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by

Adam Schmitz, Zhiguang Wang, and Jung-Han Kimn

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[*] Schmitz (Adam.Schmitz@sdstate.edu) is a PhD candidate in the Department of Mathematics and Statistics at South Dakota State University. Wang (Zhiguang.Wang@sdstate.edu) and Kimn (Jung-Han.Kimn@sdstate.edu) are assistant professors in the Department of Economics and the Department of Mathematics and Statistics at South Dakota State University, respectively. The financial support from Agricultural Experiment Station at South Dakota State University (Project H363-10) and Stahly Scholar in Financial Economics are gratefully acknowledged by Wang. We appreciate the comments by Wade Brorsen and Philip Garcia and other participants at the NCCC-134 conference.

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Stochastic volatility, price jumps, seasonality, and stochastic cost of carry, have been included separately, but not collectively, in pricing models of agricultural commodity futures and options. We propose a comprehensive model that incorporates all four features. We employ a special Markov Chain Monte Carlo algorithm, new in the agricultural commodity derivatives pricing literature, to estimate the proposed stochastic volatility (SV) and stochastic volatility with jumps (SVJ) models. Overall model fitness tests favor the SVJ model. The in-sample and out-of-sample pricing and hedging results for corn, soybeans and wheat generally, with few exceptions, lend support for the SVJ model.

Keywords: MCMC, Jump Diffusion, Bayesian Analysis, Agricultual Commodity Options

Introduction

Trading in agricultural commodities has steadily increased since the mid-1990's. There are many explanations for this including the need for investors to diversify portfolios, the increase in demand for grains because of bio-fuels, the increase in demand for grain from abroad, and the broader increase in trading that many asset classes have experienced. The increased trading of agricultural commodities and investors' need for hedging leads to the need for a sophisticated model for these instruments. An inability to accurately model the prices for and hedge a position in agricultural commodities can lead to disastrous effects. Witness Verasun's recent bankruptcy which can be, at least in part, attributable to an inability to model the dramatic price fluctuations that occur in agricultural commodities.[1]

Many previous studies modeling agricultural futures and options have included components such as jumps and seasonality separately. Our objective in this paper is to present a model which is comprehensive regarding the main characteristics of agricultural futures and options. Those characteristics are jumps, stochastic volatility, seasonality, and stochastic cost of carry. A comprehensive model that provides accurate pricing results and superior hedging performance can help farmers and agricultural businesses manage their risks more effectively. Also, a comprehensive model can help illuminate some of the more nuanced characteristics of these agricultural commodities.

The jump phenomenon found in financial markets is also present in the agricultural and broader commodities markets. Hilliard and Reis (1999) provide evidence for jumps in commodity futures prices. Koekebakker and Lien (2004) provide evidence of volatility and price jumps in wheat futures and options. Aravindhakshan (2010) finds jump-diffusion models fit the wheat futures prices well. The importance of modeling jumps becomes increasingly apparent as we witness the dramatic rise and fall of agricultural futures prices in recent years. For example, the Chicago corn spot price rose from \$3 per bushel to \$7.2 per bushel in July 2008 and

[1] VeraSun was a biofuels producer located in Aurora, SD, that filed for bankruptcy in October 2008. VeraSun entered into grain contracts at the height of the market essentially forcing them to buy grain at prices much higher than the prevailing market prices. As broader fuel prices sharply declined, the cost of production of ethanol exceeded the income from sales forcing them to go under.

subsequently dropped to $3.6 per bushel in December 2008. In order to effectively model this type of extreme movement in the spot price, the inclusion of a jump term is necessary.

In addition to the jumps in spot prices, stochastic volatility is an indispensable component in almost all derivatives asset pricing models. Agricultural commodities are no exception. In fact, recognizing the market's need to hedge against volatility risk, the Chicago Mercantile Exchange (CME) group, partnering with Chicago Board Options Exchange (CBOE), introduced the corn and soybean volatility indexes to the market in early 2011. In academia, there is a long history of modeling stochastic volatility. Heston's (1993) seminal paper provides the framework for using a mean-reverting, stochastic volatility model to price financial options. Schwartz (1997) proposes three models which increase in complexity from the general stochastic volatility model to those with stochastic convenience yield in order to model copper, oil, and gold. Trolle and Schwartz (2009) study the effects of unspanned stochastic volatility on commodity derivatives based on the Heath, Jarrow, and Morton (1992) risk-neutral measure framing. Geman and Nguyen (2005) propose a two-factor stochastic volatility model that relate the soybean stocks and scarcity to price volatility. We follow the literature by modeling the latent volatility through a mean reverting process.

Agricultural commodity prices often exhibit seasonality, a characteristic generally not shared by financial assets. Because much of the volatility of the agricultural market can be attributed to changes in the weather, it is reasonable to conclude that prices and volatility may exhibit seasonal changes. The uncertainty surrounding all relevant spot price variables tends to decrease as the growing season matures and harvest begins leading to a seasonal trend in spot price movement and volatility. Richter and Sorensen (2003) find that soybeans exhibit seasonality patterns in the spot price level and volatility. Sorensen (2002) provides a framework for modeling seasonality in commodity futures. We introduce a spot price that is seasonally changing in order to capture the empirically observed seasonality.

The spot price of an agricultural commodity described by a seasonal jump diffusion process above is not sufficient to determine the futures prices. Cost of carry information provides the missing link. We follow Gibson and Schwartz (1990) to model the stochastic cost of carry that captures the Samuelson Hypothesis (1965): the futures price volatility decreases as time to expiration increases. In a similar way, Hilliard and Reis (1998) provide a model for commodities with stochastic convenience yields, in addition to stochastic interest rates and jumps.

Although the existing literature has attempted to model some of the features for agricultural commodity prices, a more comprehensive model that incorporates all of them has yet to be considered. The most relevant studies are Geman and Nguyen (2005), Koekebakker and Lien (2004), and Aravindhakshan (2010). More specifically, Geman and Nguyen's models include seasonality and stochastic volatility for soybean futures; Koekebakker and Lien consider jumps and seasonality along with deterministic volatility for wheat options; Aravindhakshan's models incorporate jumps for wheat futures. In this paper we study the pricing and hedging effectiveness of a new comprehensive model for futures and options for three major agricultural commodities: corn, soybeans, and wheat. The model features a stochastic jump component in the deseasonalized spot price, seasonality and stochastic cost of carry with term structure consistent with the Samuelson Hypothesis, in addition to stochastic volatility. We analyze the pricing and hedging errors of the stochastic volatility (SV) and stochastic volatility with jumps (SVJ) models in order to compare the effectiveness of the jump term at modeling the stochastic behavior of the agricultural commodities.

We use a Markov Chain Monte Carlo (MCMC) method of parameter estimation as opposed to the Kalman filtering scheme used by Trolle and Schwartz (2009) for commodity options pricing and by Bakshi, Carr and Wu (2008) for financial options pricing. The sophistication of our model (with sixteen different parameters and six state variables) coupled with the need to monitor convergence of the parameter estimates necessitates the use of MCMC.[2] Karali, Power and Ishdorj (2011) utilize Bayesian MCMC for parameter estimation for their discrete time series model for the corn, soybean, and wheat futures. Our MCMC method is similar to but different from Eraker (2004), whose MCMC analysis is applied to stock options data. In this study, we need to price options and futures on the underlying commodities simultaneously. Additionally, the options pricing formulas based on our model involve a numerical solution of the Ricatti Equation that has an otherwise semi-closed-form solution for stock option prices (see Bakshi, Cao and Chen 1997; Duffie, Pan and Singleton 2000; Eraker 2004 among others). As a result, options pricing becomes more computationally intensive. We adopt a different MCMC estimation procedure offered by Damien, Wakefield and Walker (1999). Our MCMC estimation circumvents the difficulty of sampling from non-conjugate distributions that involve non-analytical options pricing procedures. Our approach is, to the best of our knowledge, new in the literature of agricultural commodity derivatives pricing.

The remainder of the paper is organized as follows: we state the general jump diffusion model and the propositions relevant to pricing futures and options; we then describe the data set and the estimation procedure, present estimation results, and analyze the in-sample and out-of-sample pricing effectiveness of the models; we also study the in-sample and out-of-sample hedging performances of the models before we conclude.

A Jump Diffusion Model for Agricultural Commodities

The model under the risk-neutral measure

We follow Gibson and Schwartz (1990), Schwartz (1997), Trolle and Schwartz (2009), and Hilliard and Reis (1999) to model the price dynamics of agricultural commodities. Our model is based on the deseasonalized spot price, X, and the cost of carry, y, hence, endogenizing the determination of futures prices. A Merton-type Poisson jump is considered in the spot price. Stochastic variance, V, is determined by a Heston-type mean-reverting square-root process with the long-run mean jointly determined by $\overline{V}$ and κ. The following system of equations are under the risk neutral measure $\mathbb{Q}$:

$$\frac{dX(t)}{X(t)} = (\delta(t) - \lambda K_J)dt + \sigma_1\sqrt{V(t)}\,dW_1(t) + J\,dN(t), \tag{1}$$

$$dy(t,T) = \mu(t,T)dt + \sigma_2(t,T)\sqrt{V(t)}\,dW_2(t), \tag{2}$$

$$dV(t) = \left(\overline{V} - \kappa V(t)\right)dt + \sigma_3\sqrt{V(t)}\,dW_3(t). \tag{3}$$

The pairwise correlation between the Wiener processes, $dW_i(t)$ for $i \in \{1,2,3\}$ are captured in the parameters ρ_{12}, ρ_{13}, and ρ_{23}, with "1" representing the deseasonalized spot price equation, "2" the cost of carry equation, and "3" the stochastic volatility. The magnitude of the

[2] An excellent general outline of methods regarding parameter estimation of stochastic volatility models is Johannes and Polson (2009). Eraker, Johannes and Polson (2003) and Eraker (2004) provide details of implementation of MCMC method.

jumps is

$$J = e^{J_x} - 1$$

with J_x being distributed $\mathrm{N}(\mu_x, \sigma_x)$. The K_j term is the jump compensator and takes the form of $e^{\mu_x + \frac{1}{2}\sigma_x^2} - 1$.

We define $F(t,T)$ as the time-t price of a futures contract maturing at the time T. Given the instantaneous cost of carry $y(t,u)$, we can then rewrite futures price as:

$$F(t,T) \equiv S(t)\exp\left\{\int_t^T y(t,u)du\right\} \tag{4}$$

with $S(t)$ being the seasonalized spot price given by

$$S(t) = X(t)e^{h(t)} \tag{5}$$

where

$$h(t) = \eta \sin\left(2\pi(t+\varphi)\right). \tag{6}$$

h introduces the spot price seasonality, a characteristic generally recognized in the commodity literature (Sorensen 2002; Richter and Sorensen 2003).

By applying Itô's lemma to Equation (4), we have the following dynamics for $F(t,T)$:

$$\begin{aligned}\frac{dF(t,T)}{F(t,T)} &= \sigma_1\sqrt{V(t)}dW_1(t) + JdN(t) + \sqrt{V(t)}\int_t^T \sigma_2(t,u)du\, dW_2(t) \\ &+ \left(\int_t^T \mu(t,u)du + \left(\frac{1}{2}\left(\int_t^T \sigma_2(t,u)du\right)^2 + \sigma_1\rho_{12}\int_t^T \sigma_2(t,u)du\right)V(t)\right)dt.\end{aligned} \tag{7}$$

(See Appendix A.1 for derivation.).

Following Trolle and Schwartz (2009), we derive the preceding item on the second line of Equation (7) as follows:

Proposition 1 *Under the risk-neutral measure,* Q*, there cannot exist any arbitrage. Therefore, the drift term in Equation (2) is given by*

$$\mu(t,T) = -\left(V(t)\sigma_2(t,T)\left(\rho_{12}\sigma_1 + \int_t^T \sigma_2(t,u)du\right)\right). \tag{8}$$

(See Appendix A.2 for derivation).

An affine model for the dynamics of the futures curve

In order to price futures, we propose a specific form for the cost of carry. We define the time-dependent coefficient of volatility in Equation (2), $\sigma_2(t,T)$, as $\alpha e^{-\gamma(T-t)}$ such that it increases as futures approach the expiration date. This feature is consistent with the Samuelson Hypothesis (see Kalev and Duong (2008)). Based on the assumed functional form for $\sigma_2(t,T)$, we derive instantaneous cost of carry as follows:

Proposition 2 *The time-t instantaneous forward cost of carry at time T, y(t,T), is given by:*

$$y(t,T) = y(0,T) + \alpha e^{-\gamma(T-t)}\chi(t) + \alpha e^{-2\gamma(T-t)}\phi(t) \tag{9}$$

with $\chi(t)$ and $\phi(t)$ following

$$d\chi(t) = \left(-\gamma\chi(t) - \left(\frac{\alpha}{\gamma} + \rho_{12}\sigma_1\right)V(t)\right)dt + \sqrt{V(t)}dW_2(t) \tag{10}$$

$$d\phi(t) = \left(-2\gamma\phi(t) + \frac{\alpha}{\gamma}V(t)\right)dt. \tag{11}$$

Proof. See Appendix A.3.

The instantaneous cost of carry term, $\delta(t)$, in Equation (1), is defined as

$$\begin{aligned} y(t,t) &= y(0,t) + \alpha e^{-\gamma(t-t)}\chi(t) + \alpha e^{-2\gamma(t-t)}\phi(t) \\ &= y(0,t) + \alpha\chi(t) + \alpha\phi(t). \end{aligned} \tag{12}$$

For $\tau = T - t$, we have the following:

$$D_\chi(\tau) = \frac{\alpha}{\gamma}\left(1 - e^{-\gamma\tau}\right) \tag{13}$$

and

$$D_\phi(\tau) = \frac{\alpha}{2\gamma}\left(1 - e^{-2\gamma\tau}\right). \tag{14}$$

From Equation (4), integrating the cost of carry term will result in

$$F(t,T) = S(t)\frac{F(0,T)}{F(0,t)}\exp\left\{D_\chi(\tau)\chi(t) + D_\phi(\tau)\phi(t)\right\}. \tag{15}$$

By taking the logarithm of Equation (15) and letting $\log S(t) \equiv s(t)$, we have:

$$\log F(t,T) = \log F(0,T) - \log F(0,t) + s(t) + D_\chi(\tau)\chi(t) + D_\phi(\tau)\phi(t). \tag{16}$$

Here, applying Itô's Lemma to $s(t) \equiv \log S(t) = \log X(t) + h(t)$ yields

$$\begin{aligned} ds(t) = &\left[y(0,t) + \alpha(\chi_t + \phi_t) - \frac{1}{2}\sigma_1^2 V_t + 2\pi\eta\cos(2\pi(t+\varphi)) + \lambda\left(1 - e^{\mu_X + \frac{1}{2}\sigma_X^2}\right)\right]dt \\ &+ \sigma_1\sqrt{V_t}\,dW_1(t) + J_x\,dN_t. \end{aligned} \tag{17}$$

Pricing options on futures contracts

We follow the approach given in Duffie, Pan, and Singleton (2000) and Trolle and Schwartz (2009) to derive the pricing formula for options on futures. Other well-known methods include Bakshi and Madan (2000) and Collin-Dufresne and Goldstein (2003).

First, we define T_0 to be the options expiration date, T_1 to be the futures expiration date, and t to be the current time. We will transform $\log(F(T_0,T_1))$ using the Laplace Transform and let

$$\Psi(u,t,T_0,T_1) = E_t^{\mathrm{Q}}\left[e^{u\log(F(T_0,T_1))}\right]. \tag{18}$$

Proposition 3 *Equation (18) has an exponential affine solution of*

$$\Psi(u,t,T_0,T_1) = e^{A(T_0-t)+B(T_0-t)V(t)+u\log(F(t,T_1))} \tag{19}$$

where $A(\tau)$ and $B(\tau)$ solve

$$\frac{dA(\tau)}{d\tau} = B(\tau)\bar{V} + \left(e^{\left(\mu_X u + \frac{1}{2}\sigma_X^2 u^2\right)} - 1\right)\lambda \tag{20}$$

$$\frac{dB(\tau)}{d\tau} = \frac{1}{2}\sigma_3^2 B(\tau)^2 + (-\kappa + u\rho_{13}\sigma_1\sigma_3)B(\tau) + \frac{1}{2}(u^2 - u)\cdot\left(\sigma_1^2 + \frac{\alpha}{\gamma}\cdot(1 + 2\rho_{12}\sigma_1)\right)$$

$$+(u^2-u)\cdot e^{-\gamma(T_1-T_0)}\cdot\frac{\alpha}{\gamma}\cdot(\rho_{12}\sigma_1-1)e^{-\gamma\tau}+\frac{1}{2}(u^2-u)\cdot e^{-2\gamma(T_1-T_0)}\cdot\frac{\alpha}{\gamma}\cdot e^{-2\gamma\tau} \tag{21}$$

with the initial conditions $A(0)=0$ and $B(0)=0$.

Proof. See Appendix A.4.

Using the solution in Proposition 3, we can calculate the price of an option with a strike price of K and an expiration date of T_0 on a futures contract expiring at time T_1 with the following proposition:

Proposition 4 *The price of a European put is given by*

$$\begin{aligned}\mathrm{P}(t,T_0,T_1,K) &= E_t^Q\left[e^{-\int_t^{T_0} r(s)ds}(K-F(T_0,T_1))1_{F(T_0,T_1)<K)}\right]\\ &= P(t,T_0)\left(KE_t^Q[1_{\log(F(T_0,T_1))<\log(K)}]-E_t^Q[e^{\log(F(T_0,T_1))}1_{\log(F(T_0,T_1))<\log(K)}]\right)\\ &= P(t,T_0)\left(KG_{0,1}(\log(K))-G_{1,1}(\log(K))\right)\end{aligned} \tag{22}$$

where $P(t,T_0)$ is the price of a risk-free financial instrument.

The price of a European call is given by

$$\mathrm{C}(t,T_0,T_1,K)=P(t,T_0)\left(G_{1,-1}(\log(K))-KG_{0,-1}(\log(K))\right). \tag{23}$$

Also,

$$G_{a,b}(y)=\frac{\Psi(a,t,T_0,T_1)}{2}-\frac{1}{\pi}\int_0^\infty\frac{Im[\Psi(a+\mathrm{i}ub,t,T_0,T_1)e^{-\mathrm{i}uy}]}{u}du \tag{24}$$

with $\mathrm{i}=\sqrt{-1}$.

Proof. See Appendix A.5.

Data and Methodology

Data

In this paper we employ daily corn, soybean and wheat futures and options data from the CME group. Since these options are of American style, we convert them into their European counterparts following Trolle and Schwartz (2009). We obtain T-Bill rates from the Federal Reserve St. Louis and use them as a proxy for risk free interest rates. We linearly interpolate over the instruments' maturities to obtain a more accurate estimate of the risk free rate. For the purpose of estimation, we choose to focus on the years from 2006 to 2010 that cover the peak of 2007-2008 financial crisis and the global food crisis. The financialization of both agricultural and non-agricultural commodities has exacerbated the interests of both the industry and the academia (Xiong and Tang 2012; Masters 2008 and the US Senate Permanent Subcommittee on Investigations 2009).

To obtain the most informative options and futures data, we apply the following filters. First, we choose the most actively traded contract month for each commodity. It is important to construct a data set that has a traded contract on each day leaving very few if any gaps. For soybean contracts, the November contract is the most actively traded. For corn and wheat, December is the most actively traded contract month. Second, we choose only those options with at least five days of maturity in order to avoid possible microstructural noise in the market. Third, we choose only those options which have positve trading volume. Finally, we choose the most near-the-money options on any given day for parameters calibration to take advantage of the greater liquidity of such options.

Table I presents the descriptive statistics for the current month futures contract and options for the years from 2006 to 2010 for the three commodities. The futures prices vary significantly across the three commodities, whereas their price volatilities are more comparable. All three futures prices show various degrees of non-normality, which motivates our consideration of jumps and stochastic volatility in the underlying commodities. We also report implied volatilities inferred from the Black (1976) model for the three comodities in Table II. It is seen that implied volatilties are comparable across the commodities, with volatilities of wheat futures being slightly higher during the sample period. Table III provides a view of options trading activities over the sample period. We find that the average daily trading volumes of the out-of-the-money (OTM) options are prodominantly greater than those of in-the-money (ITM) options. In particular, the ratios of OTM volume to ITM volume are 4.3 (call) and 3.6 (put) for corn, 5.8 (call) and 9.1 (put) for soybeans, 4.3 (call) and 5.4 (put) for wheat, respectively. Given the domininant activities of OTM options, we shall focus on them in our pricing analysis.

Markov Chain Monte Carlo

Estimation of parameters and state variables is determined by applying the Markov Chain Monte Carlo (MCMC) method to the aforementioned options and futures data. We Euler-discretize the relevant stochastic differential equations, including Equations (3), (10), (11) and (17). As in Eraker (2004), we assume that futures and options are priced with autocorrelated errors, according to the pricing formulas in Equations (16), (22) and (23). Specifically, the autocorrelations for options and futures pricing errors that are defined as the difference between theoretical and empirical prices, are denoted as ρ_C and ρ_F.[3] To avoid spreading futures pricing errors into options pricing errors, we follow Trolle and Schwartz (2009) to price options based on the actual, not fitted futures prices. Another benefit of this approach is that we can obtain a cleaner estimate of the volatility process.

Based on the modeling framework described above, we categorize all the parameters into three groups: parameters that capture autocorrelation of options and futures pricing errors, parameters associated with options pricing and those associated with futures pricing. σ_C, ρ_C, ρ_F and σ_F are the first group. $\overline{V}$, κ, σ_3, ρ_{13}, μ_x, σ_x, λ, and ρ_{12} fall into the second group. α, γ, η and φ belong to the third group. In addition to the parameters listed above, we estimate the values of the state-variables J, N, V, x, s, and ϕ. We shall now discuss our simulation procedures for each group.

We adopt different approaches to sample the three groups of parameters. For the first group of parameters, we use the conventional Gibbs sampling method. The positerior distributions of the second and the third groups of parameters are a product of the likelihood of options and futures pricing errors and the likelihood of each individual paramter Θ_i, taking the following

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[3]Denote $P_{Theo,t}$ and $P_{Emp,t}$ as theoretical model and empirically observed options (or futures) prices at time *t* respectively. We define pricing errors at time *t* as $\varepsilon_t = P_{Theo,t} - P_{Emp,t}$, which follow $\varepsilon_t : \mathrm{N}(\rho\varepsilon_{t-1}, \sigma^2)$. ρ, being either ρ_C or ρ_F where C and F indicate options and futures respectively, is autocorrelation between price errors of time *t* and *t-1*. σ, being either σ_C or σ_F, is standard deviation for the errors.

form:

$$p(\Theta_i \mid \Theta_{-i}, J, N, V, s, x, \varphi) * \prod_{t=1}^{N} p(\Theta_i \mid P_{Emp,t}) \tag{25}$$

where Θ_{-i} is the vector of all parameters Θ except Θ_i and the likelihood of the autocorrelated pricing errors reads:

$$p(\Theta_i \mid P_{Emp}) = exp\left\{-\frac{[(P_{Emp,t+1} - P_{Theo,t+1}(\Theta_i)) - \rho_c(P_{Emp,t} - P_{Theo,t}(\Theta_i))]^2}{2\sigma_c^2}\right\} \tag{26}$$

where "*Emp*" and "*Theo*" denote empirically observed market price and theoretical model price, respectively. The same function for futures data can be obtained by changing ρ_c and σ_c to ρ_F and σ_F respectively. Due to the complexity of the posterior distributions in Equation (25) and the computational intensity of options prices in Equation (26), the standard Gibbs sampling is not applicable to the last two groups of parameters. We implement a two-step procedure as described by Damien, Wakefield, and Walker (1999) (henceforth referred to as the "DWW" method) to circumvent such a difficulty.

In the first step, we draw a Gibbs sample for each parameter Θ_i iteratively from their conditional posteriors $p(\Theta_i \mid \Theta_{-i}, J, N, V, s, x, \phi)$. Detailed explanations of this sampling method are found in the finance literature (Eraker, Johannes and Polson, 2003, Eraker, 2004 and Johannes and Polson, 2009). Specifically, we follow closely Eraker's (2004) method for $\overline{V}$, κ, σ_3, μ_x, σ_x, and λ.[4] We implement a slightly different sampling schemes on the two parameters ρ_{12} and ρ_{13} in order to achieve a better convergence rate. First, we compute the sample correlation. We then draw a sample from a normal distribution with a Fisher-transformation of sample correlation as the mean and $\sqrt{N-3}$ as the standard deviation (Fisher, 1915, 1921), where N is the number of observations. Lastly, we make an inverse Fisher-transformation to obtain the sample for ρ's.

In the second step, we compare the likelihood based on the new draw from the first step to the likelihood of the sample from the previous draw. A random uniform value is generated between 0 and the likelihood of options or futures data from the previous draw. The DWW procedure recommends that the new draw is accepted if the new likelihood value is greater than the previous likelihood value based on Equation (25) and rejected otherwise (see Section 2 of Damien, Wakefield, and Walker (1999) for detailed explanations). The DWW procedure essentially decouples from the full conditional posterior the complexity of the likelihood of options and futures prices related parameters. Also, it speeds up the MCMC sampling and convergence.

Lastly, we estimate the values of the state variables using the Metropolis-Hastings random walk sampling procedure. A sample of each state variable S_i is drawn from their conditional posteriors $p(S_i \mid \Theta, S_{-i}, P_{Emp})$, where S_{-i} is the vector of all parameters S except S_i and P_{Emp} is the empirically observed price of options or futures.

Model Diagnostics

[4] Interested readers can refer to Section C of Part II of his paper.

The effectiveness of the SVJ model over the SV model will be tested using the Bayes factor, the deviance information criteria (DIC), and the modified Diebold-Mariano (MDM) test.

The Bayes factor is an odds ratio between the SV and SVJ models. It is calculated as

$$odds(sv:svj) = \frac{B(\alpha_0,\beta_0)}{B(\alpha_0,T+\beta_0)}\frac{1}{G}\sum_{g=1}^{G}\frac{B(\alpha_0+\sum_{t=0}^{T}J_t^g,\beta_0+2T-\sum_{t=0}^{T}J_t^g)}{B(\alpha_0+\sum_{t=1}^{T}J_t^g,\beta_0+T-\sum_{t=1}^{T}J_t^g)} \tag{27}$$

with G being the total number of iterations, $\mathrm{B}(\alpha,\beta)$ being the beta function, J_t^g the size of the t^{th} jump on the g^{th} iteration, and T the total number of data points. α_0 and β_0 are the priors used in the MCMC iterations for estimating the value of λ.

DIC is calculated as the difference in twice the mean of the deviance and the deviance of the mean. That is, for N iterations, letting $\theta^* = \sum_{i=1}^{N}\theta_i/N$, we have

$$DIC = 2\frac{\sum_{i=1}^{N}D(\theta_i)}{N} - D(\theta^*) \tag{28}$$

Another way to compare the SV and SVJ models is through the MDM test proposed by Harvey, Leybourne, and Newbold (1997). The test statistic is calculated by:

$$MDM = \sqrt{\frac{(T-1)}{\frac{1}{T}\sum_{t=1}^{T}(d_t/\bar{d}-1)^2}} \tag{29}$$

where d_t is the t^{th} difference in the errors for the SV and SVJ models and $\bar{d}$ is the average of the d_t's. The advantage of the MDM test over the traditional Diebold-Mariano (DM) test is that the former result is robust to autocorrelation and non-normality in the errors. We will employ both the absolute dollar errors and the absolute percentage errors to calculate d_t for both the SV and SVJ models.

Results

We first present the parameter estimates from the MCMC iterative process. We then turn to some model diagnostics in order to differentiate between the SV and SVJ models. We employ three different testing schemes for the two models and compare the outcomes. We then analyze both the in-sample and out-of-sample errors for the two models using the parameter estimates over the three products for both puts and calls. We end with an analysis of the hedging performance.

Parameter Estimates and Analysis

Tables IV, V and VI report the parameter estimates for the three products for both option types and models over the sample period from the beginning of 2006 to the end of 2010. We generate 10,000 samples, the first 5000 of which are discarded as the “burn-in” samples. For each set of values the first number is the posterior mean of the parameter estimate for the last 5,000 runs of the algorithm. The posterior standard deviation in parentheses follows the mean and finally the 95% confidence interval. In the following, we present the parameter estimates based on the five model characteristics: auto-correlated pricing errors, stochastic volatility, stochastic cost of carry, seasonality, and jump. We generally interpret the parameter estimates based on the SVJ model for each product given the better performance of the SVJ model as made clear in the subsequent sections.

Both options and futures exhibit varying degrees of statistically significant and positive autocorrelations in pricing errors. A general observation is that the autocorrelation is higher for futures than options. One interesting note is that there is asymmetry in pricing errors across commodities and option types. Namely, the corn and wheat have higher errors for the puts than the calls, while the opposite holds true for soybeans.

The three parameters for stochastic volatility are the speed of mean reversion, κ , the (scaled by κ) long run mean, $\overline{V}$ and the volatility of volatility, σ_3 . All three parameters are statistically significant at the 0.05 level for the three products, indicating the presence of mean reversion in the latent volatility process. As for κ , we convert it into the half time to gain insight into how fast the volatility reverts to its long-run mean. The average half-times of calls and puts are 292, 90, 134 days for corn, soybeans and wheat, respectively. In particular, soybean volatility shows a half time ranging from 36 to 127 days, which exceeds the value (33 days) found in Geman and Nguyen (2005). Two reasons contribute to this difference: soybean futures prices are more volatile (a standard deviation of 2.27) in our sample than in Geman and Nguyen's sample (a standard deviation of 0.94 for the most volatile contract during the 1993-1999 period) ; it takes longer time to settle back to the long run mean during a more volatile period than otherwise. The long run means of volatility (square root of $\overline{V} / \kappa$) for corn, soybeans and wheat are 42.44%, 30.03%, and 38.25%, respectively. The levels are relatively high, yet consistent with the volatile nature of the sample period. Last, the volatility of volatility (σ_3) estimates are significant for all products, lending evidence for stochastic, instead of deterministic volatility.

We find an insignificant correlation between the spot price and the cost of carry (ρ_{12}) for all three products. However, the correlation between the spot price and the volatility process is mostly positive and statistically significant. In particular, our results for soybeans are consistent with the findings in Geman and Nguyen (2005) and Richter and Sorensen (2003). The intuition behind such results is that the price increase exacerbates the uncertainty in the agricultural markets.

We examine the term structure of the cost of carry through the parameters α and γ, for which we find statistically significant and positive values for all products. As the variance of cost of carry, σ_2 , is a decreasing function of time to maturity, positive α and γ provide direct evidence for the Samuelson hypothesis: the variability of futures prices increases as the futures contract moves toward its expiration date. The result is in alignment with Kalev and Duong (2008).

The seasonal nature of the spot price is modeled using a periodic function composed of η and φ . The former quantifies the magnitude of the seasonality while the latter measures the periodicity. All of the estimates for η are significant. This result is consistent with Sorensen (2002) and Richter and Sorensen (2003) which both provide strong evidence of spot price seasonality in corn, soybean, and wheat markets. In a related note, Pereira, Ribeiro, and Securato (2012) provide evidence of seasonality in the spot price return for the Brazilian sugar market.

The values of the jump intensity, λ , vary across the products. There are around 13 (based on call options) and 5 (based on put options) jumps a year on average in corn futures during the sample period. The negative value of μ_x shows that the size of downward jumps is larger than that of upward jumps on average. More frequent jumps are seen in the soybean futures: 16 for puts and 100 for calls. The latter is positive jumps dominating the former. This finding is largely consistent with Hilliard and Reis (1999) who find 40 positive jumps annually in the soybean market. Like for soybean futures, we find relatively more frequent jumps: 11 for calls and 116 for puts. Koekkebakker and Lien (2004) report statistically significant, but rarer (less than 10), jumps for wheat futures and options based on the relatively tranquil period of 1989 through 1994.

To sum up, we find it necessary to include autocorrelation in pricing errors for both futures and options. Our MCMC-based parameter estimates are generally consistent among all three commodities. A varying degree of mean reversion is seen in all models and products. The long-run means of the latent volatility estimated from options data is high and consistent with the high standard deviations of futures prices during the sample period. Clear evidence is found for the Samuelson effect and seasonality. Jumps are observed across all three commodities during the sample period.

Parameter Comparison between Ag and Non-Ag Models

We now relate our findings to other non-ag markets, including both commodities and financial assets, to illuminate some of the similarities and differences among them. In the interest of space, we focus our comparison on two most methodologically related papers, namely Trolle and Schwartz (2009) and Eraker (2004).

Regarding non-ag commodities, we find more similarities than differences relative to Trolle and Schwartz's (2009) findings for the NYMEX crude oil futures and options. They provide strong evidence for the presence of stochastic mean-reverting volatility and stochastic time-dependent cost of carry in energy commodities, as we do for agricultural commodities. The difference lies in the speed of mean reversion in stochastic volatility: NYMEX crude oil volatility exhibits (around 4 times) faster mean reversion than do volatilities of corn, soybeans and wheat. Our parameter estimate for ρ_{12} is consistent with Trolle and Schwartz (2009) which report a negative correlation for the spot price and cost of carry. They also report positive significant values for α and γ. This result also lends support for the Samuelson hypothesis in the crude oil market. For the correlation parameters, they found consistently negative correlation among all processes. They find moderately persistent to very highly mean reverting behavior in the volatility process. We general find persistent volatility in our runs with most half-times taking more than 90 days.

For a comparison with the financial securities markets, Eraker (2004) documents significant mean reverting behavior of stochastic volatility in the S&P 500 index, which has been shown in the previous section for the three grain commodities. Another well-known feature for stock returns is the leverage effect, negative correlation between returns and latent volatility. It contrasts the positive correlation in agricultural commodities. For the jump component, Eraker reports much fewer jumps in a year in that his estimates indicate less than one jump per year. His results also consistently point to negative jumps in the security price. The differences in the parameter values can be partially attributed to the time horizons over which the data was taken. Eraker uses S&P 500 data from January 1st, 1987 to December 31st, 1990. This sample includes the notorious "Black Monday" of October 19th, 1987. It is unclear how the estimates would change if this one observation were taken out. Our data covers a time period of drastic change in price, although not as dramatic as for the Black Monday. Another factor that contributes to the difference in jumps between financial and agricultural commodity markets is that the uncertainty in grain products often corresponds to an upward jump in the grain prices, whereas the opposite is true for financial assets. However, the exact reason for the difference between financial and agricultural markets warrants further study.

Model Diagnostic Results

We present three diagnostics to evaluate the fit of the SV and SVJ models to the observed futures and options data, namely the Bayes factor, the DIC and the MDM test statistic as detailed in the methodology section. We discuss the three diagnostics in the following.

Table VII reports the Bayes factor for all six products. Using the scale originally formulated in Jeffreys (1961), we see that there is substantial evidence that the SVJ model is superior to the SV model for corn calls. The soybean calls factor number provides strong evidence in support of the SVJ model. The values for all other products provide decisive evidence that the SVJ model is superior to the SV model.

Table VIII shows the values of the DIC for both the SV and SVJ for the 2006-2010 options data for the three commodities. The deviance information criterion (DIC) measures the hierarchical structure of models. Models with more parameters get penalized ensuring that, all else being equal, the model with the fewest parameters returns a lower DIC number. The lower the DIC the better fit the model. The relatively lower DIC values for both soybean and wheat options (calls and puts) provide evidence in support of the SVJ model. The result for the corn call is ambiguous with mild support for the SV model.

Table IX presents the results of the MDM test for the three commodities. Under the "Dollar" column, MDM is calculated as the difference in the dollar errors between the SVJ and SV model. Under the "Percentage" column, MDM is calculated as the difference in the percentage errors between the SVJ and SV models. We subtract the value of the SV model from the SVJ model. A negative value shows smaller errors for the SVJ model relative to the SV model. Clearly, the MDM test provides strong evidence in favor of the SVJ model over the SV.

Pricing Error Analysis

Pricing performance is analyzed by calculating the mean and standard deviation of the difference between empirical and model prices using the absolute dollar error and the absolute percentage error. The absolute dollar error is calculated as

$$| P_{Theo,t} - P_{Emp,t} |$$

with $P_{Theo,t}$ being the theoretical model price at time t and $P_{Emp,t}$ the empirically observed price at time t. The absolute percentage error is calculated as

$$\frac{| P_{Theo,t} - P_{Emp,t} |}{P_{Emp,t}}.$$

The theoretical price for the options requires the inputs of model parameters and a vector of variances V_t. In the model analysis that follows, the "in-sample" data set is the most near-the-money options and their underlying futures that are used to run the MCMC parameter estimation. The "out-of-sample" data set is all the out-of-the-money options due to their dominant trading volumes as mentioned in the data section. We perform pricing error analysis for the at-the-money options that are used for the MCMC estimation (in-sample analysis) and for out-of-the-money options (out-of-sample analysis). We use parameter estimates reported in Tables IV - V and the average of volatility estimates V_t in the last 5000 iterations to compute the theoretical prices of in-sample and out-of-sample options.

Table X presents the average in-sample errors for the SV and SVJ models for corn, soybeans and wheat options. We emphasize the pricing errors in absolute terms in all the subsequent analysis because the signed errors may be a biased indicator if positive errors are offset by negative errors. The SVJ model produces low percentage pricing errors between 3.62% and 17.54% for calls, and between 19.88% and 23.31% for puts. The corresponding dollar amounts are all less than 17 cents. The SV model, on the other hand, generates percentage errors that are all greater than 7% and reach the level as high as 25%. The SVJ model has a lower mean for both the dollar and percentage errors than the SV model with the only exception being the soybean calls which have a slightly lower error (with statistically indistinguishable difference) in the SV than the SVJ.

Figure 1 presents the graphs of the in-sample call errors over the years 2006 to 2010 while Figure 2 presents the put in-sample errors. In the graphs of the errors, the solid line is the SVJ's errors while the dotted line is of the SV model. The graphs of the in-sample error show that most of the time the SVJ's errors are below those errors of the SV model. There are times, however, that the SVJ's errors, especially are indistinguishable from or greater than the SV’s errors. On the whole, the SVJ in-sample errors are smaller than the SV errors as evident from Table VIII.

As for the out-of-sample analysis, we price the OTM options using the NTM-based MCMC estimates and calculate the absolute dollar errors. We then separate the errors according to time to maturity and moneyness. Table XI presents the results of the out-of-sample call and put errors. For the out-of-sample put errors, we find that the SVJ dollar errors are consistently smaller than the SV dollar errors with 3 exceptions out of 30 cominations of maturity and moneyness. The same conclusion largely holds true for the out-of-sample call errors with slightly more exceptions. A further look into the overall results at the bottom panel of Table XI reveals that the average pricing errors for the SVJ model are smaller than for the SV model. Options with shorter maturities (less than 3 months) are generally priced with smaller errors by the SVJ model than by the SV model as found in stock index options (Bakshi, Cao and Chen 1997).

Hedging Performance

We implement a simple delta hedge strategy to compare the performance of SV and SVJ models. Hedging performance for calls will be analyzed using

$$\Pi_t = C_t - \Delta_C \cdot F_t$$

with

$$\Delta_C = \frac{\partial C(t,T_0,T_1,K)}{\partial F(T_0,T_1)} = e^{-\gamma(T_0-t)}\left[\frac{\partial G_{1,-1}(-\log K)}{\partial F} - \frac{K\partial G_{0,-1}(-\log K)}{\partial F}\right].$$

For puts, hedging performance will be analyzed using

$$\Pi_t = P_t + \Delta_P \cdot F_t$$

with

$$\Delta_P = \frac{\partial P(t,T_0,T,K)}{\partial F(T_0,T_1)} = e^{-\gamma(T_0-t)}\left[\frac{K\partial G_{0,1}(\log K)}{\partial F} - \frac{\partial G_{1,-1}(\log K)}{\partial F}\right].$$

Table XII presents the value of the in-sample hedging analysis for the three commodities.[5] In five out of the six cases, the SVJ model generates lower hedging errors than the SV model indicating that the former provides a more stable hedging performance. The only exception is with wheat calls, in which the average error for the SVJ model is 7 cents larger than that for the SV model. The SVJ model provides better hedging stability than does the SV model. Both pricing and hedging errors largely support the notion that the SVJ model is a better fit to the three agricultural commodities futures and options data than the SV model.

Conclusion

Stochastic volatility, jumps, price seasonality and stochastic cost of carry have been accepted as essential features of agricultural commodities futures markets. Although the four characteristics have been separately considered in agricultural commodity derivatives pricing literature, there has yet to be an attempt to model all of them collectively. We try to fill this gap by proposing a comprehensive model for pricing and hedging agricultural commodities with a focus on the major grains in the US, namely corn, soybeans and wheat.

Following Eraker (2004), we choose the Markov Chain Monte Carlo (MCMC) approach to estimate our model that includes sixteen parameters and six unobserved state variables. The Gibbs sampling method enables us to monitor the speed of convergence of each parameter. With the DWW algorithm, we further improve the sampling efficiency and speed. An alternative method commonly seen in the financial options pricing literature is maximum likelihood estimation (MLE) with Kalman filtering (Carr and Wu, 2007; Bakshi, Carr and Wu, 2008). The number of parameters and state variables therein is significantly smaller than that in our model. Fast Fourier Transform (FFT) can be applied to speed up the computation of financial options prices. We would face both challenges in the estimation of the proposed agricultural commodity model if MLE were implemented.

To evaluate the overall fitness of the SV and SVJ models, we conduct such diagnostic tests as BIC, DIC, and MDM. On the whole, these tests indicate that the SVJ model outperforms the SV model meaning that the addition of a stochastic jump term in the spot price equation is significant.

We also perform detailed pricing error analyses along with in-sample hedging error analysis for the SV and SVJ models. For pricing error analysis, we compute both the in-sample and out-of-sample errors. We find that both the in-sample and out-of-sample errors for the SVJ model are generally smaller than those for the SV model. The in-sample hedging errors for the SVJ model are of smaller magnitude than those for the SV model. This implies that the SVJ model also provides more stability than the SV model in terms of delta hedge. Both pricing and hedging errors largely support the notion that the SVJ model is a better fit to the three agricultural commodities futures and options data than the SV model.

Our findings imply that agricultural commodities prices exhibit the jump phenomenon previously found in other commodities markets and in equites. All of our diagsnotic tools indicate the inclusion of a jump term in the stochastic pricing process produces much better fits

[5] We also performed the out-of-sample delta-hedging analysis. The dollar pricing errors for both the SV and SVJ models are indistinguishable and large, which indicates the simple delta hedge is not suited for the models with stochastic volatility and/or jumps. One may consider testing more sophisticated hedging schemes, such as delta-gamma-vega hedge. To make a perfect hedging mechanism is not the purpose of the paper. We leave it for future research.

to the empircal pricing data than without the jump term. For example, when we apply our model to the pricing of in-sample corn options, the SVJ model generates about 50% the errror of the SV model. The SVJ percentage errors are lower than the SV percentage errors for soybeans and wheat as well indicating that the SVJ model is a better fit to the data across agricultural products. Therefore, when pricing agricultural commodity options, having a jump term to model the spot price process is essential.

Accuracy of options and futures pricing is relevant to agricultural businesses (buyers) and market makers/dealers (sellers). Our comprehensive SVJ model prices options better than the SV model. For instance, the at-the-money corn calls had a pricing error of 2.53 cents for the SV model and an error of 1.16 cents for the SVJ model. This is a difference of 1.37 cents. For soybean puts the SV model has a pricing error of 21.52 cents and the SVJ model has an error of 14.65 cents. This is a difference of 6.87 cents. This difference has an effect on a grain elevator's bottom line because this small difference has a magnified affect when millions of bushels are bought and sold.

Further study in the pricing of agricultural futures and options could include yet a second jump term in the volatility equation. This jump term could be correlated to the jump term in the spot price model analogous to the models proposed by Duffie, Pan and Singleton (2000) and Eraker (2004). In addition to this second jump term, one could study the correlation between options with the same expiry but different strikes. What we have found in this paper is that the inclusion of a jump term in the spot price, in addition to modeling the seasonality, stochastic volatility, and term structure of the cost of carry, is necessary in order to realistically model the futures price. Also, a further study could add in a seasonality component for the latent volatility process.

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Appendix

A Proofs

A.1 Derivation of $\dfrac{dF(t,T)}{F(t,T)}$.

$$
\begin{aligned}
& dF(t,T) = dF(t, X(t), Y(t,T)) \\
& = F_t dt + F_X dX + F_Y dY + \frac{1}{2} F_{tt}(dt)^2 + \frac{1}{2} F_{XX}(dX)^2 \\
& + \frac{1}{2} F_{YY}(dY)^2 + F_{tX} dtdX + F_{tY} dtdY + F_{XY} dXdY \\
& + \Delta F(t,T) dN(t)
\end{aligned}
$$

Because

$$
(dt)^2 = dtdX(t) = dtdY = F_{XX} = 0
$$

and

$$
\begin{aligned}
& \Delta F(t,T) = (X_{t-} + J * X_{t-}) e^{h(t)+Y(t,T)} - X_{t-} e^{h(t)+Y(t,T)} \\
& = J * X_{t-} e^{h(t)+Y(t,T)}
\end{aligned}
$$

we get the following:

$$
\begin{aligned}
& dF(t,T) = dF(t, X(t), Y(t,T)) \\
& = F_t dt + F_X dX + F_Y dY + \frac{1}{2} F_{YY}(dY)^2 + F_{XY} dXdY \\
& + \left[(X_{t-} + JX_{t-}) e^{h(t)+Y(t,T)} - X_{t-} e^{h(t)+Y(t,T)} \right] dN(t) \\
& = F_t dt + F_X dX + F_Y dY + \frac{1}{2} F_{YY}(dY)^2 + F_{XY} dXdY \\
& + X_{t-} e^{h(t)+Y(t,T)} J dN(t)
\end{aligned}
$$

Now divide $dF(t,T)$ by $F(t,T) = X(t) e^{h(t)+Y(t,T)}$ to get the following:

$$
\begin{aligned}
& \frac{dF(t,T)}{F(t,T)} = \frac{\left(dX(t) e^{h(t)+Y(t,T)} + X(t) e^{h(t)+Y(t,T)} (h'(t) dt + dY) \right) dt}{X(t) e^{h(t)+Y(t,T)}} \\
& + \frac{e^{h(t)+Y(t,T)} dX(t)}{X(t) e^{h(t)+Y(t,T)}} + \frac{X(t) e^{h(t)+Y(t,T)} dY}{X(t) e^{h(t)+Y(t,T)}} \\
& + \frac{1}{2} \frac{X(t) e^{h(t)+Y(t,T)} (dY)^2}{X(t) e^{h(t)+Y(t,T)}} + \frac{e^{h(t)+Y(t,T)} dXdY}{X(t) e^{h(t)+Y(t,T)}} \\
& + \frac{X_{t-} e^{h(t)+Y(t,T)} J_1^{(i)} dN(t)}{X(t) e^{h(t)+Y(t,T)}}
\end{aligned}
$$

$$= \frac{dX(t)}{X(t)}dt + \frac{dX(t)}{X(t)} + dY + \frac{1}{2}(dY)^2 + \frac{dX(t)}{X(t)}dY$$
$$+ JdN(t)$$

Because

$$\frac{dX(t)}{X(t)}dt = 0,$$
$$\frac{dX(t)}{X(t)} = \delta(t)dt + \sigma_1\sqrt{V(t)}dW_1(t) + JdN(t),$$
$$dY(t,T) = \left(-\delta(t) + \int_t^T \mu(t,u)du\right)dt + \sqrt{V(t)}\int_t^T \sigma_2(t,u)du\ dW_2(t),$$
$$(dY)^2 = \left(\sqrt{V(t)}\int_t^T \sigma_2(t,u)du\ dW_2(t)\right)^2,$$

and

$$\frac{dX^c(t)}{X(t)}dY = \sigma_1 V(t)dW_1(t)\int_t^T \sigma_2(t,u)du\ dW_2(t)$$

we will now have:

$$\frac{dF(t,T)}{F(t,T)} = \delta(t)dt + \sigma_1\sqrt{V(t)}dW_1$$
$$+\left(-\delta(t) + \int_t^T \mu(t,u)du\right)dt + \sqrt{V(t)}\int_t^T \sigma_2(t,u)du dW_2(t)$$
$$+\frac{1}{2}\left(\sqrt{V(t)}\int_t^T \sigma_2(t,u)du\ dW_2(t)\right)^2$$
$$+\sigma_1 V(t)dW_1(t)dW_2(t)\int_t^T \sigma_2(t,u)du$$
$$+ JdN(t)$$

This then becomes:

$$\frac{dF(t,T)}{F(t,T)} = \delta(t)dt + \sigma_1\sqrt{V(t)}dW_1(t)$$
$$-\delta(t)dt + \int_t^T \mu(t,u)du \cdot dt + \sqrt{V(t)}\int_t^T \sigma_2(t,u)du\ dW_2(t)$$
$$+\frac{V(t)}{2}\left(\int_t^T \sigma_2(t,u)du\right)^2 dt$$
$$+\sigma_1 V(t)\rho_{12}dt\int_t^T \sigma_2(t,u)du$$
$$+ JdN(t)$$

Finally,

$$\frac{dF(t,T)}{F(t,T)} = \sigma_1\sqrt{V(t)}dW_1(t) + JdN(t) + \sqrt{V(t)}\int_t^T \sigma_2(t,u)du\ dW_2(t)$$

$$+\left(\int_t^T \mu(t,u)du + \left(\frac{1}{2}\left(\int_t^T \sigma_2(t,u)du\right)^2 + \sigma_1\rho_{12}\int_t^T \sigma_2(t,u)du\right)V(t)\right)dt.$$

A.2 Proof of Proposition 1.

Proposition 1 *Under the risk-neutral measure,* $\mathbb{Q}$*, there cannot exist any arbitrage. Therefore, the drift term in Equation (2) is given by*

$$\mu(t,T) = -\left(V(t)\sigma_2(t,T)\left(\rho_{12}\sigma_1 + \int_t^T \sigma_2(t,u)du\right)\right).$$

The drift term in Itô's Lemma is all terms with an associated dt. Therefore,

$$\left(\int_t^T \mu(t,u)du + \left(\frac{1}{2}\left(\int_t^T \sigma_2(t,u)du\right)^2 + \rho_{12}\sigma_1\int_t^T \sigma_2(t,u)du\right)V(t)\right)dt$$

is the drift term. Under the assumption of no arbitrage, we assume the drift term is 0. Then, we solve for $\mu(t,T)$ as follows:

$$0 = \left(\int_t^T \mu(t,u)du + \left(\frac{1}{2}\left(\int_t^T \sigma_2(t,u)du\right)^2 + \rho_{12}\sigma_1\int_t^T \sigma_2(t,u)du\right)V(t)\right)dt$$

$$\int_t^T \mu(t,u)du = -V(t)\left(\frac{1}{2}\left(\int_t^T \sigma_2(t,u)du\right)^2 + \rho_{12}\sigma_1\int_t^T \sigma_2(t,u)du\right).$$

Now we take the derivative with respect to T (via F.T.C.) to get the following:

$$\frac{d}{dT}\int_t^T \mu(t,u)du = -V(t)\frac{d}{dT}\left(\frac{1}{2}\left(\int_t^T \sigma_2(t,u)du\right)^2 + \rho_{12}\sigma_1\int_t^T \sigma_2(t,u)du\right)$$

$$\mu(t,T) = -V(t)\left(\left(\int_t^T \sigma_2(t,u)du\right)\sigma_2(t,T) + \rho_{12}\sigma_1\sigma_2(t,T)\right)$$

$$= -\left(V(t)\sigma_2(t,T)\left(\rho_{12}\sigma_1 + \int_t^T \sigma_2(t,u)du\right)\right)$$

A.3 Proof of Proposition 2.

We have,

$$\int_0^t y(u,T)du = y(t,T) - y(0,T)$$

and we get the following:

$$y(t,T) = \int_0^t \left(\mu(s,T)ds + \sigma_2(s,T)\sqrt{V(s)}dW_2(s)\right) + y(0,T)$$

$$= \int_0^t \mu(s,T)ds + \int_0^t \sigma_2(s,T)\sqrt{V(s)}dW_2(s) + y(0,T)$$

$$= \int_0^t \left[-\left(V(s)\alpha e^{-\gamma(T-s)}\left(\rho_{12}\sigma_1 + \frac{\alpha}{\gamma}\left[1 - e^{-\gamma(T-s)}\right]\right)\right)\right]ds$$

$$+\int_0^t \alpha e^{-\gamma(T-s)}\sqrt{V(s)}dW_2(s) + y(0,T)$$

$$= -\alpha\int_0^t V(s)\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)e^{-\gamma(T-s)}ds$$
$$+\alpha\int_0^t e^{-\gamma(T-s)}\sqrt{V(s)}dW_2(s)$$
$$+\alpha\int_0^t V(s)\frac{\alpha}{\gamma}e^{-2\gamma(T-s)}ds + y(0,T)$$

Now, $T > s$ implies that $e^{-\gamma(T-s)} = e^{-\gamma(T-t)}e^{-\gamma(t-s)}$ for some t with $T > t > s$. Then we have the following:

$$y(t,T) = -\alpha\int_0^t V(s)\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)e^{-\gamma(T-s)}ds$$
$$+\alpha\int_0^t e^{-\gamma(T-s)}\sqrt{V(s)}dW_2(s)$$
$$+\alpha\int_0^t V(s)\frac{\alpha}{\gamma}e^{-2\gamma(T-s)}ds + y(0,T)$$
$$= e^{-\gamma(T-t)}\left(-\alpha\int_0^t V(s)\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)e^{-\gamma(t-s)}ds\right)$$
$$+e^{-\gamma(T-t)}\left(\alpha\int_0^t e^{-\gamma(t-s)}\sqrt{V(s)}dW_2(s)\right)$$
$$+e^{-\gamma(T-t)}\left(\alpha\int_0^t V(s)\frac{\alpha}{\gamma}e^{-2\gamma(t-s)}ds\right) + y(0,T)$$

Finally, we have

$$y(t,T) = y(0,T) + \alpha e^{-\gamma(T-t)}\chi(t) + \alpha e^{-2\gamma(T-t)}\phi(t)$$

where

$$\chi(t) = -\int_0^t V(s)\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)e^{-\gamma(t-s)}ds + \int_0^t e^{-\gamma(t-s)}\sqrt{V(s)}dW_2(s)$$
$$= -e^{-\gamma t}\int_0^t V(s)\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)e^{\gamma s}ds$$
$$+e^{-\gamma t}\int_0^t e^{\gamma s}\sqrt{V(s)}dW_2(s)$$

and

$$\phi(t) = \int_0^t V(s)\frac{\alpha}{\gamma}e^{-2\gamma(t-s)}ds.$$

We also have

$$d\chi(t) = \gamma e^{-\gamma t}\left[\int_0^t V(s)\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)e^{\gamma s}ds\right]$$
$$-e^{-\gamma t}V(t)\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)e^{\gamma t}dt$$

$$+e^{-\gamma t}\left[\int_0^t e^{\gamma s}\sqrt{V(s)}dW_2(s)\right]dt$$
$$+e^{-\gamma t}e^{\gamma t}\sqrt{V(t)}dW_2(t)$$
$$=\left(-\gamma\chi(t)-\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)V(s)\right)dt+\sqrt{V(t)}dW_2(t).$$

Finally, after finding out how to get $\sqrt{V(t)}dW_2(t)$ we have the following:

$$d\chi(t)=\left(-\gamma\chi(t)-\left(\frac{\alpha}{\gamma}+\rho_{12}\sigma_1\right)V(t)\right)dt+\sqrt{V(t)}dW_2(t)$$

and

$$d\phi(t)=\left(-2\gamma\phi(t)+\frac{\alpha}{\gamma}V(t)\right)dt.$$

A.4 Proof of Proposition 3.

Derivation of $\frac{d\Psi(t)}{\Psi(t)}$:

The stochastic variables of Ψ are t, V, and F. Therefore, applying Itô's Lemma to $\Psi(t,V,F)$ results in the following:

$$d\Psi(t,V,F)=\Psi_t dt+\Psi_V dV+\Psi_F dF+\frac{1}{2}\Psi_{VV}(dV)^2$$
$$+\frac{1}{2}\Psi_{FF}(dF^c)^2+\Psi_{VF}(dVdF^c)+\Delta\Psi$$

with

$$\Delta\Psi=\Psi_t+\Psi_{t-}.$$

For ease of notation, let $e^*\equiv e^{A(T_0-t)+B(T_0-t)V(t)+u\log(F(t,T_1))}$ and $\tau\equiv T_0-t$. Here T_0 and T_1 are the expiration dates for the option and futures respectively.

Then we get the following equations:

$$\Psi_t dt=e^*\cdot\left(-\frac{dA(\tau)}{d\tau}-\frac{dB(\tau)}{d\tau}V(t)\right)dt$$
$$\Psi_V dV=e^*\cdot B(\tau)dV$$
$$\Psi_F dF=e^*\cdot\frac{u}{F(t,T_1)}dF$$
$$\frac{1}{2}\Psi_{VV}(dV)^2=\frac{1}{2}e^*\cdot B(\tau)^2(dV)^2$$
$$\frac{1}{2}\Psi_{FF}(dF)^2=\frac{1}{2}e^*\cdot\left[\left(\frac{u}{F(t,T_1)}\right)^2+\frac{-u}{F(t,T_1)^2}\right](dF)^2$$

$$\Psi_{VF} dVdF = \frac{u \cdot e^{*} \cdot N(\tau) dVdF}{F}$$

Dividing by $\Psi(t,V,F)$ and then replacing with ξ, we get the following:

$$\begin{aligned}
\frac{d\Psi(t)}{\Psi(t)} &= \left(-\frac{dA(\tau)}{d\tau} - \frac{B(\tau)}{d\tau} V(t) \right) dt \\
&+ B(\tau) dV(t) + u \frac{dF(t,T_1)}{F(t,T_1)} \\
&+ \frac{1}{2} B(\tau)^2 (dV(t))^2 + \frac{1}{2}(u^2 - u) \left(\frac{dF(t,T_1)}{F(t,T_1)} \right)^2 \\
&+ B(\tau) u dV(t) \frac{dF(t,T_1)}{F(t,T_1)} + \left(e^{uJ} - 1 \right) dN.
\end{aligned}$$

Finding $\frac{1}{dt} E_t^Q \left[\frac{d\Psi(t)}{\Psi(t)} \right]$ requires knowing $E_t^Q [dV(t)]$, $E_t^Q \left[\frac{dF(t,T_1)}{F(t,T_1)} \right]$, and $E_t^Q \left[\frac{dF(t,T_1)}{F(t,T_1)}^2 \right]$.

Then $E_t^Q [dV(t)]$ is the following:

$$\begin{aligned}
E_t^Q [dV(t)] &= E_t^Q \left[(\overline{V} - \kappa V(t)) dt + \sigma_3 \sqrt{V(t)} dW_3(t) \right] \\
&= E_t^Q \left[(\overline{V} - \kappa V(t)) dt \right] + E_t^Q \left[\sigma_3 \sqrt{V(t)} dW_3(t) \right] \\
&= (\overline{V} - \kappa V(t)) dt
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{1}{dt} E_t^Q \left[\frac{d\Psi(t)}{\Psi(t)} \right] \\
&= -\frac{dA(\tau)}{d\tau} - \frac{dB(\tau)}{d\tau} V(t) + B(\tau)(\overline{V} - \kappa V(t)) + \frac{1}{2} B(\tau)^2 \sigma_2^2 V(t) \\
&+ \frac{1}{2}(u^2 - u) \left(\sigma_1^2 + D_\chi (T_1 - t)^2 + 2\rho_{12} \sigma_1 D_\chi (T_1 - t) \right) V(t) + B(\tau) u \sigma_2 V(t) \\
&= -\frac{dA(\tau)}{d\tau} + B(\tau) \overline{V} + \left(e^{\left(\mu_\chi u + \frac{1}{2} \sigma_\chi^2 u^2 \right)} - 1 \right) \lambda \\
&+ \left[-\frac{dB(\tau)}{d\tau} + B(\tau)(-\kappa) + \frac{1}{2} B(\tau)^2 \sigma_3^2 \right] V(t) \\
&+ \left[\frac{1}{2}(u^2 - u) \left(\sigma_1^2 + D_\chi (T_1 - t)^2 + 2\rho_{12} \sigma_1 D_\chi (T_1 - t) \right) \right] V(t),
\end{aligned}$$

where

$$D_x(T_1-t)=\frac{\alpha}{\gamma}\left[1-e^{-\gamma(T_1-t)}\right]$$

From this equation, we get the ordinary differential equations

$$\frac{dA(\tau)}{d\tau}=B(\tau)\overline{V}+\left(e^{\left(\mu_x u+\frac{1}{2}\sigma_x^2u^2\right)}-1\right)\lambda$$

and

$$\begin{aligned}\frac{dB(\tau)}{d\tau}&=B(\tau)(-\kappa+u\rho_{13}\sigma_1\sigma_3)+\frac{1}{2}B(\tau)^2\sigma_3^2\\&+\frac{1}{2}(u^2-u)\cdot\left(\sigma_1^2+\left[D_\chi(T_1-t)\right]^2+2\rho_{12}\sigma_1\left[D_\chi(T_1-t)\right]\right)\\&=\frac{1}{2}\sigma_3^2B(\tau)^2+(-\kappa+u\rho_{13}\sigma_1\sigma_3)B(\tau)+\frac{1}{2}(u^2-u)\sigma_1^2\\&+\frac{1}{2}(u^2-u)\cdot\left(\frac{\alpha}{\gamma}\left[1-e^{-\gamma(T_1-T_0+\tau)}\right]^2+2\rho_{12}\sigma_1\frac{\alpha}{\gamma}\left[1-e^{-\gamma(T_1-T_0+\tau)}\right]\right)\\&=\frac{1}{2}\sigma_3^2B(\tau)^2+(-\kappa+u\rho_{13}\sigma_1\sigma_3)B(\tau)\\&+\frac{1}{2}(u^2-u)\cdot\left(\sigma_1^2+\frac{\alpha}{\gamma}\cdot(1+2\rho_{12}\sigma_1)\right)\\&+(u^2-u)\cdot e^{-\gamma(T_1-T_0)}\cdot\frac{\alpha}{\gamma}\cdot(\rho_{12}\sigma_1-1)e^{-\gamma\tau}\\&+\frac{1}{2}(u^2-u)\cdot e^{-2\gamma(T_1-T_0)}\cdot\frac{\alpha}{\gamma}\cdot e^{-2\gamma\tau}\end{aligned}$$

A.5 Proof of Proposition 4.

To evaluate $G_{a,b}(y)$, note that its Fourier transform is given by

$$\begin{aligned}\mathsf{G}_{a,b}(y)&=\int_{\mathsf{R}}e^{\mathrm{i}uy}dG(a,b)(y)\\&=E_t^Q\left[e^{(a+\mathrm{i}ub)\log(F(T_0,T_1))}\right]\\&=\Psi(a+\mathrm{i}ub,t,T_0,T_1)\end{aligned}$$

where $\mathrm{i}=\sqrt{-1}$. Applying the Fourier inversion theorem, we have

$$G_{a,b}(y)=\frac{\Psi(a,t,T_0,T_1)}{2}-\frac{1}{\pi}\int_0^\infty\frac{Im[\Psi(a+\mathrm{i}ub,t,T_0,T_1)e^{-\mathrm{i}uy}]}{u}du.$$

Table I: Descriptive Statistics

This table reports the first four moments, minimum, and maximum for the futures price and at-the-money options price for corn, soybeans and wheat for the years from 2006 to 2010.

	Mean	Std. Dev	Skewness	Kurtosis	Min.	Max.
			Corn			
Futures	4.0905	1.1031	0.8271	3.6464	2.3550	7.8800
Calls	0.3713	0.2045	0.7122	2.8937	0.0399	1.3294
Puts	0.4086	0.2112	0.5658	2.5154	0.0426	1.0079
			Soybeans			
Futures	9.3809	2.2717	0.3857	2.9851	5.3850	16.3100
Calls	0.7460	0.4416	1.1454	8.2739	0.0626	4.9169
Puts	0.8439	0.4482	0.4822	2.2665	0.0676	2.1216
			Wheat			
Futures	6.1723	1.6887	0.7672	3.0542	3.6600	12.5750
Calls	0.5388	0.3497	1.8796	9.7021	0.0507	3.3243
Puts	0.6657	0.4382	1.9916	9.4397	0.0593	4.0337

Table II: Parameter Estimates for the Black Model

This table reports the parameter estimates for the Black model for corn, soybeans and wheat for the years from 2006 to 2010. The standard deviation is in parenetheses and follows the mean.

Calls	Corn	0.3177	(0.0959)
	Soybean	0.2714	(0.1072)
	Wheat	0.3268	(0.1043)
Puts	Corn	0.2934	(0.1779)
	Soybean	0.3257	(0.0776)
	Wheat	0.3721	(0.0640)

Table III: Total Option Volume

This table reports the total option volume in 1000s for the in-the-money (ITM), near-the-money (NTM), and out-of-the-money (OTM) options for the years from 2006 to 2010.

		CALLS			PUTS	
	ITM	NTM	OTM	ITM	NTM	OTM
Corn	3252	1213	13879	2740	978	9998
Soybeans	958	487	5515	435	443	3938
Wheat	523	222	2224	297	228	1606

Table IV: Parameter Estimates for Corn

This table reports means, standard deviations (in parenthesis) and 95% confidence intervals (in square brackets) of parameter estimates in the stochastic volatility (SV) and stochastic volatility with jumps (SVJ) models. Parameters are estimated by applying the MCMC method to the years from 2006 to 2010 at-the-money corn data.

	SV		SVJ	
Parameter	**Calls**	**Puts**	**Calls**	**Puts**
$\bar{V}$	0.0310	0.6313	0.0388	0.8549
	(0.0190)	(0.2110)	(0.0250)	(0.2270)
	[0.0305,0.0315]	[0.6255,0.6372]	[0.0381,0.0395]	[0.8486,0.8611]
κ	0.2980	2.1270	0.3917	3.2170
	(0.2390)	(1.2720)	(0.2250)	(1.1800)
	[0.2914,0.3046]	[2.0918,2.1623]	[0.3855,0.3979]	[3.1843,3.2498]
ρ_{12}	0.0577	0.0010	-0.0047	0.0097
	(0.5530)	(0.5860)	(0.2500)	(0.5760)
	[0.0424,0.0730]	[-0.0152,0.0173]	[-0.0117,0.0022]	[-0.0062,0.0257]
ρ_{13}	-0.0002	0.0035	-0.0007	0.0108
	(0.0300)	(0.0630)	(0.0310)	(0.1170)
	[0.0011,0.0006]	[0.0017,0.0052]	[-0.0016,0.0002]	[0.0075,0.0140]
σ_3	0.0301	1.0645	0.0311	1.3615
	(0.0010)	(0.1660)	(0.0010)	(0.2510)
	[0.0301,0.0301]	[1.0598,1.0691]	[0.0311,0.0312]	[1.3545,1.3684]
α	0.4863	0.5231	0.4946	0.5050
	(0.2870)	(0.2840)	(0.2890)	(0.2890)
	[0.4784,0.4943]	[0.5152,0.5310]	[0.4866,0.5026]	[0.4970,0.5131]
γ	7.1537	7.5510	6.9250	7.5115
	(4.3490)	(4.3480)	(4.4340)	(4.3820)
	[7.0332,7.2743]	[7.4304,7.6715]	[6.8020,7.0479]	[7.3900,7.6329]
η	0.0105	0.2120	0.0081	0.3041
	(0.0140)	(0.2110)	(0.0070)	(0.2750)
	[0.0101,0.0109]	[0.2061,0.2179]	[0.0079,0.0083]	[0.2965,0.3117]
φ	0.0771	0.0040	-0.0254	-0.0144
	(0.2930)	(0.2940)	(0.2980)	(0.2880)
	[0.0690,0.0852]	[-0.0042,0.0121]	[-0.0337,-0.0172]	[-0.0224,-0.0064]
σ_C	0.0431	0.0596	0.0412	0.0591
	(0.0010)	(0.0020)	(0.0010)	(0.0270)
	[0.0431,0.0432]	[0.0595,0.0596]	[0.0412,0.0412]	[0.0584,0.0599]
ρ_C	0.8838	0.8155	0.7967	0.7656
	(0.0210)	(0.0530)	(0.0140)	(0.0430)

	[0.8832,0.8843]	[0.8140,0.8170]	[0.7963,0.7971]	[0.7644,0.7668]
σ_F	0.0607	0.1100	0.2007	0.0953
	(0.0040)	(0.0190)	(0.0130)	(0.0300)
	[0.0606,0.0608]	[0.1095,0.1105]	[0.2004,0.2011]	[0.0945,0.0961]
ρ_F	0.9946	0.9741	0.9642	0.9916
	(0.0030)	(0.0090)	(0.0080)	(0.0060)
	[0.9945,0.9946]	[0.9739,0.9744]	[0.9640,0.9645]	[0.9914,0.9917]
μ_x			-0.1579	-0.0364
			(0.0760)	(0.1310)
			[-0.1600,-0.1558]	[-0.0401,-0.0328]
σ_x			0.6925	0.5059
			(0.0540)	(0.2030)
			[0.6910,0.6940]	[0.5003,0.5115]
λ			0.0519	0.0207
			(0.0070)	(0.0280)
			[0.0517,0.0521]	[0.0199,0.0214]

Table V: Parameter Estimates for Soybeans

This table reports means, standard deviations (in parenthesis) and 95% confidence intervals (in square brackets) of parameter estimates in the stochastic volatility (SV) and stochastic volatility with jumps (SVJ) models. Parameters are estimated by applying the MCMC method to the 2006 to 2010 at-the-money soybean data.

	SV		**SVJ**	
Parameter	**Calls**	**Puts**	**Calls**	**Puts**
$\bar{V}$	0.2688	0.1869	0.2398	0.0358
	(0.0080)	(0.0480)	(0.0150)	(0.0468)
	[0.2685,0.2690]	[0.1856,0.1882]	[0.2394,0.2403]	[0.0345,0.0371]
κ	1.9959	1.3744	1.8021	3.5420
	(0.0570)	(0.3370)	(0.1010)	(0.5938)
	[1.9943,1.9974]	[1.3651,1.3838]	[1.7993,1.8049]	[3.5255,3.5585]
ρ_{12}	-0.0128	-0.0092	-0.0111	0.0139
	(0.5770)	(0.5600)	(0.5460)	(0.5961)
	[-0.0287,0.0032]	[-0.0247,0.0063]	[-0.0262,0.0041]	[-0.0026,0.0305]
ρ_{13}	-0.0006	0.0001	-0.0012	-0.9983
	(0.0290)	(0.0300)	(0.0310)	(0.0017)
	[-0.0014,0.0002]	[-0.0008,0.0009]	[-0.0021,-0.0003]	[-0.9983,-0.9982]
σ_3	0.0295	0.0300	0.0293	0.1000
	(0.0010)	(0.0010)	(0.0010)	0.0000
	[0.0295,0.0295]	[0.0299,0.0300]	[0.0293,0.0294]	[0.1000,0.1000]
α	0.5064	0.4927	0.5160	0.4988
	(0.3000)	(0.2890)	(0.2890)	(0.2935)
	[0.4981,0.5147]	[0.4847,0.5007]	[0.5080,0.5240]	[0.4906,0.5069]
γ	7.3937	7.5040	7.7428	7.6367
	(4.3290)	(4.2970)	(4.3380)	(4.3452)
	[7.2737,7.5137]	[7.3848,7.6231]	[7.6225,7.8630]	[7.5163,7.7572]
η	0.0638	0.0093	0.0100	0.3717
	(0.1140)	(0.0110)	(0.0090)	(0.2467)
	[0.0607,0.0670]	[0.0091,0.0096]	[0.0097,0.0103]	[0.3649,0.3785]
φ	0.0252	0.0030	-0.0180	-0.0143
	(0.2980)	(0.2840)	(0.2920)	(0.2848)
	[0.0170,0.0335]	[-0.0049,0.0109]	[-0.0261,-0.0099]	[-0.0222,-0.0064]
σ_C	0.0617	0.1687	0.0629	0.1352
	(0.0030)	(0.0040)	(0.0030)	(0.3335)
	[0.0616,0.0617]	[0.1686,0.1689]	[0.0628,0.0630]	[0.1259,0.1444]
ρ_C	0.9659	0.7042	0.9679	0.8206
	(0.0080)	(0.0350)	(0.0080)	(0.0589)
	[0.9657,0.9661]	[0.7033,0.7052]	[0.9677,0.9681]	[0.8190,0.8222]
σ_F	0.1085	0.1102	0.2206	0.1113
	(0.0100)	(0.0060)	(0.0240)	(0.0133)

	[0.1082,0.1087]	[0.1101,0.1104]	[0.2199,0.2212]	[0.1110,0.1117]
ρ_F	0.9960	0.9960	0.9872	0.9955
	(0.0020)	(0.0020)	(0.0050)	(0.0027)
	[0.9960,0.9961]	[0.9959,0.9960]	[0.9870,0.9873]	[0.9955,0.9956]
μ_x			-0.2863	0.2961
			(0.0690)	(0.0726)
			[-0.2882,-0.2844]	[0.2941,0.2981]
σ_x			0.5731	1.2733
			(0.0640)	(0.0585)
			[0.5713,0.5749]	[1.2717,1.2749]
λ			0.0639	0.3891
			(0.0070)	(0.0409)
			[0.0637,0.0641]	[0.3879,0.3902]

Table VI: Parameter Estimates for Wheat

This table reports means, standard deviations (in parenthesis) and 95% confidence intervals (in square brackets) of parameter estimates in the stochastic volatility (SV) and stochastic volatility with jumps (SVJ) models. Parameters are estimated by applying the MCMC method to the 2006 to 2010 at-the-money wheat data.

	SV		**SVJ**	
Parameter	**Calls**	**Puts**	**Calls**	**Puts**
$\bar{V}$	0.3800	0.1322	0.1894	2.0988
	(0.1440)	(0.0820)	(0.0530)	(0.5990)
	[0.3760,0.3840]	[0.1299,0.1345]	[0.1879,0.1908]	[2.0822,2.1154]
κ	1.7260	0.6986	0.9524	64.8063
	(0.9620)	(0.4390)	(0.4230)	(19.1630)
	[1.6993,1.7527]	[0.6864,0.7108]	[0.9407,0.9641]	[64.2751,65.3376]
ρ_{12}	0.0222	-0.0227	-0.0043	(0.0139)
	(0.5760)	(0.5660)	(0.5680)	(0.5760)
	[0.0062,0.0382]	[-0.0384,-0.0070]	[-0.0200,0.0115]	[-0.0299,0.0021]
ρ_{13}	-0.0039	-0.0003	0.0077	-0.9970
	(0.0460)	(0.0300)	(0.0460)	(0.0090)
	[-0.0051,-0.0026]	[-0.0011,0.0005]	[0.0064,0.0090]	[-0.9973,-0.9968]
σ_3	0.9961	0.0299	1.3582	0.1000
	(0.0920)	(0.0010)	(0.0980)	0.0000
	[0.9935,0.9987]	[0.0299,0.0299]	[1.3554,1.3609]	[0.1000,0.1000]
α	0.5011	0.5129	0.4965	0.4978
	(0.2920)	(0.2880)	(0.2850)	(0.2910)
	[0.4930,0.5092]	[0.5049,0.5209]	[0.4886,0.5044]	[0.4897,0.5058]
γ	7.5949	7.4854	7.4277	7.9799
	(4.2430)	(4.3670)	(4.3020)	(4.1650)
	[7.4773,7.7126]	[7.3643,7.6065]	[7.3084,7.5469]	[7.8644,8.0953]
η	0.1277	0.0101	0.2365	0.4979
	(0.1860)	(0.0240)	(0.2450)	(0.3030)
	[0.1225,0.1328]	[0.0094,0.0108]	[0.2297,0.2433]	[0.4895,0.5063]
φ	0.0124	0.0120	0.0165	0.0801
	(0.2880)	(0.2850)	(0.2970)	(0.2930)
	[0.0044,0.0204]	[0.0041,0.0199]	[0.0083,0.0248]	[0.0719,0.0882]
σ_C	0.0934	0.3388	0.0836	0.3111
	(0.0060)	(0.0070)	(0.0850)	(0.0070)
	[0.0932,0.0936]	[0.3386,0.3390]	[0.0812,0.0860]	[0.3109,0.3113]
ρ_C	0.4542	0.2964	0.2430	0.3161
	(0.1300)	(0.0480)	(0.1200)	(0.0450)
	[0.4506,0.4578]	[0.2951,0.2978]	[0.2397,0.2463]	[0.3149,0.3174]

σ_F	0.1128	0.1494	0.3336	0.3249
	(0.0770)	(0.0170)	(0.0180)	(0.0540)
	[0.1107,0.1150]	[0.1489,0.1499]	[0.3331,0.3340]	[0.3234,0.3264]
ρ_F	0.9815	0.9895	0.9140	0.9113
	(0.0380)	(0.0050)	(0.0190)	(0.0530)
	[0.9804,0.9825]	[0.9893,0.9896]	[0.9134,0.9145]	[0.9098,0.9127]
μ_x			-0.2768	-0.1218
			(0.0890)	(0.0410)
			[-0.2793,-0.2743]	[-0.1230,-0.1207]
σ_x			0.9609	1.7434
			(0.0760)	(0.0060)
			[0.9588,0.9630]	[1.7432,1.7436]
λ			0.0643	0.4597
			(0.0080)	(0.0190)
			[0.0641,0.0645]	[0.4592,0.4602]

Table VII: Bayes Factors

This table reports the calculated Bayes factors for corn, soybeans and wheat for the years 2006 to 2010. The Bayes factors are the odds ratios of the SVJ vs. SV models. A higher Bayes factor favors the SVJ model.

	Corn	**Soybeans**	**Wheat**
Calls	4.3588	10.9091	∞
Puts	∞	∞	∞

Table VIII: Deviance Information Criterion (DIC)

This table presents the DIC for corn, soybeans and wheat for the years from 2006 to 2010. The DIC values are reported for the SV and SVJ models.

	Corn		**Soybeans**		**Wheat**	
	SV	**SVJ**	**SV**	**SVJ**	**SV**	**SVJ**
Calls	1839	1842	2738	2431	232	288
Puts	2172	1464	2548	2090	2147	999

Table IX: Modified Diebold-Mariano (MDM) Test

This table presents the results of the MDM test. The test is the difference in pricing errors between the SV and SVJ models. The negative values indicate that the SVJ model produces smaller errors than does the SV model.

	Corn		**Soybeans**		**Wheat**	
	Dollar	**Percentage**	**Dollar**	**Percentage**	**Dollar**	**Percentage**
Calls	-25.8579	-21.1318	-0.3460	-10.7981	-21.1420	-18.0441
Puts	-13.7040	-8.2437	-10.8351	-6.6331	-1.8293	-4.2181

Table X: In-Sample Dollar and Percentage Option Pricing Errors

This table presents the in-sample pricing errors for corn, soybeans and wheat options. At-the-money options from 2006 to 2010 are used for estimation. "Dollar errors" are defined as the absolute difference between the theoretical model and empirically observed option prices. The "percentage" errors are defined as the Dollar errors divided by the observed options price. The first row for each product is in absolute terms whereas the second row is with (positive/negative) signs.

	SV		**SVJ**	
	Dollar	**Percentage**	**Dollar**	**Percentage**
		Corn		
Calls	0.0253	0.07	0.0116	0.0362
	-0.0185	-0.0402	-0.0105	-0.0383
Puts	0.0812	0.2253	0.0626	0.1988
	-0.0671	-0.2236	-0.0472	-0.2417
		Soybeans		
Calls	0.1029	0.1702	0.1031	0.1754
	-0.08	-0.1484	-0.0864	-0.1574
Puts	0.2152	0.257	0.1465	0.2092
	-0.2029	-0.1996	-0.1424	-0.2124
		Wheat		
Calls	0.0544	0.1363	0.0307	0.0769
	-0.088	-0.2014	-0.0738	-0.1509
Puts	0.1713	0.2504	0.1677	0.2331
	-0.3097	-0.2307	-0.2817	-0.2181

Table XI: Out-of-Sample Dollar Errors

This table presents out-of-sample dollar errors for corn, soybeans and wheat put options based on time to maturity and moneyness. "Out-of-sample" observations are those options contracts which are not at-the-money during the sample period of 2006-2010. "#" denotes the number of options. "Dollar errors", abbreviated as "Dol", are defined as the absolute difference between the theoretical model and empirically observed option prices.

		Moneyness			
Product		**PUT**		**CALL**	
Maturity		**<0.90**	**0.90-1.0**	**1.0-1.1**	**>1.1**
< 1 m	Corn #	748	590	550	789
	SV Dol	1.02	2.51E-01	3.13E-02	1.10E-02
	SVJ Dol	1.02	2.41E-01	3.12E-02	8.92E-03
	Soybean #	551	672	610	840
	SV Dol	1.78	4.97E-01	6.34E-02	1.71E-02
	SVJ Do	1.73	4.15E-01	6.38E-02	1.54E-02
	Wheat #	403	498	512	764
	SV Dol	1.06	3.68E-01	3.45E-01	1.37E-01
	SVJ Dol	1.04	3.67E-01	3.49E-01	1.39E-01
1-2 m	Corn #	1203	677	662	1764
	SV Dol	1.08	2.36E-01	5.47E-02	2.09E-02
	SVJ Dol	1.08	2.27E-01	5.48E-02	1.70E-02
	Soybean #	734	713	650	1348
	SV Dol	2.01	4.70E-01	5.93E-02	2.49E-02
	SVJ Dol	1.91	4.00E-01	6.16E-02	2.08E-02
	Wheat #	825	574	585	1259
	SV Dol	1.17	5.85E-01	3.31E-01	1.64E-01
	SVJ Dol	1.19	5.86E-01	3.35E-01	1.69E-01
2-3 m	Corn #	982	412	431	1841
	SV Dol	1.08	2.44E-01	3.58E-02	2.13E-02
	SVJ Dol	1.07	2.25E-01	3.59E-02	1.38E-02
	Soybean #	791	464	464	1416
	SV Dol	2.23	5.02E-01	8.22E-02	4.27E-02
	SVJ Dol	2.08	4.01E-01	8.39E-02	3.56E-02
	Wheat #	825	488	482	1212
	SV Dol	1.23	6.42E-01	3.42E-01	1.52E-01
	SVJ Dol	1.23	6.28E-01	3.29E-01	1.45E-01
3-6 m	Corn #	3023	1321	1350	6196
	SV Dol	1.24	2.80E-01	5.13E-02	3.36E-02
	SVJ Dol	1.21	2.49E-01	5.06E-02	2.34E-02
	Soybean #	2979	1207	1235	3768
	SV Dol	2.73	6.21E-01	9.79E-02	6.81E-02
	SVJ Dol	2.48	3.95E-01	9.82E-02	5.81E-02

	Wheat #	2035	1052	1203	3406
	SV Dol	1.12	3.39E-01	3.04E-01	1.52E-02
	SVJ Dol	1.1	3.47E-01	2.94E-01	1.44E-02
>6 m	Corm #	6037	2340	2397	9565
	SV Dol	1.02	2.07E-01	4.60E-02	4.98E-02
	SVJ Dol	9.90E-01	1.71E-01	3.94E-02	3.46E-02
	Soybean #	4243	1751	1614	5509
	SV Dol	2.24	5.49E-01	1.60E-01	8.84E-02
	SVJ Dol	1.96	3.65E-01	1.69E-01	8.73E-02
	Wheat #	1210	521	585	2335
	SV Dol	1.58	2.32E-01	2.60E-01	1.82E-01
	SVJ Dol	1.47	2.18E-01	2.46E-01	1.95E-01
All	Corn#	11993	5342	5390	20155
	SV Dol	1.09	2.44E-01	4.38E-02	2.73E-02
	SVJ Dol	1.07	2.23E-01	4.24E-02	1.95E-02
	Soybean #	9298	4807	4573	12881
	SV Dol	2.2	5.28E-01	9.26E-02	4.82E-02
	SVJ Dol	2.03	3.95E-01	9.54E-02	4.34E-02
	Wheat #	5298	3133	3367	8976
	SV Dol	1.23	4.33E-01	3.16E-01	1.57E-01
	SVJ Dol	1.21	4.29E-01	3.11E-01	1.58E-01

Table XII: In-Sample Hedging Performance

This table presents the in-sample hedging performance of the SV and SVJ models for corn, soybeans and wheat options. The at-the-money options and futures data for the years from 2006 to 2010 are employed for parameter estimation and in-sample analysis. Values in parantheses are standard deviations.

	Corn		Soybeans		Wheat	
	SV	**SVJ**	**SV**	**SVJ**	**SV**	**SVJ**
Calls	2.233	2.221	5.012	5.001	3.426	3.4934
	(0.63)	(0.63)	(1.35)	(1.35)	(1.06)	(1.08)
Puts	-1.4417	-1.4242	-3.3816	-2.865	-1.9487	-1.7466
	(0.43)	(0.43)	(0.93)	(0.91)	(0.80)	(0.93)

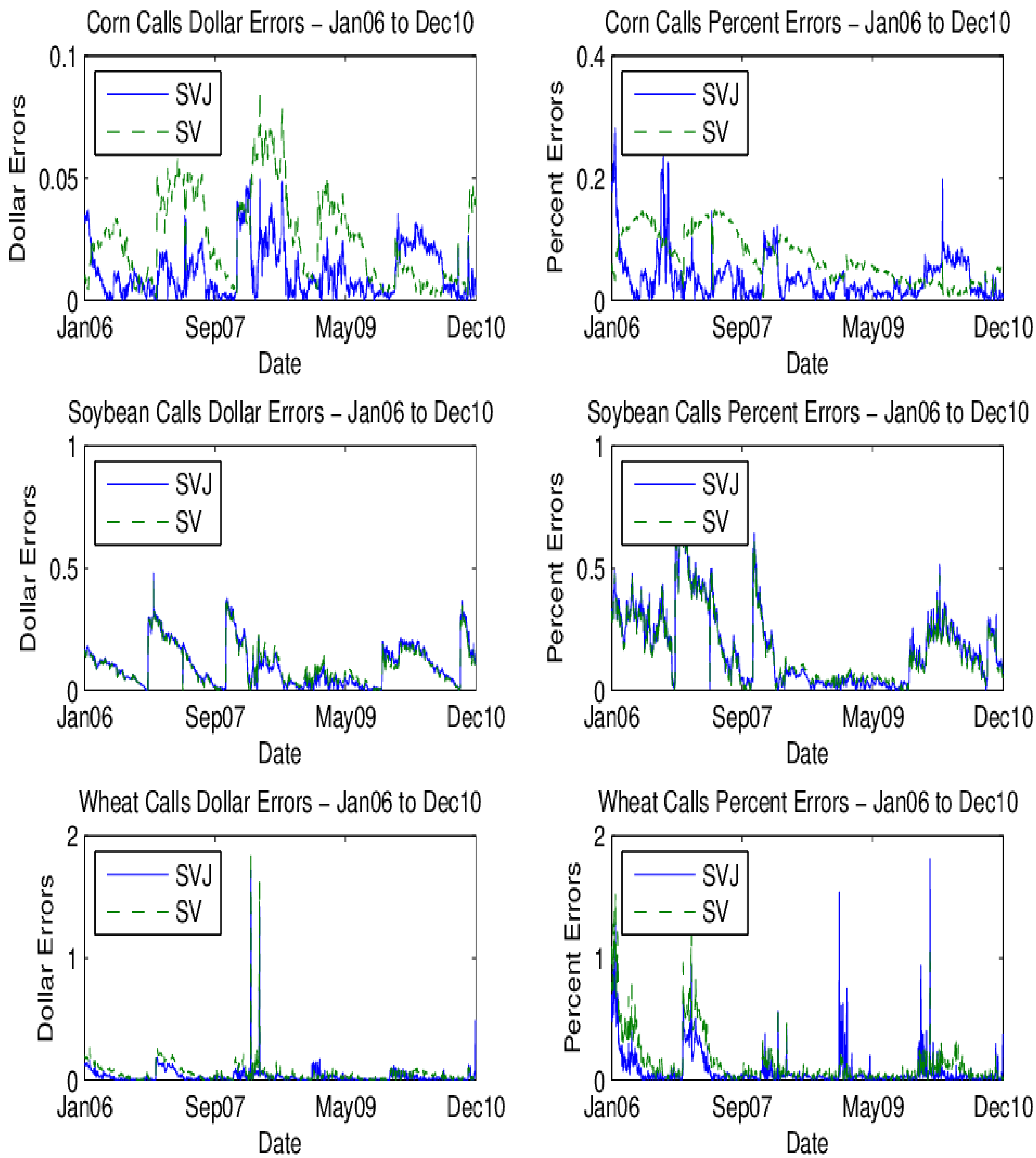

Figure 1: In-Sample SV and SVJ Calls Errors

These graphs show the values of the in-sample pricing errors for both the SV and SVJ models for corn, soybeans and wheat call options during the sample period of 2006-2010. "Dollar errors" are the absolute difference between the theoretical and empirically observed options prices. "Percent errors" are the dollar errors divided by the observed options prices.

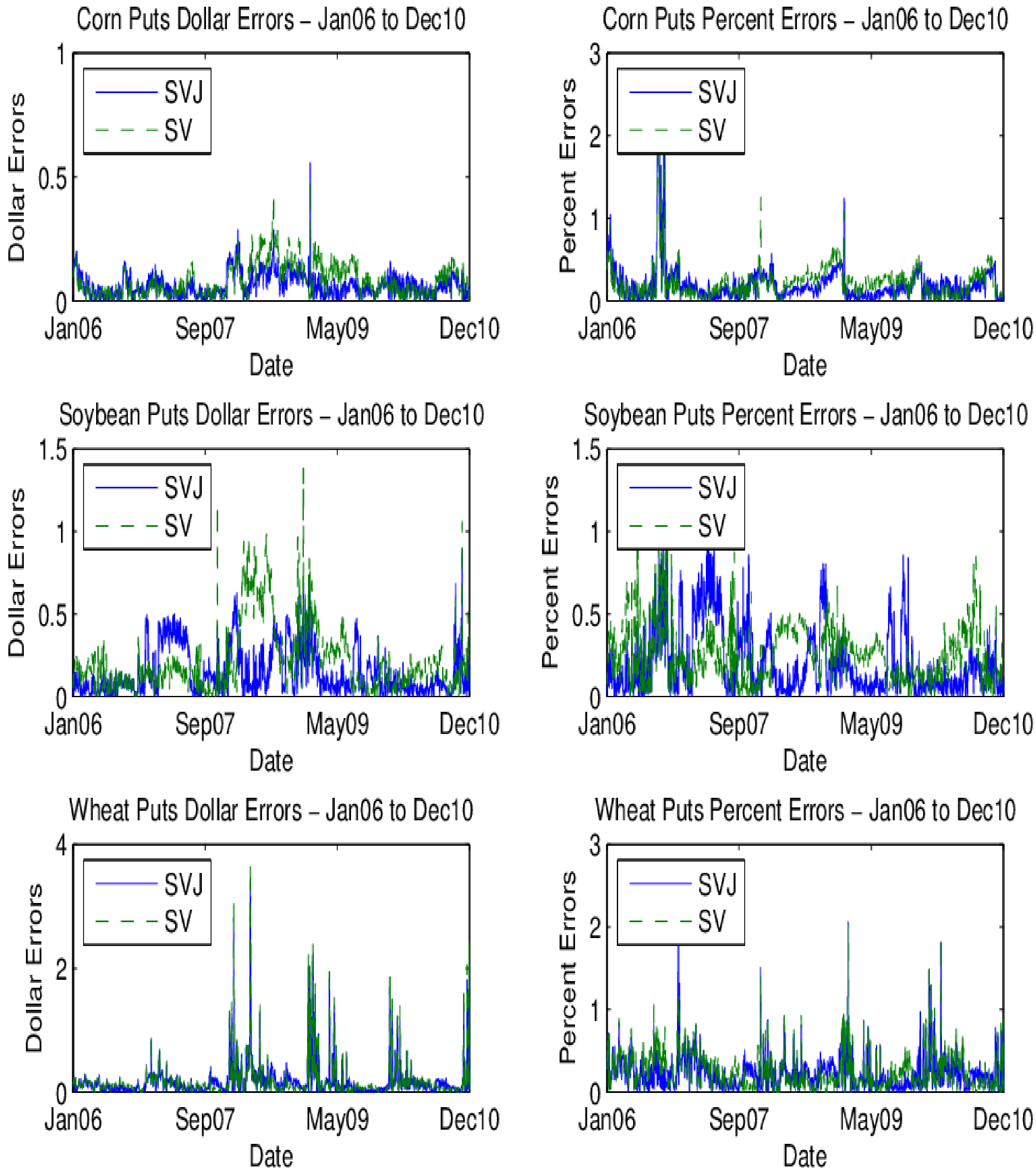

Figure 2: In-Sample SV and SVJ Puts Errors

These graphs show the in-sample pricing errors for the SV and SVJ models for corn, soybeans and wheat put options during the sample period of 2006-2010. "Dollar errors" are the absolute difference between the theoretical and empirically observed options prices. "Percent errors" are the dollar errors divided by the observed options prices.