insurance. Under rational expectations, the equilibrium period 1 spot commodity price is

$$P_1 = \frac{E(P_2) - \pi}{1 + r} - \delta + y(I_1) . \qquad (1)$$

This representation is essentially the same as in the seminal paper by Brennan (1958), except that we specify δ and π as constants. In the next section, we allow these two parameters and the cost of capital to vary over time.

We denote the futures price in period i for delivery in period j as $F_{i,j}$. Futures contracts are settled by issuing delivery instruments, which can only be issued by the regular firm. Delivery instruments issued in period 1 may be held by the financial firm until period 2 through payment of a storage fee γ to the regular firm, or the financial firm may convert the instruments immediately into grain at zero cost. Converting the delivery instrument into grain (i.e., loading out) would mean that the regular firm had effectively sold grain on the spot market at the expiring futures price, $F_{1,1}$, which it would never do if $F_{1,1}$ were less than the spot price. Thus, the absence of arbitrage requires $F_{1,1} \geq P_1$. The point of our model is to show that the conditions under which the equilibrium outcome is to hold delivery instruments also imply non-convergence in period 1, i.e., $F_{1,1} > P_1$. In period 2, outstanding delivery instruments are automatically converted into grain after markets clear. Thus, the absence of arbitrage requires $F_{2,2} = P_2$.

There are three potential ways for the regular firm to realize profits from storing the commodity: (i) wait and sell at the prevailing spot price in period 2 (store in the physical market), (ii) take a short futures position in period 1 for delivery in period 2 (store in the futures market) or (iii) deliver a delivery instrument in period 1 that will be converted into grain in period 2 (store in the delivery instrument market). Combining (i) and (ii), the absence of arbitrage between the physical spot and futures markets ensures that the period 1 futures price for delivery in period 2 is

$$F_{1,2} = (1+r)(P_1 + \delta - y(I_1)). \tag{2}$$

Below, we combine (ii) and (iii) to explain how non-convergence can occur in period 1.

The delivery instrument market is the market for an expiring futures contract. The financial firm can take delivery of a delivery instrument in period 1 at price $F_{1,1}$. After taking delivery, the firm can enter a futures contract to deliver the certificate back to the regular firm in period 2 at price $F_{1,2}$. Thus, the discounted payoff to taking delivery equals $F_{1,2}/(1+r^f) - F_{1,1}$. The firm must pay the storage rate γ to hold the delivery instrument, so it will be willing to engage in this transaction if

$$\frac{F_{1,2}}{1+r^f} - F_{1,1} \geq \gamma . \tag{3}$$

The regular firm incurs a cost from issuing delivery instruments in period 1, which we term the inconvenience cost. This cost can arise because futures exchange rules require the firm to store any grain that backs delivery instruments in a more costly location than it would use for unconstrained storage. The inconvenience cost can also represent transactions costs associated with issuing delivery instruments. We specify the inconvenience cost as a monotonically non-decreasing function of the number of certificates that are issued and held, C_1. We denote this function by $x(C_1)$.

The payoff to issuing a delivery instrument that is held equals $F_{1,1} + \gamma - x(C_1)$, and the cost is the discounted price of buying the certificate back next period, i.e., $F_{1,2}/(1+r)$. Thus, the regular firm would be willing to issue C_1 delivery instruments if

$$\frac{F_{1,2}}{1+r} - F_{1,1} \leq \gamma - x(C_1). \tag{4}$$

Given the no-arbitrage condition $F_{1,1} \geq P_1$, (4) implies that the expiring futures price must satisfy the condition

$$F_{1,1} \geq \max\left(P_1, \frac{F_{1,2}}{1+r} - \gamma + x(C_1)\right) \tag{5}$$

Inserting (2) into (5), we see that the supply curve for delivery instruments is given by

$$F_{1,1} = P_1 + \max\left(0, \delta - y(I_1) - \gamma + x(C_1)\right). \tag{6}$$

This curve is weakly upward sloping because $x(C_1)$ is monotonically nondecreasing in C_1.

The delivery instrument market clears if there exists a value C_1 such that supply equals demand, i.e., if there exists C_1 such that

$$P_1 + \max\left(0, \delta - y(I_1) - \gamma + x(C_1)\right) = \frac{F_{1,2}}{1+r^f} - \gamma$$

which, using (2), we can re-write as

$$\begin{aligned} \max\left(y(I_1) - \delta + \gamma, x(C_1)\right) &= \frac{F_{1,2}}{1+r^f} - P_1 + y(I_1) - \delta \\ &= \frac{(1+r)(P_1 + y(I_1) - \delta)}{1+r^f} - P_1 + y(I_1) - \delta . \\ &= (r - r^f)\frac{P_1 + \delta - y(I_1)}{1+r^f} \end{aligned} \tag{7}$$

This market clearing condition describes a competitive equilibrium in which expected profit equals zero for both firms. It implies that delivery instruments will not be issued if the inconvenience cost to the regular firms exceeds the difference in capital cost between the two firms. By issuing a delivery instrument, the regular firm incurs the inconvenience cost, but by selling today at price $F_{1,1}$ and buying back next period at the price $F_{1,2}/(1+r^f)$, it essentially gains access to credit at rate r^f. Thus, assuming a nonzero inconvenience cost, the delivery instrument market clears at positive C_1 only if the financial firm has a lower cost of capital than the regular firm.

Equations (6) and (7), along with the condition that $F_{1,1} \geq P_1$, imply that

$$\begin{aligned} F_{1,1} - P_1 &= \max\left(0, \delta - y(I_1) - \gamma + x(C_1)\right) \\ &= \max\left(0, \delta - y(I_1) - \gamma + (r - r^f)\frac{P_1 + \delta - y(I_1)}{1+r^f}\right) \end{aligned} \tag{8}$$

Thus, the following two market conditions can cause convergence failure in period 1:

1. *The cost of storing the physical commodity exceeds the cost of holding certificates.* If γ is set too low relative to the market price of physical grain storage, $\delta - y(I_1)$ then non-convergence may arise.

2. *Difference in capital costs between the holders of delivery instruments and the holders of grain inventory.* A large credit spread provides a means for the regular firm to access cheaper credit. This difference can be accentuated by low interest rates and high spot prices.

Equation (8) shows that non-convergence arises from a wedge between the cost of carrying the commodity and the cost of holding delivery instruments. To elucidate this result, we place it in the classic supply of storage framework of Working (1948 1949)) and Brennan (1958). Equation

(9) shows that the supply of commodity storage is an increasing function of inventories because the convenience yield is decreasing in inventories, i.e.,

$$E(P_2) - P_1 = rP_1 + \pi + (1+r)(\delta - y(I_1)) \tag{9}$$

The demand for commodity storage is determined by the relative market net demand curves in the two periods because of the price tradeoff between selling the commodity in period 1 versus period 2. The demand for storage is

$$E(P_2) - P_1 = E\left(f(I_1, \varepsilon_2)\right) - f(I_0 - I_1). \tag{10}$$

From (6), we can write the supply of delivery instruments as a function of the term spread in the futures market, i.e.,

$$F_{1,2} - F_{1,1} = F_{1,2} - P_1 - \max\left(0, \delta - y(I_1) - \gamma + x(C_1)\right). \tag{11}$$

From (3), the demand for delivery instruments is $F_{1,2} - F_{1,1} = r^f F_{1,1} + (1+r^f)\gamma$.

Figure 2 plots the supply and demand for commodity storage and delivery instruments. Panel A displays a case with high inventory (I_1) and therefore low convenience yield and a high price of storing the physical commodity. The demand for delivery instruments sets the futures term spread below the price of physical grain storage. This wedge creates non-convergence, i.e., a positive basis. To understand this result, recall that a delivery instrument issued in period 1 and held until period 2 becomes equivalent to owning the physical commodity in period 2. Thus, for the regular firm, issuing such a certificate is identical to taking a short futures position in period 1 to deliver the commodity in period 2. It follows that the returns to storage must be identical under the two approaches. The expected returns to storage for the regular firm are

$$\underbrace{E(P_2) - P_1}_{\text{expected price of physical storage}} = \underbrace{F_{1,2} - F_{1,1}}_{\text{futures term spread}} + \underbrace{F_{1,1} - P_1}_{\text{basis}} + \underbrace{\pi}_{\text{risk premium}} \tag{12}$$

where we use the equilibrium condition $F_{1,2} = E[P_2] - \pi$. The regular firm is compensated for a low futures term spread by a positive basis.

Panel B of Figure 2 shows a case with low inventory carryover and therefore a large convenience yield that drives the price of physical storage below the level required for the financial firm to hold delivery instruments. There is no positive wedge between the futures term spread and the price of physical storage, and the futures market converges. Another way to see that no delivery instruments are issued is to note that the high convenience yield drives down the supply curve for delivery instruments so that it does not cross the demand curve for positive C_1. As drawn in Panel B, the supply of delivery instruments becomes the convenience yield is too large at all values of C_1 to produce $\max\left(0, \delta - y(I_1) - \gamma + x(C_1)\right) > 0$.

The demand for delivery instrument curve defines "full carry" in the futures market as defined by Irwin et al (2011). Full carry occurs when the futures term spread is pushed to its maximum value given the fixed storage rate for delivery instruments (γ). The term spread cannot exceed this maximum value; otherwise a riskless arbitrage opportunity is created. We thus generate the regularity Irwin et al. identified; namely that the futures market is at full carry when the basis is inflated. Our model shows that full carry occurs when plentiful inventories drive the price of physical storage above the cost of holding delivery instruments.

The contrast between panels A and B in Figure 2 illustrates the important role of inventory levels in producing non-convergence. Other components of the model are important as well. An increase in the storage rate for delivery instruments (γ) or the cost of capital faced by the financial firm (r^f) raises the demand for certificates and therefore reduces the basis. Thus, convergence could be achieved by raising the storage rate high enough that holding delivery instruments is unattractive. Similarly, a smaller value for the fixed warehouse cost (δ) or the cost of capital faced by the regular firm (r) moves the supply of storage curve down and reduces the basis.

Increasing the inconvenience cost reduces the willingness of the regular firm to supply delivery instruments, thus moving the supply of certificates curve down. The effect of high inconvenience cost on the delivery instrument market depends on the level of inventories. If I_1 is sufficiently small, the storage market clears at a price below the demand for certificates plus the risk premium, i.e., if $E(P_2) - P_1 \leq r^f F_{1,1} + (1 + r^f)\gamma + \pi$, and the demand to hold delivery instruments will not exist. Any issued certificates would be immediately turned into grain and the basis would be zero. If inventories are sufficiently large so that $E(P_2) - P_1 > r^f F_{1,1} + (1 + r^f)\gamma + \pi$, the financial firm would demand delivery instruments to hold, but the regular firm would not find it profitable to issue any. The delivery market would fail to clear. Conversely, lowering the inconvenience cost could enable the delivery instrument market to clear and a nonzero basis to arise. In a market such as in Panel A of Figure 2, which exhibits non-convergence, making it less costly for the regular firm to issue delivery instruments may increase the number of outstanding certificates, but it would not affect the basis.

An increase in the risk premium (π) reduces the willingness of the regular firm to hold inventory thereby shifting the supply of storage curve up. This change reduces the equilibrium amount of inventory held as the market slides up the demand for storage curve. Thus, to the extent that risk aversion reduces inventory holdings, the market moves closer to convergence. This result is akin to the standard result that the producer bears some of the incidence of a sales tax if demand is less than perfectly inelastic. The converse point is that the basis may expand as risk aversion declines. However, risk aversion affects the basis only through its effect on inventory; it does not affect the basis directly.

We model both firms as behaving competitively. A relevant empirical alternative could be that the regular firm has monopoly power. In the model as presented above, the regular firm faces a perfectly elastic demand for delivery instruments, so it could not extract monopoly rents by reducing supply in this market. However, if the demand for certificates were less than perfectly elastic, which could arise if the financial firm faced an increasing marginal cost of capital, then the firm would have an incentive to issue fewer delivery instruments. The effect on the model would be the same as the inconvenience cost because issuing an additional certificate would lower the price received on all certificates. If the regular firm were to have market power in the grain storage market, then the effect would be to shift the supply of storage curve up, thereby raising the expected cost of storage. This change would expand the basis.

Dynamic Infinite Horizon Model

In this section we expand the model to an infinite horizon to accommodate the empirical reality that convergence is not forced at any particular future date. Other than the provision that the two firms maximize over an infinite horizon, the structure of the model remains the same as in the previous section. The results and intuition gained there continue to apply. We add time subscripts to π, δ, γ, r, and r^f to allow the possibility that they vary over time, but for simplicity we treat these quantities as exogenous and deterministic.

Uncertainty enters the model through the spot market net demand $P_t = f(I_{t-1} - I_t, \varepsilon_t)$, where we specify the shock sequence $\{\varepsilon_t\}$ to be stationary and ergodic. This shock is the only source of uncertainty in the model. Following Williams and Wright (1991) and Routledge, Seppi, and Spatt (2000) among others, a stationary rational expectation equilibrium exists and implies

$$P_t = \frac{E_t(P_{t+1}) - \pi_t}{1 + r_t} - \delta_t + y(I_t). \tag{13}$$

Futures market equilibrium implies $F_{t,t+1} = E_t(F_{t+1,t+1}) - \pi_t$.

The financial firm's period t demand for delivery instruments is $F_{t,t+1} = (1 + r_t^f)(F_{t,t} + \gamma_t)$, as in the previous section. Because the option always exists to immediately convert a delivery instrument into grain, the absence of arbitrage implies $F_{t,t} \geq P_t$. Thus,

$$F_{t,t} = \max\left(\frac{F_{t,t+1}}{1 + r_t^f} - \gamma_t, P_t\right) \tag{14}$$

The basis is:

$$\begin{aligned}
F_{t,t} - P_t &= \max\left(0, \frac{E_t(F_{t+1,t+1}) - \pi_t}{1 + r_t^f} - P_t - \gamma_t\right) \\
&= \max\left(0, \frac{E_t(F_{t+1,t+1}) - \pi_t}{1 + r_t^f} - \frac{E_t(P_{t+1}) - \pi_t}{1 + r_t} + \delta_t - y(I_t) - \gamma_t\right) \\
&= \max\left(0, \frac{E_t(F_{t+1,t+1} - P_{t+1})}{1 + r_t^f} + \delta_t - y(I_t) - \gamma_t + (r_t - r_t^f)\frac{E_t(P_{t+1}) - \pi_t}{(1 + r_t)(1 + r_t^f)}\right) \\
&= \max\left(0, \frac{E_t(F_{t+1,t+1} - P_{t+1})}{1 + r_t^f} + \delta_t - y(I_t) - \gamma_t + (r_t - r_t^f)\frac{P_t + \delta_t - y(I_t)}{1 + r_t^f}\right)
\end{aligned} \tag{15}$$

where we use $F_{t,t+1} = E_t(F_{t+1,t+1}) - \pi_t$, and $P_t = (E_t(P_{t+1}) - \pi_t)/(1 + r_t) - \delta_t + y(I_t)$. Apart from the term $E_t(F_{t+1,t+1} - P_{t+1})/(1 + r_t^f)$, this is the same expression as the second line in equation (8).

Equation (15) presents the expiring futures market basis assuming that the delivery instrument market clears. As was the case in equations (4)-(7), the regular firm would be willing exit period t with C_t outstanding delivery instruments if

$$\frac{F_{t,t+1}}{1 + r_t} - F_{t,t} \leq \gamma_t - x(C_t) \tag{16}$$

which implies a delivery instrument supply curve of

$$F_{t,t} = \max\left(P_t, \frac{F_{t,t+1}}{1+r_t} - \gamma_t + x(C_t) \right) \quad (17)$$

and the same market clearing condition as in (7).

We define the "excess" spread in the futures market as

$$\begin{aligned}
S_t &\equiv \frac{F_{t,t+1}}{1+r_t^f} - F_{t,t} - \gamma_t \\
&= \frac{E_t(F_{t+1,t+1}) - \pi_t}{1+r_t^f} - \gamma_t - \max\left(\frac{E_t(F_{t+1,t+1}) - \pi_t}{1+r_t^f} - \gamma_t, \frac{E_t(P_{t+1}) - \pi_t}{1+r_t} - \delta_t + y(I_t) \right) \\
&= -\max\left(0, -\frac{E_t(F_{t+1,t+1}) - \pi_t}{1+r_t^f} + \delta_t - \gamma_t - y(I_t) + \frac{E_t(P_{t+1}) - \pi_t}{1+r_t} \right) \\
&= \min\left(0, \frac{E_t(F_{t+1,t+1} - P_{t+1})}{1+r_t^f} + \delta_t - \gamma_t - y(I_t) + (r_t - r_t^f)\frac{P_t + \delta_t - y(I_t)}{1+r_t^f} \right).
\end{aligned} \quad (18)$$

This variable measures the extent to which the futures market departs from full carry. We also define the basis as $B_t \equiv F_{t,t} - P_t$, and the wedge between the cost of carrying the commodity and the cost of holding certificates as $W_t \equiv \delta_t - y(I_t) - \gamma_t + (r_t - r_t^f)(P_t + \delta_t - y(I_t))/(1+r_t^f)$. With these definitions, we can write the basis and the excess spread as

$$\begin{aligned}
B_t &= \max\left(\frac{E_t(B_{t+1})}{1+r_t^f} + W_t, 0 \right) \\
S_t &= \min\left(\frac{E_t(B_{t+1})}{1+r_t^f} + W_t, 0 \right)
\end{aligned} . \quad (19)$$

The wedge W_t is identical to that in equation (8) from the two-period framework. Equation (19) says that, if $E_t(B_{t+1})/(1+r_t^f) + W_t > 0$, then we have a futures market at full carry (S_t=0) and non-convergence. Conversely, if $E_t(B_{t+1})/(1+r_t^f) + W_t < 0$, then we have a futures market at less than full carry (S_t<0) and convergence.

Equation (19) also shows that the level of the basis in t depends on the expected basis in t+1, which depends on the expected basis in t+2, etc. Specifically,

$$\begin{aligned}
B_t &= \max\left(\frac{E_t(B_{t+1})}{1+r_t^f} + W_t, 0 \right) \\
&= \max\left(E_t\left(\max\left(\frac{E_{t+1}(B_{t+2})}{(1+r_{t+1}^f)(1+r_t^f)} + \frac{W_{t+1}}{1+r_t^f}, 0 \right) \right) + W_t, 0 \right) \\
&= \max\left(E_t\left(\max\left(E_{t+1}\left(\ldots \max\left(\frac{E_{t+s}(B_{t+s+1})}{\prod_{i=0}^{s}(1+r_{t+i}^f)} + \frac{W_{t+s}}{\prod_{i=0}^{s-1}(1+r_{t+i}^f)}, 0 \right) \ldots \right) + \frac{W_{t+1}}{1+r_t^f}, 0 \right) \right) + W_t, 0 \right)
\end{aligned} \quad (20)$$

Hence, the basis equals the expected present discounted value of future positive wedges. This equation is the main addition to the model from the two-period case. It implies that a relatively small wedge term in period *t* can have a large effect on the basis if it is expected to persist for an extended period.

The stationary equilibrium in (13) ensures that the sequence in (20) converges as $s \to \infty$. This result follows from the fact that with probability one the model will enter a state with a negative wedge, i.e., convergence will occur someday. Nonetheless, consider the possibility that the basis contains a bubble component. Specifically, suppose the basis is

$$F_{t,t} - P_t = R_t + N_t \tag{21}$$

where $R_t = \max\left(E_t(R_{t+1})/(1+r_t^f) + W_t, 0\right)$ denotes the rational component of the basis and N_t denotes a nonnegative noise component. A little algebra shows that the equilibrium condition $F_{t,t+1} = E_t(F_{t+1,t+1}) - \pi_t$ holds if $N_t = E_t(N_{t+1})/(1+r_t^f)$.

At prices characterized by $N_t>0$, the regular firm would be willing to issue delivery instruments, and the financial firm would be willing to hold them as long as it could hedge at the price $F_{t,t+1} = E_t(F_{t+1,t+1}) - \pi_t = E_t(P_{t+1} + R_{t+1}) - \pi_t + (1+r_t^f)N_t$. Thus, the noise term could perpetuate itself if both firms believed it would continue, and it is an example of a rational bubble (Diba and Grossman, 1988). However, because the firms in our model are infinitely lived, neither would be willing to take the other side of this hedge. Both firms know that the bubble will burst at some future date and at that time the firm on the other side of this hedge would be left holding delivery instruments or grain for which it had overpaid. To the extent that firms do not display such rationality, a bubble could arise and persist.

In summary, our model shows that convergence failure occurs because of a wedge between the price of physical commodity storage and the cost of holding delivery instruments. The observed basis equals the present value of a nonlinear function of expected future wedges. In the next section, we develop econometric methods to assess the model and understand the causes of the recent episode of non-convergence in the CBOT and KCBOT grain futures markets.

Empirical Analysis

We begin this section by describing our data. Next, we estimate regression models to quantify the driving forces behind the basis. Then we provide some graphical evidence in support of our model using CBOT wheat as an example. Convergence failures in this market have been greater in magnitude than those in other grain markets.

Data

Using data from 1990-2010, we estimate regression models to determine drivers of the wedge and to assess the prediction of our theory that aggregate inventory levels at deliverable locations have strong explanatory power. Variables other than inventory that may cause changes in the wedge include credit spreads between commercial paper and the T-Bill yield, the exchange established storage rate γ_t, and the ratio of the inventories and supplies for food product manufacturing firms to their sales which could affect convenience yield on a more macro level. We also include the open interest held by commodity index traders to capture the possibility of a bubble induced by the limited ability of the market to arbitrage against the index investment.

To compute basis for delivery locations, we use the settlement price of the expiring futures contract on the first day of the delivery month. The source for the futures prices is barchart.com. Cash (spot) prices are from the Agricultural Marketing Service of the U.S. Department of Agriculture (USDA) for Chicago, Illinois River North of Peoria, Illinois River South of Peoria, Toledo, and Kansas City. The USDA reports the range of spot bids at the specified location after 1:30 pm CST (closely after the close of the futures markets.) The data is generally available by 3:00 pm CST. Basis is calculated as the midrange of the settlement futures price minus the cash bid. Delivery location and grade differentials from the CBOT and KCBOT Rulebooks are applied as necessary. Contract storage rates are also collected from the Rulebooks.

For the interest rate faced by financial firms, we use the 3-month London Interbank Offered Rate (LIBOR). It is the most widely used "benchmark" or reference rate for short-term interest rates and is compiled by the British Bankers Association in conjunction with Reuters and released to the market shortly after 11am London time each day. To approximate differences in the cost of capital between regular and financial firms, we use the spread between yield on 3-month non-financial commercial paper and 3-month Treasury Bills. Prior to 1997, financial and non-financial commercial paper yields were not reported separately; for this period we use the reported overall commercial paper rate. We obtained these data from the Federal Reserve Bank of St Louis.

Inventories of grain at deliverable locations are collected from Registrar Reports available from the CME and KCBOT. The reported inventories include deliverable grades, non-deliverable grades/ungraded, and Commodity Credit Corporation (CCC) stocks. Deliverable grades of grain meet the exchange quality requirements for futures delivery, excluding CCC-owned grain but including all non-CCC deliverable grades regardless of whether receipted and/or registered. Non-deliverable grades/ungraded is graded grain not meeting exchange quality requirements for futures delivery and ungraded grain, excluding CCC-owned grain. CCC stocks are owned by the CCC of the USDA and not deliverable. We tested deliverable grades only and total stocks in the regressions reported in the next section and found nearly identical results (not surprising given the small differences between the two series). We report only results for total stocks at deliverable locations.

In recent years, manufacturing firms have developed more efficient inventory management terms using information technology. This change has reduced the willingness of these firms to hold inventory. In food markets, this change may manifest in a reduced convenience yield for grains. To approximate this component of convenience yield we use the ratio of inventories of materials and supplies held by food products manufacturing firms to sales of those firms (source: Bureau of Economic Analysis).

Positions of commodity index traders are drawn from the Supplemental *Commitments of Traders* report, which is generated by the Commodity Futures Trading Commission (CFTC). The report is commonly referred to as the Commodity Index Trader (CIT) report. The CIT data are released each Friday in conjunction with the traditional Commitment of Traders report and show the combined futures and options positions as of the previous Tuesday's market close. Positions are also aggregated across all contract maturities for a given commodity. The publically-available CIT data starts in 2006. The CFTC collected additional data for CBOT corn, soybeans, and wheat and KCBOT wheat over 2004-2005 at the request of the U.S. Senate Permanent Subcommittee on Investigations (USS/PSI, 2009) and these additional observations are used in the analysis.[5] CIT positions are measured as the net long position (long minus short contracts) on the report date closest to the first day of delivery for the relevant futures contract. We assume zero values for CIT net long positions before 2004. We are unaware of any data on CIT positions before this date, which is likely a reflection of their very small position size (Sanders, Irwin, and Merrin 2010).

Regressions to Explain Variation in the Wedge

Our theory predicts that the basis is driven by a wedge between the price of physical grain storage and the cost of holding delivery instruments. The wedge is

$$W_t \equiv \delta_t - y(I_t) - \gamma_t + (r_t - r_t^f)\frac{P_t + \delta_t - y(I_t)}{1 + r_t^f} \tag{22}$$

The term $\delta_t - y(I_t)$ in (12) captures the expected price of physical storage, apart from the interest cost. Similarly, γ_t is the cost of carrying delivery instruments apart from the interest cost. The term $(r_t - r_t^f)\frac{P_t + \delta_t - y(I_t)}{1 + r_t^f}$ measures the difference in capital costs for grain storage compared to the delivery instrument storage.

Most of the terms in (22) are unobservable, but the model in Section II suggests a way to approximate it. We add the basis to the excess term spread in (18) which yields the linear expression:

$$B_t + S_t = \frac{E_t(B_{t+1})}{1 + r_t^f} + W_t. \tag{23}$$

This can be easily re-arranged to develop a measure of the wedge,

$$B_t + S_t - \frac{B_{t+1}}{1 + r_t^f} = W_t + \varepsilon_{t+1} \tag{24}$$

where $\varepsilon_{t+1} = \left(E_t(B_{t+1}) - B_{t+1}\right)/(1+r_t^f)$ denotes the prediction error in the basis. We observe B_t and essentially observe S_t because we observe the storage rate (γ_t), and can approximate the capital cost of financial firms (r_t^f) with the LIBOR. Thus, we can calculate a noisy version of the wedge in (24) as $S_t + B_t - B_{t+1}/(1 + r_t^f)$.

We seek to explain the wedge using a set of explanatory variables Z_t, i.e., we would like to estimate the following regression equation:

$$W_t = \beta' Z_t + u_t \tag{25}$$

This equation is not estimable because we do not observe W_t. Combining (24) and (25), we obtain a regression equation

$$S_t + B_t - \frac{B_{t+1}}{1+r_t^f} = \beta' Z_t + v_{t+1} \tag{26}$$

where $v_{t+1} = u_t + \varepsilon_{t+1}$ has the property that $E(v_{t+1} \mid Z_t) = 0$. The left-hand-side variable in (26) is essentially the excess spread minus the change in the basis to the next contract expiration. Due to differences in time-to-expiration, we scale the wedge measure by m_t, the number of months until the next expiration. In our data, m_t is either 1, 2, or 3. The adjusted wedge and the left-hand-side variable in our regressions are

$$\tilde{W}_t = \frac{1}{m_t}\left(S_t + B_t - \frac{B_{t+1}}{1+r_t^f} \right). \tag{27}$$

This adjusted wedge is measured in cents per bushel per month.

In addition to the variables described in the previous section, we include fixed effects for each contract month, as well as trend variables to purge the analysis of any systematic time-variant factors. Our dependent variable exhibits occasional large negative values when the futures term structure becomes deeply inverted. The observations are large enough to dominate in a regression model, but are irrelevant to our modeling because we are interested in the determinants of positive values of the wedge. Thus, we add a dummy variable for each dependent variable observation that is more than four standard deviations below the mean.

The estimated results for the three CBOT contracts individually and together are provided along with the KCBT wheat contract in Table 1. For corn and soybeans, we use Toledo as the basis location until the end of 1999, when Toledo ceased to be a delivery location. Beginning in 2000, we switch to the Illinois River location. We include a dummy variable in the corn and soybeans regressions to allow a level shift in the wedge between these two periods. The use of Toledo and Illinois River prices for the Chicago corn and soybean contract reflects the limited cash trade that flows through Chicago and concerns about the representativeness of reported Chicago cash prices. However, our results are similar if we use Chicago cash prices to measure the basis.

Consistent with our theoretical model, the primary driver in the relationship is stocks in deliverable locations, which are strongly related to the wedge in all cases and take similar coefficient values across commodities. The inventory variable enters in logs, so a coefficient of 3.95 (CBT wheat) implies that a 10% increase in inventory leads to a 0.4 cent increase the wedge. During the 2004 to 2008 period, deliverable stocks approximately doubled which corresponds to an increase in log inventory of 0.69. In response to such a doubling of inventories, the coefficient implies an increase in the wedge of 2.73 cents per month. For comparison, the contract storage rate was 5 cents per month during this period, so the inventory effect is substantial.

To assess the possibility that a high wedge draws inventory into delivery locations rather than the other way around, we use instrumental variables estimation. We use national crop-year beginning stocks of the commodity as an instrument. This variable is determined by the size of the most recent harvest and prior-year aggregate storage decisions, which are unlikely to be caused by an anticipated future wedge in storage costs. In all cases, this proved to be a strong instrument (1st stage F>10). The instrumental variables estimates were close to the OLS estimated reported in table 1 and in no case did a Hausman test suggest the presence of endogeneity.

In almost all equations, the contract-month fixed effects show that the wedge is lowest late in the crop year when inventories are low and highest around the harvest when inventories are plentiful. The corn and soybean harvest occurs around October. Corn and soybeans exhibit the lowest month effects in July and August, and the highest in September and November, respectively. (November is the omitted category in the definition of the dummy variables for soybeans.) The wheat harvest occurs in June, which is consistent with the wedge being smallest in March and largest in July for KCBT wheat and September for CBT wheat.

The coefficients for the other variables in the model are not statistically significant although they often possess the expected signs. The storage rate for corn and soybeans is negatively correlated with the wedge, and not statistically different from −1, which is anticipated from the model. The storage rate coefficients for wheat are positive but they demonstrate large standard errors. This coefficient is not well identified because of limited changes in the storage rate during the period (e.g., only once during the sample period for KCBOT wheat), and when it did change in wheat it occurred in response to large basis movements which leads to a positive sign. For the inventory-sales ratio, which enters the model in log first difference form, only the wheat coefficients approach any modest level of statistical significance. These estimates suggest that the convenience yield for wheat in food manufacturing may have declined over time.

The credit spread only emerges modestly significant in the corn equation with a positive sign, indicating that a difference in interest rates would widen the wedge. However, with a coefficient of 0.03 and the small differentials that have existed between these interest rates, it is unlikely that its effect on the wedge would be anything but small. This result is consistent with the theory, which shows that credit spreads are a small determinant of the magnitude of the wedge even though they are an important determinant of the equilibrium number of delivery instruments issued and the possible profits earned by the participating firms.

Finally, the findings do not support that notion the CIT trader open interest which might have limited ability of the market to arbitrage against the funds influenced the dynamics of the wedge. Overall, these results provide evidence of the importance of inventories at the deliverable locations as the key factor in explaining wedge behavior over time. As anticipated from the theory and earlier evidence, inventories have a pervasive effect on the behavior of the wedge in all the markets.

Tests for breaks in the estimated structures provide mixed findings. The Elliott-Muller (2006) break tests consistently indicate stable structures in all cases. However, specific tests of a break in 2006 suggest a structural change in both the CBT and the KCBT wheat markets at this time. In the next section, we explore the model predictions graphically for CBT wheat, which exhibited particularly dramatic behavior in 2008-09.

Graphical Evidence in Support of the Model

The econometric results highlight the key role of inventories and the price of physical storage in explaining non-convergence. The results are consistent with our theory which predicts that the basis expands when grain inventory is plentiful and collapses when inventories are scarce. Specifically, non-convergence develops as cash prices drop after a temporary price shock. The temporary nature of the shock is important, because only temporary shocks such as a bad harvest cause inventories to be run down and convenience yield to rise.

Figure 3 shows for CBOT wheat, the basis, the wedge and inventory in Toledo, which was typically the cheapest delivery location during this period. Similar to the data for the econometric model, each curve in Figure 3 is compiled from data measured on the first day of delivery for each contract expiration between 1990-2010. Panel A reveals three distinct episodes of non-convergence: 1999-2001, 2006-07, and 2009-10. The first and third of these episodes occurred after prices had descended from a peak and inventory had accumulated. The 2006-07 episode occurred during a period of rising prices, but Panel C shows that inventory was also relatively high in this period, consistent with the theory that the basis expands when inventory is high. The fact that prices were high in 2006-07 in spite of high inventory suggests that a persistent demand shock caused the price increase. This period was characterized by a strong increase in the demand for grain due to the massive expansion of corn-ethanol production.

Panel B decomposes the wedge into two components. The first component is the storage rate on delivery instruments, γ_t. Aside from a small drop in 2000 and two separate increases in 2009-10, the storage rate remained constant at about 5 cents per bushel per month. The second line shows the remaining components of the wedge, i.e., it is

$$W_t + \gamma_t = B_t + S_t - \frac{B_{t+1}}{1+r_t^f} + \gamma_t = \delta_t - y(I_t) + (r_t - r_t^f)\frac{P_t + \delta_t - y(I_t)}{1+r_t^f} + \varepsilon_{t+1} \qquad (28)$$

We plot a three-period, centered moving average of this quantity to smooth out the shocks ε_{t+1}. We label this curve the price of physical grain storage although it also includes a credit spread component. Comparing Panels B and C, we see that this second component is closely related to inventory, which suggests that the credit spread component plays a small role in determining the wedge. Rather, the dominant factor is inventories, as we found with the econometric model.

With the aid of Figure 3, the reasons for the recent and dramatic failures of convergence come into sharper focus. Basically, demand and/or supply shocks created a surge in inventories, which in turn drove up the price of physical grain storage. This market price of storage substantially exceeded the storage rate being paid on CBOT and KCBOT grain futures contracts. In the case of CBOT wheat, Panel B shows that the gap between the two was very large for much of 2007-2010, and hence, the very large delivery location basis. The gap between the two series only began to narrow with the implementation of the CBOT's variable storage rate rule starting with the September 2010 contract (Seamon 2009).

Summary and Conclusions

In a well-functioning futures market, the futures price on the expiration date equals the price of the underlying asset on that date. An unprecedented episode of non-convergence in Chicago

Board of Trade (CBOT) corn, soybeans, and wheat began in late 2005, and with the exception of some brief periods, largely persisted through 2010. Most recently, the Kansas City Board of Trade (KCBOT) wheat contract also has demonstrated convergence problems. During this unprecedented and extended episode of non-convergence, futures contracts have expired at prices up to 35 percent greater than the prevailing cash grain price. This failure in the most basic of futures market functions would appear to create an extraordinary opportunity for grain traders to make massive arbitrage profits by acquiring inexpensive grain in the cash market and delivering it at much higher futures contract prices.

We develop a dynamic rational expectations commodity storage model that explains the observed convergence failures. Specifically, we show that non-convergence arises in equilibrium when the market price of physical grain storage exceeds the cost of holding delivery instruments. The storage fee on delivery instruments is set by the futures exchange and does not vary much over time. However, the price of physical grain storage varies substantially over time as the level of inventory changes. Plentiful inventories generate a high price of physical storage and small inventories cause the price of storage to become negative as the market is willing to pay a convenience yield to store the commodity. We call the difference between the cost of carrying physical grain and the cost of carrying delivery instruments the *wedge*. We show that the magnitude of the non-convergence equals the expected present discounted value of future positive wedges.

We estimate an econometric model to test the predictions of the theoretical model for CBOT corn, soybeans, and wheat, and the KCBOT wheat markets. The empirical evidence strongly supports our rational expectations model. Specifically, we find that the storage rate correlates negatively with the wedge, and that the wedge is greatest early in the crop year when inventory is at its largest. We also find evidence that high stocks in deliverable locations correlate strongly with the wedge as the convenience yield has dropped which contributes to a larger wedge for corn and wheat. We find no evidence of a futures bubble caused by commodity index traders.

Graphical analysis highlights the important role that the difference between the futures storage fee and the price of physical grain storage played in explaining recent non-convergence. Basically, demand and/or supply shocks in recent years created a surge in inventories, which in turn drove up the price of physical grain storage. This market price of storage substantially exceeded the storage rate being paid on CBOT and KCBOT grain futures contracts. Two important implications follow. First, futures exchanges need to either closely monitor developments in the market for storage in order to appropriately adjust fixed contract storage rates or adopt a flexible storage rate rule, like the CBOT's variable rate rule for wheat, to avoid non-convergence problems in the future. Second, research is needed to better understand the forces driving the rise in the market price of storage in recent years.

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Endnotes

[1] Additional details on the CBOT and KCBOT delivery systems can be found in Irwin et al. (2011) and the Exchange Rule Books located at http://www.cmegroup.com/rulebook/CBOT/I/7/7.pdf and http://www.kcbt.com/rule_book_kcbt.html.

[2] CBOT corn and soybean delivery was based on warehouse receipts prior to the March 2000 contract. CBOT wheat delivery was via a warehouse receipt prior to the July 2008 contract.

[3] In the case of a shipping certificate, title to the grain does not change hands until load out of grain occurs at the shipping station.

[4] The non-convergence of futures prices since 2005 shown in Figure 1 far exceeds any reasonable estimates of the cost of delivery arbitrage. See Irwin et al. (2011) for further discussion on this point.

[5] The authors are indebted to the staff of the U.S. Senate Permanent Subcommittee on Investigations for providing the 2004-2005 index trader position data.

Table 1. Wedge Regressions for CBOT Corn, Soybeans, and Wheat and KCBOT Wheat, 1990-2010.

Commodity		Corn		Soybeans		CBT Wheat		All CBOT		KCBT Wheat	
Basis Location		Toledo/IL River		Toledo/IL River		Toledo		Toledo/IL River		Kansas City	
variable		**coeff**	**t-stat**	**coeff**	**t-stat**	**coeff**	**t-stat**	**coeff**	**t-stat**	**coeff**	**t-stat**
Inventory (Total)		3.17	3.25	4.56	3.13	3.95	2.80	3.20	5.66	3.55	4.16
Storage Rate		-0.91	-0.70	-0.78	-0.27	1.49	1.18	1.02	0.83	5.79	0.82
Inventory-Sales Ratio		0.29	0.45	0.87	1.08	-1.20	-1.54	-0.08	-0.18	-0.77	-1.54
Credit Spread		0.03	1.49	-0.02	-0.50	-0.03	-0.74	0.01	0.35	-0.03	-0.92
CIT		0.00	0.02	-0.46	-0.86	0.03	0.12	-0.05	-0.33	-0.03	-0.04
Constant		-858	-2.60	-475	-0.93	-166	-0.34	-534	-2.24	-772	-3.03
Trend		0.42	2.57	0.22	0.87	0.06	0.25	0.25	2.09	0.36	2.69
January				-4.66	-2.37			0.42	0.27		
March		0.37	0.40	-4.57	-2.38	-6.52	-2.93	-2.14	-1.96	-1.62	-0.96
May		-1.75	-1.42	-5.93	-2.66	-1.90	-0.77	-2.33	-1.88	1.87	1.04
July		-2.35	-1.27	-7.15	-2.02	0.86	0.32	-2.05	-1.59	4.25	2.17
August				-15.62	-3.41			-11.61	-2.46		
September		3.31	2.19	-10.38	-2.48	2.92	1.76	2.07	1.86	3.33	2.74
Corn Dummy								-3.36	-2.11		
Soybean Dummy								-2.39	-1.08		
Toledo Dummy (Corn)		-1.50	-0.91					3.85	2.43		
Toledo Dummy (Soy)				-3.21	-1.00			2.80	1.44		
Outlier Dummies											
	19960701	-44.95	-25.71					-45.97	-34.80		
	20040701			-135.95	-39.17			-137.62	-76.87		
	20090701			-100.21	-14.32			-106.69	-42.22		
	20090901			-80.74	-20.34			-93.04	-46.78		
Diagnostics		**stat**	**crit. val.**	**stat**	**crit. val.**	**stat**	**crit. val.**	**stat**	**crit. val.**	**stat**	**crit. val.**
t_test: Storage Rate = -1		0.07	±1.96	0.07	±1.96	1.97	±1.96	1.64	±1.96	0.96	±1.96
1st Stage F-Stat		11.72	10.00	11.88	10.00	20.45	10.00	110.79	10.00	74.44	10.00
Hausman Test		0.37	3.84	0.17	3.84	0.06	3.84	0.01	3.84	0.40	3.84
Break in 2006		1.58	5.99	1.35	5.99	88.29	5.99	1.10	5.99	11.07	5.99
Elliott-Muller Break Test		-10.44	-14.32	-6.38	-14.32	-9.09	-14.32	-3.56	-14.32	-11.87	-14.32
R squared		0.53		0.63		0.29		0.47		0.26	
Sample Size		124		174		124		422		102	

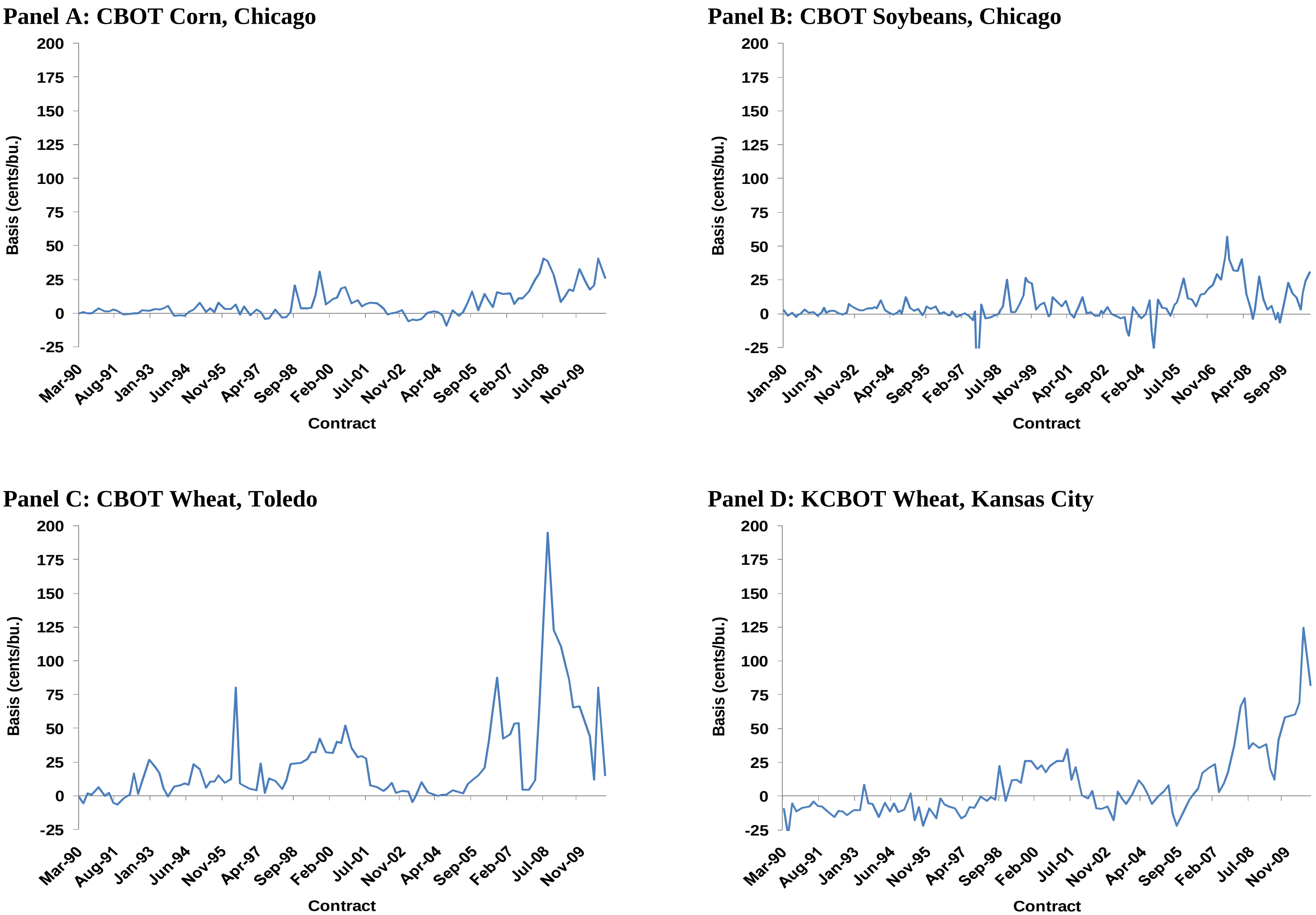
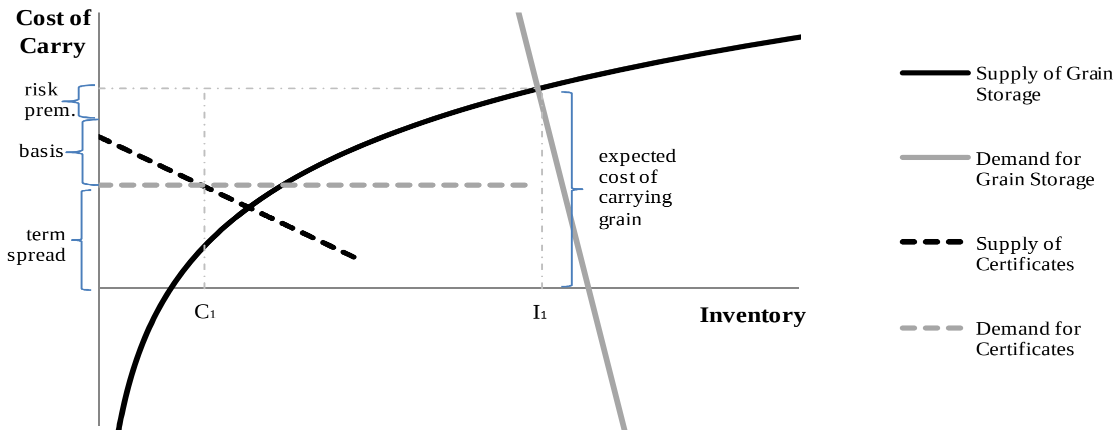


Figure 1. Delivery Location Basis on the First Day of Delivery for CBOT Corn, Soybeans, and Wheat and KCBOT Wheat, 1990 - 2010

Panel A: Non-convergence (High Inventory)



Panel B: Convergence (Low Inventory)

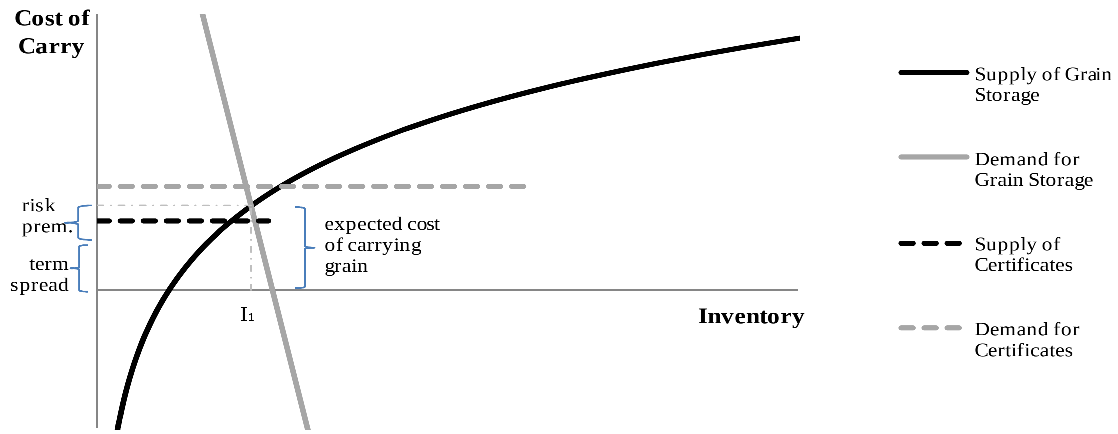


Figure 2. Equilibrium in the Delivery Instrument, Futures and Spot Markets

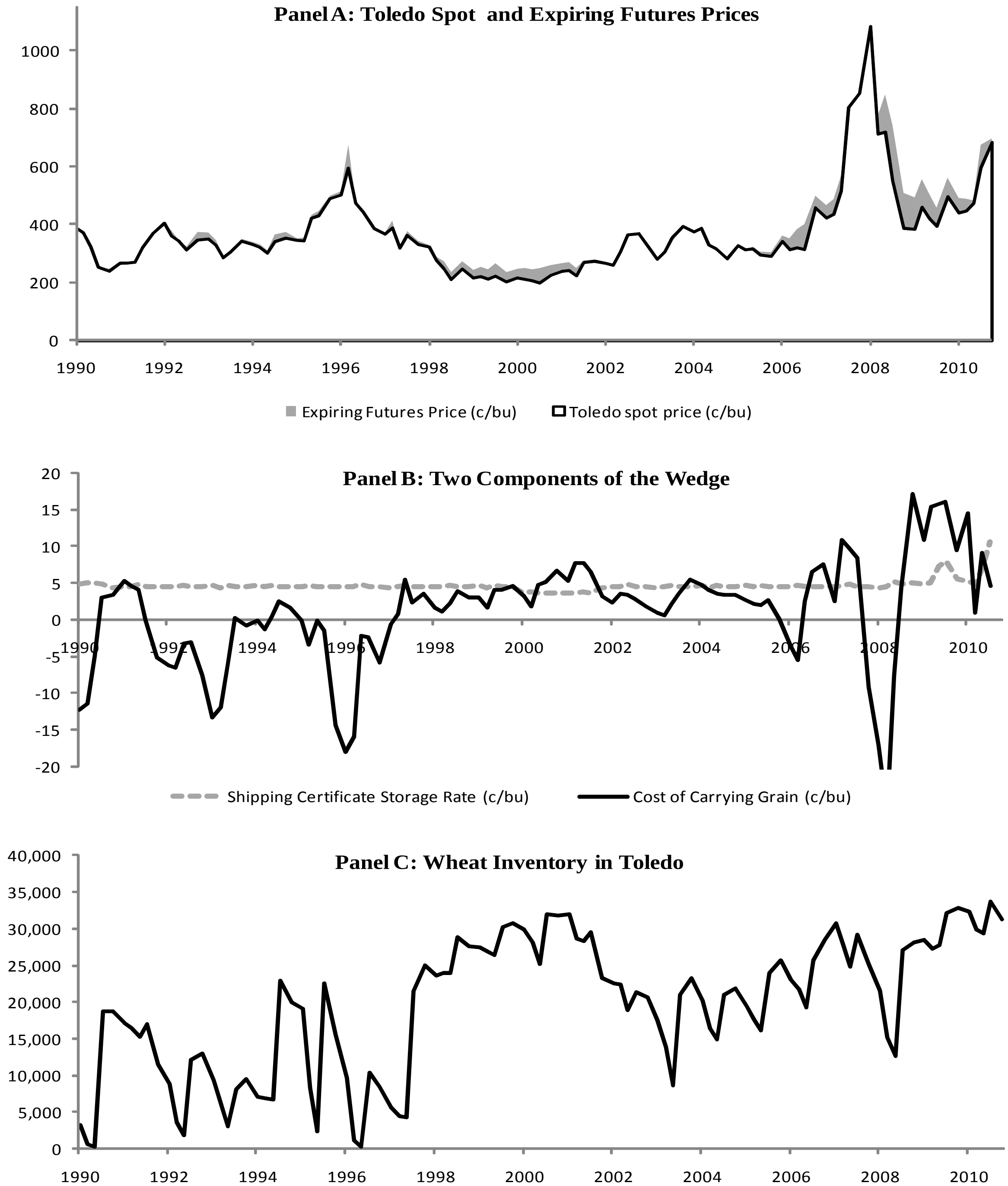


Figure 3. Elements of Non-convergence in CBOT Wheat, 1990-2010