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Suggested citation format:

Trujillo-Barrera, A., P. Garcia, and M. Mallory. 2012. "Density Forecasts of Lean Hog Futures Prices." Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. St. Louis, MO. [http://www.farmdoc.illinois.edu/nccc134].

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Paper presented at the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management St. Louis, Missouri, April 16-17, 2012

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## **Density Forecasts of Lean Hog Futures Prices**

#### Abstract

High price variability in agricultural commodities increases the importance of accurate forecasts. Density forecasts estimate the future probability distribution of a random variable, offering a complete description of risk. In this paper we investigate density forecast of lean hog prices for the 2002-2012 period for two weeks horizons. We estimate historical densities using GARCH models with different error distributions and generate forward looking implied distributions, obtaining risk-neutral densities from the information contained in options prices. Real-world densities, which incorporate risk, are obtained by parametric and non parametric calibration of the risk-neutral densities. Then the predictive accuracy of the forecasts is evaluated and compared. Goodness of fit and out of sample log-likelihood comparisons indicate that real-world densities outperform risk-neutral and historical densities, suggesting the presence of risk premiums in the lean hog markets. For the historical density forecasts, GED error distributions for the GARCH estimations show an adequate predictive accuracy. Meanwhile, historical densities with normal and t-distributions show a discrete performance.

Keywords: Density Forecast, Lean Hog prices, Options, Futures Prices.

### Introduction

Increasing price variability in agricultural markets and the introduction of new risk management instruments such as Volatility Index (VIX) contracts that allow investors to buy and sell volatility like any other asset heighten the importance of developing accurate forecasting techniques. Isengildina, Irwin, and Good (2004) argue that volatility of agricultural prices causes many individuals to rely on forecasts in their decision making and that the value of agricultural forecasts is substantial. Adam, Garcia, and Hauser (1996) also demonstrate the value of improved agricultural forecasts of the mean and variance in the presence of futures and options for the live hog contract.

However, traditional forecasting procedures based on a mean-variance framework may not fully characterize the nature of risk in agricultural markets. There is evidence that agricultural prices and returns exhibit non-Gaussian and non-linearity properties. Further the preferences of agents in these markets are unlikely to be quadratic (Deaton and Laroque, 1992; Myers and Hanson, 1993; Koekebakker and Lien, 2004; Peterson and Tomek, 2005). In this context, by estimating the future conditional probability distribution of prices, density forecasts offer a thorough description of future uncertainty, providing decision makers with more information than point forecasts of expected returns and volatilities.(Tay and Wallis, 2000; Timmermann, 2000).

Moreover, recent studies by Wang, Fausti, and Qasmi (2012) and Wilson and Dahl (2009) identify that the increased commodity price volatility has considerable implications on production, marketing and risk management practices. Higher price volatility reduces the effectiveness of traditional risk managerial tools which may not be able to capture variance and tail risk directly. As a consequence, instruments from the financial markets such as volatility index options and futures, which are used to trade and hedge short term market volatility, are being implemented in the agricultural markets. For instance, the CME introduced VIX (volatility index) contracts for corn and soybeans in 2011. Wang, Fausti, and Qasmi (2012) claim that these kinds of products will enhance market participants' ability to accurately gauge price risk and manage volatility risk. Accurately pricing these instruments require knowledge of higher moments of the price distributions, therefore density price forecasting may provide important insights in the analysis, management, and pricing of these new tools.

Density forecast estimation techniques are not new, but it was not until the 1990s that significant interest in the economics literature began to emerge. Applications to macroeconomic forecasting by central banks, the development of Value at Risk measures for financial institutions, and the increasing computational power stimulated their use. Furthermore, pioneering work by Diebold, Gunther, and Tay (1998) promoted the development of density forecasting evaluation, which has been a fast growing area of research with widespread applications in econometrics, asset pricing, and portfolio selection (Amisano and Giacomini, 2007; Gneiting, 2008).

The importance of density forecasts for agricultural commodity prices was identified as early as Bottum (1966) and Timm (1966), who recommended that probabilistic outlook forecasts be developed in the manner of weather forecasts. Yet, the use of density forecasts for agricultural commodity prices is relatively scarce. Some papers have looked at estimation procedures (i.e. (Sherrick, Garcia, and Tirupattur, 1996; Silva and Kahl, 1993)), but there is a lack of applications in the areas of density forecast evaluation, comparison, and combination, although price volatility forecasting has been an active area of research.

In this setting, the paper has two objectives, to estimate forecast densities for lean hog futures prices using several alternative procedures, and to assess their predictive power using recently developed evaluation and comparison measures. To generate the density forecasts we use two general procedures: one is based on historical data using GARCH models, and the second is a forward-looking procedure based on the information content of options prices which provides risk-neutral and risk adjusted densities. To evaluate the forecast performance, we use the probability integral transforms (*PIT*) adopted by Diebold, Gunther, and Tay (1998), and the Berkowitz test introduced by Berkowitz (2001). For model comparison we use the out-of-sample log likelihood based on the Kullback-Leiber information criterion as suggested by Bao, Lee, and Saltoglu (2007). The analysis is performed with a two week forecasting horizon using daily settlement futures prices and a set of options prices of lean hogs from December 1996 to February 2012. The starting date of analysis corresponds to the switch in futures and options contracts from live to lean hog contracts, and from physical delivery to cash settlement.

We focus on the hog market because considerable predictive performance research already exists, often comparing econometric procedures to market generated forecasts. For instance, the reliability

of hog futures prices to accurately reflect subsequent cash prices has been a traditional area of market research. More recently researchers have begun to investigate the degree to which the implied volatilities from the hog options reflect subsequent realized volatility.

While the recent evidence is mixed, the empirical findings using monthly and bimonthly observations (e.g., two and four months) suggest futures prices provide long-run unbiased forecasts, but that short-run inefficiencies in forecasting may exist (McKenzie and Holt, 2002; Carter and Mohapatra, 2008; Frank and Garcia, 2009). In terms of the options market, Szakmary et al. (2003) and Egelkraut and Garcia (2006) identify biases in implied forward volatility forecasts of subsequent realized volatility. Historical volatilities also add information to the market generated implied volatilities in predicting realized volatility, implying options prices do not contain all available information or may not account adequately for risk.

Similarly, McKenzie, Thomsen, and Phelan (2007) show that long hog straddle positions exited on Hogs and Pigs Report days are profitable if transaction costs are under certain levels. However, Urcola and Irwin (2010) analyze market efficiency of lean hog options contract looking at several trading strategies such as options straddles and strangles. They find that returns on options are often small, and even large returns are not statistically significant. They conclude that returns are not sufficiently large enough to allow for consistent speculative profits for off-floor traders. Hence the bulk of the evidence suggests that short-term biases in market prices and their volatilities are likely to exist, but that developing selective strategies to take advantage of them may indeed prove challenging for market participants.

In this context, short horizon density forecasts may offer useful information to decision makers by providing them insights into the presence of added volatility, skewness, and kurtosis. Such information could play an important role in understanding spreads and assist traders in managing their daily risk. Accurate density forecasts also can help exchanges to determine appropriate margins and daily price limits and permit a clearer understanding of the existence and magnitude of volatility and tail risk premiums. To date no research has investigated the ability to generate accurate forecast densities in the hog market using either historical information or market generated forecasts.

## **Density Forecast Estimation**

Following Taylor (2005), Liu et al. (2007), and Høg and Tsiaras (2011) densities are derived using two approaches, historical and implied. We obtain historical densities by estimating GARCH models and allowing the distributions of their standard errors to be characterized by several functional forms. Implied densities rely on extracting the information contained in the prices of option contracts, which should reflect aggregated risk-neutral market expectations on the underlying asset when the option contracts expire.

### **Historical Densities**

#### **Estimation**

GARCH models of daily returns of lean hog futures prices are simulated in order to provide historical densities. For the in-sample specification of the mean and variance dynamics, we consider the GJR-GARCH specification proposed by Glosten et al. (1993), which permits asymmetric volatility response to news and has been shown in various studies to reflect market reaction (e.g., (Wu, Guan, and Myers, 2011)). The model is:

$$r_t = \mu_0 + \sum_{i=1}^m \delta_i r_{t-i} + \varepsilon_t \tag{1}$$

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 I(\varepsilon_{t-1} < 0) + \beta h_{t-1}$$

$$\tag{2}$$

$$\varepsilon_t = \sqrt{h_t} \eta_t, \eta_t \sim i.i.d \ D(0,1) \tag{3}$$

In equation 1,  $r_t = log(P_t) - log(P_{t-1})$  corresponds to the logarithmic return of lean hog price  $P_t$ , which is equal to the sum of m lagged returns and the error term  $\varepsilon_t$ . In equation 2 the conditional variance of price returns  $h_t$  is the sum of past innovations  $\varepsilon_{t-1}^2$  plus the lagged conditional variance  $h_{t-1}$ . The asymmetric response emerges through the indicator function ( $I(\varepsilon_{t-1} < 0)$ ) that takes a value of 1 if ( $\varepsilon_{t-1} < 0$ ) and 0 otherwise. Equation 3 describes the error term as the product of the conditional standard deviation  $\sqrt{h_t}$  by a random error  $\eta_t$ , where D(0,1) is a zero mean unit variance probability distribution.

In addition to the standard normal (N), we consider different families of error distributions such as and the standardize t (T), the generalized error distribution (GED), the normal inverse Gaussian (NIG), and the generalized hyperbolic (GH). Since these last distributions allow for skewness and kurtosis, they provide a more flexible and comprehensive simulation of density forecasts. For model selection, we use AIC and BIC criteria and tests misspecification of the standardized residuals of the estimated models such as test of autocorrelation, and LM-ARCH homoscedasticity. Tests suggest the use of a AR(5)-GJR-GARCH(1,1).<sup>1</sup> Although we found a few estimations for which different order models in the GARCH component were selected by the information criteria, we maintain the GJR-GARCH(1,1) specification for model consistency, following the procedure of Høg and Tsiaras (2011). Furthermore, Bao, Lee, and Saltoglu (2007) found that the accuracy of density forecasts depends more on the choice of the distribution of the standardized innovations than on lags of the conditional variance.

#### Simulation

The AR(5)-GJR-GARCH-based forecast densities are constructed using a procedure suggested by Taylor (2005). First, for a particular date, t, we use the 5 most recent years of daily logarithmic

<sup>&</sup>lt;sup>1</sup>The models are estimated in R using the package rugarch version 1.0.9. Mispecification tests of the 406 GARCH estimations are available from the authors. Those correspond to 81 forecasts of each of the 5 specifications.

returns to estimate the parameters of the model by maximum likelihood. By drawing a random number from the D distribution and multiplying it by  $\sqrt{h_t}$  a set of new residuals  $\varepsilon_t$  are generated. These are used to update the conditional variance and then calculate simulated returns. This is repeated from time t until the forecast horizon t + n. In this paper n corresponds to ten business days, since we are looking to obtain a density prediction of the final price of the futures/options contract two weeks before expiration. The simulated returns are compounded and are multiplied by the price at time t to generate the forecast,  $P_{t+n} = P_t exp(r_{t+1} + r_{t+2} + ... + r_{t+n})$ . To create the density forecast we repeat this process 100,000 times.<sup>2</sup> To produce a smoother distribution we apply a Gaussian kernel density with bandwidth equal to  $0.9N^{-\frac{1}{5}}\sigma$ , where  $\sigma$  is the standard deviation of the forecast value and N the number of simulations.<sup>3</sup>

### **Risk-neutral Densities from Options**

An option contract gives the holder the right to make a transaction on an underlying asset at a later date for a specific price (strike price). The owner of a call option has the right but not the obligation to buy the underlying asset, while the owner of a put option has the right but not the obligation to sell. Option prices contain useful information about aggregate market expectations that can be used to extract the implied distribution of future commodity prices. The price of a European call option is equal to the present value of its final payoffs, therefore:

$$c(X) = e^{-r_f T} E^Q[(S_T - X)]$$

$$= e^{-r_f T} \int_0^\infty max(x - X, 0) f_Q(x) dx$$

$$= e^{-r_f T} \int_x^\infty max(x - X) f_Q(x) dx$$
(4)

where X is the strike price, c(X) is the price of the call option,  $S_T$  is the price of the underlying contract,  $r_f$  is the free risk rate, T is the time to maturity,  $f_Q$  is the risk-neutral probability distribution, and  $E^Q$  is an expectation. This holds for a complete set of exercise prices  $X \ge 0$ , and  $\int_0^{\infty} f_Q(x) dx = 1$ . Breeden and Litzenberger (1978) show that the existence and uniqueness of a risk neutral density  $f_Q$  can be inferred from European call prices c(X) from contracts with continuous strike prices and lack of arbitrage opportunities. The risk-neutral density (RND) is then given by:

$$f(x) = e^{r_f T} \frac{\partial^2 C}{\partial X^2} \tag{5}$$

The estimation task is to find a RND  $f_Q(x)$  that provides a reasonable approximation to observed market prices. Several methods have been proposed to recover risk-neutral densities from option

<sup>&</sup>lt;sup>2</sup>Bootstraping techniques have also been used, examples include Rosenberg (2002) and Pascual, Romo, and Ruiz (2006).

<sup>&</sup>lt;sup>3</sup>The differences in the results before and after applying the Gaussian kernel density are almost negligible.

prices as reviewed by Jackwerth (2000) and Taylor (2005). For instance Shimko (1993) estimates interpolations for the volatility smile, Melick and Thomas (1997) use log normal mixtures, and Aït-Sahalia and Lo (1998) follow non-parametric estimations. Examples in the agricultural economics literature include Fackler and King (1990), Sherrick, Garcia, and Tirupattur (1996), and Egelkraut, Garcia, and Sherrick (2007). We follow a similar approach but using the Generalized Beta distribution of the second kind (GB2) as the implied density as in Liu et al. (2007) and Høg and Tsiaras (2011).<sup>4</sup> In addition to its flexibility, Taylor (2005) advocates the use of the GB2 because it has several desirable characteristics including: the tails are fat relative to lognormal distributions, estimates are not sensitive to the discreteness in options prices, it has closed-form expressions for the probability density and cumulative distribution functions, and solutions and calibrations are relatively easy to obtain.

The GB2 density has four parameters  $\theta = (a, b, p, q)$ , allowing for the estimation of the mean, variance, skewness, kurtosis, and its probability distribution function is defined as:

$$f_{GB2}(x|a,b,p,q) = \frac{a}{b^a B(p,q)} \frac{x^{ap-1}}{[1+(x/b)^{p+q}]}, x > 0$$
(6)

with  $B(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$  where  $\Gamma$  is the gamma function. The density is risk-neutral when the underlying futures price F is

$$F = E^{Q}[S_{T}] = bB(p + \frac{1}{a}, q - \frac{1}{a})/B(p, q)$$
(7)

To obtain the RND for the GB2, we find the paramater vector  $\theta$  that minimizes the sum of the squared differences between the observed market and a panel of theoretical option prices (Ji and Brorsen, 2009):

$$\min h(\theta) = \sum_{i=1}^{n} (C_{market}(x_i) - C(X_i|\theta))^2 + (P_{market}(x_i) - P(X_i|\theta))^2$$
(8)

where  $C_{market}(x_i)$  and  $P_{market}(x_i)$  the call and put prices at strikes  $X_i$ , and the theoretical prices are structured in the following manner. Replace  $f_q$  by  $f_{GB2}(x|a, b, p, q)$  in equation (4) and applying the constraint in equation (7) then the European call option price is given by

$$c = (X|\theta) = e^{-r_f T} \int_x^\infty (x - X) f_{GB2}(x|\theta) dx$$
(9)  
$$F e^{-r_f T} [1 - F_{GB2}(x|a, b, p + \frac{1}{a}, q - \frac{1}{a})] - X e^{-r_f T} [1 - F_{GB2}(x|\theta)]$$

where  $F_{GB2}$  is the cumulative distribution function of GB2 density. The functional form in equation (9) is used in the minimization problem, and the put is calculated using the put-call parity condition.

<sup>&</sup>lt;sup>4</sup>Sherrick, Garcia, and Tirupattur (1996) use the Burr-3 distribution which is a special case of the GB2 when q = 1.

## From Risk-Neutral to Real-World Densities

A fundamental idea in pricing theory is that the value of an asset is equal to its expected discounted cash flows. Risk-neutral densities assume that risk is irrelevant for pricing future cash flows, but if an investor is risk-averse and rational then risk-neutral implied densities from option contracts are likely to provide inaccurate forecasts. In fact, the difference between the risk-neutral-density and the objective forecast can be used to infer the degree of risk aversion of the representative agent (Bliss and Panigirtzoglou, 2004).

A possible approach to adjust densities from risk-neutral to real-world, which incorporate risk, is to assume a particular utility function and degree of a risk aversion for the agent, as implemented for equity markets in Bakshi, Kapadia, and Madan (2003), and Liu et al. (2007). According to Høg and Tsiaras (2011) using such simple transformations is usually problematic since the estimated stochastic discount factors generally do not match the expected risk aversion behavior of investors.

In the case of agricultural commodity futures the situation seems even more complex than in equities because it has been difficult to establish if a risk premium exists. For instance, Frank and Garcia (2009) found no evidence of time varying risk premium on corn, soybean meal, and lean hogs at two and four month horizons. However, Egelkraut and Garcia (2006) looked at different forecasting horizons for volatility found evidence that the lean hog markets may demand a risk premium for bearing volatility risk when volatility becomes less predictable. How volatility risk affects risk premium is also a puzzling question. Han (2011) argues that risk premiums are positively related to volatility and negatively related to volatility risk, and it is the volatility risk premium that distorts the positive relation between the market risk premium and market systematic risk.

An alternative approach that avoids some of the previous difficulties involves the use of statistical methods. Real-world densities are obtained via statistical calibration of the risk-neutral densities that are considered to be misspecified. Fackler and King (1990) describe the calibration process as one that improves a set of densities judged against the assumption that the random variables defined by their cumulative distribution functions (cdf) are uniformly distributed. In this section we follow this strategy, using the approach of Shackleton, Taylor, and Yu (2010) to perform parametric and non-parametric density calibration.

Let  $f_Q(v)$  and  $F_Q(v)$  be the risk-neutral density and the cumulative distribution function of the underlying asset v at time  $T, v_T$ . Denote G(u) as the real-world cumulative distribution of random variable  $U = F_Q(v_t)$ , and g(u) its first derivative. Then the real-world cumulative distribution function  $F_p(v)$  and probability density function  $f_p(v)$  of  $v_t$  are

$$F_p(v) = G(F_Q(v)) \tag{10}$$

$$f_p(v) = \frac{dF_p(v)}{dv} = \frac{dG(F_Q(v))}{dv} = \frac{dG}{dF_Q}\frac{dF_Q}{dv} = g(F_Q(v))f_Q(v)$$
(11)

Therefore the real-world density is a function of the calibration function, and the cdf and pdf of the risk-neutral density. In order to estimate the real-world densities we use the risk-neutral densities obtained from the solution of equation (8),  $\theta_{GB2}$ , and the Beta distribution as a calibration function. Fackler and King (1990) recommended using the cdf of the Beta distribution because this parametric distribution is defined on the interval [0, 1], has a flexible shape, and the parameters can be easily estimated by applying maximum likelihood.

If G(.) is the cumulative distribution function of the Beta distribution defined as

$$G(u|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} \int_0^u s^{\alpha-1} (1-s)^{\beta-1} ds$$
(12)
where  $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ 

then the calibration density g(.) is its derivative

$$g(u|\alpha,\beta) = \frac{u^{\alpha-1}(1-u)^{\beta-1}}{B(\alpha,\beta)}$$
(13)

The parameters of the Beta density  $\alpha$  and  $\beta$  are estimated by maximizing the following loglikelihood function:

$$log(L(v_1, v_2, ..., v_t)) = \sum_{t=1}^n log(f_p(v_t | \theta_{GB2}, \alpha, \beta))$$
(14)

Fackler and King (1990) acknowledge that a disadvantage of a parametric approach is that the form chosen may not represent the calibration function well in particular cases. Therefore, as an alternative we also employ non-parametric calibration. The non-parametric calibration allows multimodal shapes in the density, a stylized fact that is common in practice. Following Shackleton, Taylor, and Yu (2010), we construct the real-world density using the past realizations of  $u_t = F_{Q,t}(v_t)$  then the series is transformed into a new series  $z_t = \Phi^{-1}(u_t)$ , where  $\Phi(.)$  is the cdf of the standard normal. A normal kernel density h(z) is obtained with empirical distribution H(z). The empirical calibration of  $u_t$  is then  $G(u) = H(\Phi^{-1}(u_t))$ , therefore the real-world cdf and pdf of the forecasted quantity is

$$F_p(v) = G(F_Q(v)) \text{ and } f_p(v) = \frac{f_q(v)h(z)}{\Phi(z)}$$
 (15)

### **Evaluation of the Density Forecasting Performance**

Diebold, Gunther, and Tay (1998) introduced the probability integral transform (*PIT*) developed by Rosenblatt (1952) as a method to evaluate whether the density forecast correctly specifies the actual realizations of the underlying random variable. Let  $f(y_t)$  and  $F(y_t)$  denote the probability and cumulative density function forecast of a random variable  $y_t$  at time t, and  $Y_{t+n}$  correspond to the actual realization of the random variable at the forecast horizon. The probability integral transform (*PIT*) is given by:

$$PIT_t = \int_{-\infty}^{Y_{t+n}} f(y_t) dy \equiv F(Y_{t+n})$$
(16)

The *PIT* is the value that the predictive cdf attains at the observation  $Y_{t+n}$ . Although the true random variable distribution is often unobservable, Diebold, Gunther, and Tay (1998) and the subsequent literature exploit the fact that when the forecast density equals the true density, then the *PIT* follows a uniform variable in the [0, 1] interval (U(0, 1)) and is independent and identically distributed (iid). In this context, evaluation of whether the conditional forecast density matches the true conditional density can be performed by a test of the joint hypothesis of independence and uniformity of the sequence of *PIT*. Similar to the idea behind the calibration of Fackler and King (1990).

Berkowitz (2001) suggests a further transformation of the *PIT* distribution from Uniform to Normal. This transformation offers the advantage of working with normally distributed variables which facilitates testing of the hypothesis. Suppose  $\phi^{-1}$  denote the inverse of the standard normal distribution, Berkowitz (2001) shows that for any sequence of *PIT* that is iid U(0, 1), it follows that  $z_t = \phi^{-1}(PIT_t)$  is an iid N(0, 1). Under the Berkowitz transformation, independence and normality are tested jointly by using a likelihood ratio test on the following model:

$$z_t - \mu = \rho(z_{t-1} - \mu) + \varepsilon_t, \quad \varepsilon_t \sim i.i.d \ N(0, \sigma^2)$$
(17)

The null hypothesis is that  $z_t$  follows an uncorrelated Gaussian process with zero mean unit variance against an AR(1) with unspecified mean and variance. Therefore, the likelihood ratio can be set as  $LR_3 = -2(L(0, 1, 0) - L(\hat{\mu}, \hat{\sigma}^2, \hat{\rho}))$ , that follows a  $\chi^2$  distribution with three degrees of freedom.

### **Out-of-Sample Forecast Comparisons**

The preceding methods offer measures of the reliability of density forecasts relative to the data generating process; however, in practice we are also interested in comparing competing forecasting methods. We implement that comparison by assigning scoring rules, which are defined by Gneiting and Raftery (2007) as functions of predictive distributions and realized outcomes used to evaluate predictive densities. In this paper as a scoring rule we use the out-of-sample log likelihood values (OLL), in a similar fashion as Bao, Lee, and Saltoglu (2007), Shackleton, Taylor, and Yu (2010) and Mitchell and Wallis (2011). As explained in Bjørnland et al. (2011) logarithmic scores are linked to the Kullback-Leibler information criterion (*KLIC*), the *KLIC* of the ith model is given by:

$$KLIC_{i} = E\left(\log\left(\frac{h(y_{t})}{f_{i}(y_{t})}\right)\right)$$
(18)

where this expectation is taken with respect to the true unknown density  $h(y_t)$ . For a continuous distribution the expectation can be expressed as:

$$KLIC_{i} = \int_{-\infty}^{\infty} \log\left(\frac{h(y_{t})}{f_{i}(y_{t})}\right) h(y_{t})dy$$
(19)

$$\int_{-\infty}^{\infty} \log\left(h(y_t)\right) h(y_t) dy - \int_{-\infty}^{\infty} \log\left(f_i(y_t)\right) h(y_t) dy$$

The *KLIC* represents the expected divergence of the model density relative to the true unobservable density across the domain of the true density. Therefore, the *KLIC* would attain a lower bound of zero only if  $h(y_t) = f_i(y_t)$ . Furthermore, although the expected value of  $h(y_t)$  is unknown, it is considered as a fixed constant. Therefore, the *KLIC* is minimized by maximizing  $\int_{-\infty}^{\infty} \log (f_i(y_t)) h(y_t) dy$  (Bao, Lee, and Saltoglu, 2007; Bjørnland et al., 2011) Assuming ergodicity this expression can be expressed by:

$$OLL = \sum_{t=0}^{n-1} log(f_t(y_t))$$
 (20)

The out-of-sample log-likelihood statistic (*OLL*) can be used to rank predictive accuracy of alternative procedures. The best forecast method yields the highest value which corresponds to the procedure that produces the closest to the true but unknown density.

### Data

The data set consists of daily settlement prices of lean hog futures and options traded at the Chicago Mercantile Exchange (CME) obtained from the Commodity Research Bureau (CRB). The futures data start on January 31, 1996 and end on February 14, 2012; the options data start on January 16, 2002 and end on the same day as the futures data. For the estimation of GARCH models, logarithmic returns calculated as  $r_t = [ln(P_t) - ln(P_{t-1})]$ , and are obtained using the nearby contract, except when there are ten days or less to delivery, in which case the returns are calculated using the next closest delivery contract. Returns are always calculated using the same delivery contract. We proxy the short-run interest rate  $(r_f)$  with the 3-month Treasury Bill rate that is obtained from the Federal Reserve Bank.

The options in the dataset are American-style written on futures contracts of lean hogs. The underlying futures contract expires on the tenth business day of the month of expiration, the same day as the option contract. There are eight contracts in a calendar year for lean hog options and futures, with expirations in February, April, May, June, July, August, October, and December. The lean hog future contract uses cash settlement to the CME Lean Hog Index,<sup>5</sup> that is a two-day weighted average of lean hog values collected by USDA from the Western Cornbelt, Eastern Cornbelt, and MidSouth regions, this ensures convergence between futures and cash prices.

<sup>&</sup>lt;sup>5</sup>Settlement procedures can be found at http://www.cmegroup.com/rulebook/CME/II/150/152/152.pdf.

We collect option prices ten business days before expiration, which usually corresponds to fifteen calendar days. The final dataset consists of 81 panels of option prices. In order to construct real-world densities from risk-neutral ones, previous data are required to estimate the calibration function. We start using the first 2 years of data (16 observations) for the initial calibration, after which calibration is done recursively by adding observations to the calibration set. As a result we generate 65 real-world densities. Since the dataset corresponds to call prices of American options and our estimation requires the use of European options, the Barone-Adesi and Whaley (1987) approximation is used. We filtered the options data eliminating strikes with no volume trade, and not complying with the put-call parity conditions.

## Results

Table 1 presents summary statistics of daily prices and returns of lean hogs from December 1996 to February 2012. Lean hog prices moved in a range of \$86 from \$21.10 to \$107.45. However the prices observed between the 25th and 75th percentile only move within \$22.12 range. Similarly for returns, while the overall range moves between -7.6 and 6.3 percent, the interquartile range only moved within the range of -0.83 to 0.83 percent. Mean and median for returns are close to zero as frequently observed in commodity prices. The price distribution for the whole period is slightly negatively skewed and shows some excess kurtosis.

Figure 1 shows the price and returns during the period. Prices exhibit an overall positive trend, however strong swings can be observed in several periods. Since 2006 lean hog prices seem to follow the pattern seen in other agricultural commodities. A strong price increase until 2008, a sharp decrease in late 2008 and beginning of 2009 during the financial crisis, followed by a swift recovery that lasted at least until the end of 2011.

### **Densities**

Eighty one density forecasts are generated for the contracts expiring from January 2002 until February 2012, and calibration of risk-neutral densities leads to generating sixty five real-world densities starting in January 2004. Figures 2 and 3 show examples of two density forecasts generated in October, 2009 and August, 2011.<sup>6</sup>

Even though the GJR-GARCH density forecasts do vary with time, several patterns emerged across the period. The normal and the standardize t-distribution exhibit very similar patterns. In a few occasions all the distributions generate nearly the same shape, however the GARCH estimations that allow higher moments often exhibit a more leptokurtic distribution and also are slightly skewed to the right. In the case of the risk-neutral distributions the variation is more pronounced. Although the RND and the GARCH-GJR densities often produce similar looking distributions, the RND are usually more leptokurtic and exhibit mass concentrated in the right tail, perhaps reflecting a market sentiment of increasing prices. The real-world density calibrated parametrically show patterns that

<sup>&</sup>lt;sup>6</sup>The eighty one density forecast figures can be found in the electronic appendix at: http://bit.ly/JEaWh3

do not seem to deviate from the risk-neutral density, but the non-parametric calibrated densities exhibit less leptokurtosis than the risk-neutral densities.

#### **PIT Histograms and Berkowitz Test**

Histograms of *PIT* values are used as a preliminary assessment of uniformity, in a similar way that ACF are used to explore autocorrelation or QQ plots in the case of normality. If the *PIT* values are spread evenly in the [0, 1] interval, then the bins in *PIT* histogram would be uniform. We present the *PIT* histograms of the GARCH models and the RND in in Figure 2 that corresponds to eighty one observations from January 2002 to February 2012. In Figure 3 we include the real-world densities; recall calibration requires a training period, therefore real-world densities are from January 2004 to February 2012 corresponding to sixty five observations. The densities for the GARCH models and RND are also presented for that period.

The histograms in Figures 4 and 5 have been divided in 10 bins, corresponding to deciles. Although somewhat uniform, the *PIT* series exhibit under-dispersion since observations are clustered in the first and the last bins. Høg and Tsiaras (2011) indicate this means that the variance or kurtosis (of the target density) is underestimated.

To evaluate the uniformity and independence of the *PITs* we use the Berkowitz test. We evaluate the same two periods used to construct the PIT histograms in Figures 4 and 5. Results of the test presented in Table 2 indicate that for the sixty five observations the real-world parametric, real-world non-parametric, risk-neutral density, GARCH-GED, and GARCH-NIG are satisfactory forecasts since tests fails to reject the null hypothesis at the 10% level. Real-world densities outperform the risk neutral and forward looking estimated densities exhibit a better goodness of fit than the historical models. For the eighty one observations, the GARCH-NIG becomes significant at 10%, and the GARCH-GH is in a gray zone since it is not significant at 5% but it is significant at 10%. The GARCH-T and in particular the GARCH-N are close to the critical value at the 5% level, rejecting the null hypothesis in both periodsindicating their density forecast performance is inferior.

### **Out-of-Sample Log-Likelihood**

Table 3 presents the results of the out-of-sample log-likelihood. According to the Kullback-Leibler information criterion the densities which are closer to the true density have the highest out of sample log likelihood. Following this criterion the results for the sixty five observations starting in 2004 show that as in the goodness of fit test the real-world densities are the preferred methods. Those are followed by the GJR-GARCH-GED that outperforms the risk-neutral densities. The historical models assuming normality and t-distribution display the worst performance. Results for the eighty one observations starting in January 2002 that do not include the evaluation of the calibrated real-world densities are consistent with the results from observations starting in 2004.

## Conclusions

In this paper we estimate and evaluate density forecasts of lean hog futures prices using two approaches. The first method generates forecasts based on historical data, using an AR(5)-GJR-GARCH(1,1) model and different error distributions. The second method is a forward looking approach that obtains an implied risk-neutral density from options prices assuming a generalized beta distribution (GB2). Assuming the RNDs fail to adequately account for risk, the RND functions also are adjusted parametrically and non-parametrically.

Overall, the findings suggest the risk-neutral and real-world density functions generally provide the most accurate representations of the price distributions, with the non-parametrically real-world adjusted model exhibiting the best out-of-sample performance. Among the historical GARCH models, only the GED error structure seems to reflect the price distributions reasonably well. Clearly, using the most current market information from options prices improves the density forecasts and suggests the historical forecasts may not contain much additional information. Interestingly, adjusting the risk-neutral densities does seem to improve the forecasts, indicating that the RND do not completely reflect the underlying densities. This is consistent with results found in other markets, for instance Shackleton, Taylor, and Yu (2010) for equities, and Høg and Tsiaras (2011) for crude oil, show that real-world densities outperform RND and historical densities.

Our results show that historical approach models using the Normal or Std-t distributions do not work well and deviate from the true density. The Generalized Error distribution (GED) captures the skewness of the price distribution and does perform better. The density forecasts obtained from the forward looking approach are correctly specified since risk-neutral and real-world densities exhibit satisfactory goodness of fit. Out of sample log likelihood allows to compare which distributions are closer to the true but unobserved distribution of prices. Again the GED and the calibrated distributions exhibit the best performance, and for the horizon of two weeks, the realworld densities are superior to the historical densities.

Improvements to goodness of fit and accuracy of the forecasts are obtained by the calibration from risk-neutral to real-world densities. This implies that risk premiums exist in the lean hog futures markets, a finding consistent with Szakmary et al. (2003), Egelkraut and Garcia (2006), McKenzie, Thomsen, and Phelan (2007) for volatility risk. The markets value not only risk premiums on the mean levels, but also in volatility, and in the tails. New instruments such as Volatility Index (VIX) and Skew Index, developed in the equities derivative markets, acknowledge these dimensions of risk and the need to price, trade, and hedge them. Agricultural markets have already started to adopt such instruments and density forecasting is a tool that should guide decision making on these markets. Traditional options markets can also benefit from more accurate forecasts that incorporate higher moments.

## References

- Adam, B. D., P. Garcia, and R. J. Hauser. 1996. The Value of Information to Hedgers in the Presence of Futures and Options. *Applied Economic Perspectives and Policy* 18(3): 437–447.
- Aït-Sahalia, Y., and A. W. Lo. 1998. Nonparametric Estimation of State-Price Densities Implicit in Financial Asset Prices. *The Journal of Finance* 53(2): 499–547.
- Amisano, G., and R. Giacomini. 2007. Comparing Density Forecasts via Weighted Likelihood Ratio Tests. *Journal of Business & Economic Statistics* 25(2): 177–190.
- Bakshi, G., N. Kapadia, and D. Madan. 2003. Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. *Review of Financial Studies* 16(1): 101–143.
- Bao, Y., T. H. Lee, and B. Saltoglu. 2007. Comparing Density Forecast Models. *Journal of Forecasting* 26(3): 203–225.
- Barone-Adesi, G., and R. E. Whaley. 1987. Efficient Analytic Approximation of American Option Values. *Journal of Finance* 42(2): 301–320.
- Berkowitz, J. 2001. Testing Density Forecasts, with Applications to Risk Management. *Journal of Business and Economic Statistics* 19(4): 465–474.
- Bjørnland, H. C., K. Gerdrup, A. S. Jore, C. Smith, and L. A. Thorsrud. 2011. Weights and Pools for a Norwegian Density Combination. *The North American Journal of Economics and Finance* 22(1): 61–76.
- Bliss, R. R., and N. Panigirtzoglou. 2004. Option-Implied Risk Aversion Estimates. *The Journal* of *Finance* 59(1): 407–446.
- Bottum, J. C. 1966. Changing Functions of Outlook in the U.S. *Journal of Farm Economics* 48(5): 1154–1159.
- Breeden, D. T., and R. H. Litzenberger. 1978. Prices of State-Contingent Claims Implicit in Option Prices. *The Journal of Business* 51(4): 621–651.
- Carter, C. A., and S. Mohapatra. 2008. How Reliable are Hog Futures as Forecasts? *American Journal of Agricultural Economics* 90(2): 367–378.
- Deaton, A., and G. Laroque. 1992. On the Behaviour of Commodity Prices. *The Review of Economic Studies* 59(1): 1–23.
- Diebold, F. X., T. A. Gunther, and A. S. Tay. 1998. Evaluating Density Forecasts with Applications to Financial Risk Management. *International Economic Review* 39(4): 863–883.
- Egelkraut, T. M., and P. Garcia. 2006. Intermediate Volatility Forecasts Using Implied Forward Volatility: The Performance of Selected Agricultural Commodity Options. *Journal of Agricultural and Resource Economics*: 508–528.

- Egelkraut, T. M., P. Garcia, and B. J. Sherrick. 2007. The Term Structure of Implied Forward Volatility: Recovery and Informational Content in the Corn Options Market. *American Journal of Agricultural Economics* 89(1): 1–11.
- Fackler, P. L., and R. P. King. 1990. Calibration of Option-Based Probability Assessments in Agricultural Commodity Markets. *American Journal of Agricultural Economics* 72(1): 73–83.
- Frank, J., and P. Garcia. 2009. Time-varying Risk Premium: Further Evidence in Agricultural Futures Markets. *Applied Economics* 41(6): 715–725.
- Gneiting, T. 2008. Editorial: Probabilistic Forecasting. *Journal of the Royal Statistical Society: Series A (Statistics in Society)* 171(2): 319–321.
- Gneiting, T., and A. E. Raftery. 2007. Strictly Proper Scoring Rules, Prediction, and Estimation. *Journal of the American Statistical Association* 102(477): 359–378.
- Han, Y. 2011. On the Relation between the Market Risk Premium and Market Volatility. *Applied Financial Economics* 21(22): 1711–1723.
- Høg, E., and L. Tsiaras. 2011. Density Forecasts of Crude-Oil Prices using Option-Implied and ARCH-type Models. *Journal of Futures Markets* 31(8): 727–754.
- Isengildina, O., S. H. Irwin, and D. L. Good. 2004. Evaluation of USDA Interval Forecasts of Corn and Soybean Prices. *American Journal of Agricultural Economics* 86(4): 990–1004.
- Jackwerth, J. 2000. Recovering Risk Aversion from Option Prices and Realized Returns. *Review* of *Financial Studies* 13(2): 433–451.
- Ji, D., and B. W. Brorsen. 2009. A Relaxed Lattice Option Pricing Model: Implied Skewness and Kurtosis. Agricultural Finance Review 69(3): 268–283.
- Koekebakker, S., and G. Lien. 2004. Volatility and Price Jumps in Agricultural Futures Prices– Evidence from Wheat Options. *American Journal of Agricultural Economics* 86(4): 1018–1031.
- Liu, X., M. B. Shackleton, S. J. Taylor, and X. Xu. 2007. Closed-Form Transformations from Risk-Neutral to Real-World Distributions. *Journal of Banking & Finance* 31(5): 1501–1520.
- McKenzie, A., and M. Holt. 2002. Market Efficiency in Agricultural Futures Markets. *Applied Economics* 34(12): 1519–1532.
- McKenzie, A., M. Thomsen, and J. Phelan. 2007. How do you Straddle Hogs and Pigs? Ask the Greeks! *Applied Financial Economics* 17(7): 511–520.
- Melick, W. R., and C. P. Thomas. 1997. Recovering an Asset's Implied PDF from Option Prices: An Application to Crude Oil during the Gulf Crisis. *The Journal of Financial and Quantitative Analysis* 32(1): 91–115.

- Mitchell, J., and K. F. Wallis. 2011. Evaluating Density Forecasts: Forecast Combinations, Model Mixtures, Calibration and Sharpness. *Journal of Applied Econometrics* 26, pages(6): 1023– 1040.
- Myers, and S. Hanson. 1993. Pricing Commodity Options When the Underlying Futures Price Exhibits Time-Varying Volatility. *American Journal of Agricultural Economics* 75(1): 121–130.
- Pascual, L., J. Romo, and E. Ruiz. 2006. Bootstrap Prediction for Returns and Volatilities in GARCH Models. *Computational Statistics & Data Analysis* 50(9): 2293–2312.
- Peterson, H. H., and W. G. Tomek. 2005. How Much of Commodity Price Behavior can a Rational Expectations Storage Model Explain? *Agricultural Economics* 33(3): 289–303.
- Rosenberg, J. 2002. Empirical Pricing Kernels. Journal of Financial Economics 64(3): 341–372.
- Rosenblatt, M. 1952. Remarks on a Multivariate Tansformation. *The Annals of Mathematical Statistics* 23(3): 470–472.
- Shackleton, M. B., S. J. Taylor, and P. Yu. 2010. A Multi-Horizon Comparison of Density Forecasts for the S&P 500 Using Index Returns and Option Prices. *Journal of Banking & Finance* 34(11): 2678–2693.
- Sherrick, B. J., P. Garcia, and V. Tirupattur. 1996. Recovering Probabilistic Information from Option Markets: Tests of Distributional Assumptions. *Journal of Futures Markets* 16(5): 545–560.
- Shimko, D. 1993. Bounds of Probability. Risk (6): 33-37.
- Silva, E. M. d. S., and K. H. Kahl. 1993. Reliability of Soybean and Corn Option-Based Probability Assessments. *Journal of Futures Markets* 13(7): 765–779.
- Szakmary, A., E. Ors, J. Kyoung Kim, and W. N. Davidson. 2003. The Predictive Power of Implied Volatility: Evidence from 35 Futures Markets. *Journal of Banking & Finance* 27(11): 2151– 2175.
- Tay, A. S., and K. F. Wallis. 2000. Density Forecasting: A Survey. *Journal of Forecasting* 19(4): 235–254.
- Taylor, S. J. 2005. *Asset Price Dynamics, Volatility, and Prediction*, 544. Princeton, NJ: Princeton Univ Press.
- Timm, T. R. 1966. Proposals for Improvement of the Agricultural Outlook Program of the United States. *Journal of Farm Economics* 48(5): 1179–1184.
- Timmermann, A. 2000. Density Forecasting in Economics and Finance. *Journal of Forecast-ing* 19(4): 231–234.

- Urcola, H. A., and S. H. Irwin. 2010. Hog Options: Contract Redesign and Market Efficiency. *Journal of Agricultural & Applied Economics* 42(4): 773–790.
- Wang, Z., S. W. Fausti, and B. A. Qasmi. 2012. Variance Risk Premiums and Predictive Power of Alternative Forward Variances in the Corn Market. *Journal of Futures Markets* 32(6): 587–608.
- Wilson, W., and B. Dahl. 2009. Grain Contracting Strategies to Induce Delivery and Performance in Volatile Markets. *Journal of Agricultural and Applied Economics* 41(2): 363–367.
- Wu, F., Z. Guan, and R. J. Myers. 2011. Volatility Spillover Effects and Cross Hedging in Corn and Crude Oil Futures. *Journal of Futures Markets* 31(11): 1052–1075.

# **Tables and Figures**

	Prices	Returns
Num observations	3806	3806
Minimum	21.10	-7.63
Maximum	107.45	6.31
1st Quartile	54.83	-0.83
3rd Quartile	72.95	0.83
Mean	63.90	-0.04
Median	62.80	0.00
Variance	179.69	2.26
SD	13.40	1.50
Skewness	0.20	-0.23
Excess kurtosis	1.05	1.68
Coef. of Variation	0.21	37.40

Table 1: Descriptive Statistics

Notes: Returns are multiplied by 100

Density Forecasting Method	$LR_3$ 81 observations	n-value	$LR_3$ 65 observations	n-value
		p vuide		p varae
GARCH-N	7.8454	0.0493	7.9121	0.0478
GARCH-T	7.7475	0.0515	7.5110	0.0572
GARCH-GED	6.1092	0.1064	6.0109	0.1111
GARCH-NIG	6.5920	0.0861	6.1084	0.1064
GARCH-GH	6.8206	0.0778	6.6192	0.0851
RND	4.6765	0.1970	4.8400	0.1838
RWD-P			3.7712	0.2872
RWD-NP			3.4711	0.3245

Table 2: Berkowitz Test

Notes: 81 observations start in January 2002, 65 observations start in January 2004, both series end in February 2012.

Density Forecasting Method	81 observations	65 observations
GJR-GARCH-N	-220.35	-180.06
GJR-GARCH-T	-220.14	-180.05
GJR-GARCH-GED	-215.36	-172.89
GJR-GARCH-NIG	-217.86	-174.49
GJR-GARCH-GH	-218.35	-176.18
RND	-216.21	-173.11
RWD-P		-169.46
RWD-NP		-167.42

Table 3: Out-of-Sample Log-Likelihood

Notes: 81 observations start in January 2002, 65 observations start in January 2004, both series end in February 2012.



Figure 1: Lean Hog Price and Returns



Figure 2: Fifteen Day Ahead Density Forecasts for GJR-GARCH Models, Risk-Neutral Density, and Real-World Density on October 14, 2009



Figure 3: Fifteen Day Ahead Density Forecasts for GJR-GARCH Models, Risk-Neutral Density, and Real-World Density on August 12, 2011







Figure 4: Probability Integral Transforms (PIT) Histograms January 2002 to February 2012 (81 observations)









Figure 5: Probability Integral Transforms (PIT) Histograms January 2004 to February 2012 (65 observations)