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by

Juheon Seok and B. Wade Brorsen

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and

B. Wade Brorsen*

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* Juheon Seok is an Associate Research Fellow in the Korea Energy Economics Institute (juheon@keei.re.kr) ; B. Wade Brorsen is a Regents Professor and A.J. & Susan Jacques Chair in the Department of Agricultural Economics at Oklahoma State University (wade.brorsen@okstate.edu)

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Previous studies provide pricing models of options on futures spreads. However, none fully reflect the economic reality that spreads can stay near full carry for long periods of time. A new option pricing model is derived that assumes convenience yield follows arithmetic Brownian motion that is truncated at zero. The new models as well as alternative models are tested by testing the truth of their distributional assumptions for calendar spreads and convenience yield with Chicago Board of Trade corn calendar spreads. Panel unit root tests fail to reject the null hypothesis of a unit root and thus support our assumption of arithmetic Brownian motion as opposed to a mean-reverting process as is assumed in much past research. The assumption that convenience yield follows a normal distribution truncated at zero is only approximate as the volatility of convenience yield never goes to zero. Estimated convenience yields can be negative, which is presumably due to measurement error. Option payoffs are estimated with the four different models and the relative performance of models is determined using bias and root mean squared error (RMSE). The new model outperforms three other models and that the other models overestimate actual payoffs. There is no significant difference in error variance for Hinz and Fehr, Poitras, and the new model, and the error variance of the new model is smaller than that of Gibson and Schwartz.

Key words: calendar spreads, corn, futures, panel unit root tests, options, bias, RMSE, significance tests

Introduction

The Chicago Board of Trade (CBOT) offers trading of calendar spread options on futures in wheat, corn, soybean, soybean oil, and soybean meal and the New York Mercantile Exchange offers trading of calendar spread options on cotton and crude oil. Calendar spread options are a new risk management tool. For example, storage facilities can purchase a calendar spread call option to hedge the risk of futures spread narrowing or inverting. Grain elevators can use calendar spread options to partially offset the risk of offering hedge-to-arrive contracts.

Calendar spreads are the difference between futures prices of the same commodity with different delivery dates. The CBOT definition of calendar spread is the nearby futures minus distant futures. Options on calendar spreads cannot be replicated by combining two futures options with different maturity dates. The reason is that calendar spread options are affected only by volatility and value of the price relationship while any strategy to replicate the spread using futures options is also sensitive to the value of the underlying commodity (CME Group). Despite such benefits, so far the volume of calendar spread options traded has been low. Table 1 presents the volume of CBOT futures, options, and calendar spread options on December 2, 2013. The volume year to date 2013 across all agricultural calendar spread option markets was 361,597 contracts, compared to the volume in the corresponding futures contracts of 168,076,317. The small volume may at least be partly due to a lack of understanding of how to value such options.

A more precise pricing formula for calendar spread options would allow option traders to offer lower bid-ask spreads as has occurred with the adoption of the Black-Scholes model.

Earlier papers model the relationship between spot and futures prices by assuming a mean reverting convenience yield (Gibson and Schwartz 1990; Shimko 1994). However, such an assumption is doubtful for storable agricultural commodities since convenience yield may not follow a mean reversion process. Gold does not have strong mean reversion (Schwartz 1997). Gold is typically stored continually with no convenience yield so its spreads tend to remain at full carry[2]. Spreads for agricultural markets could be close to full carry for long periods. Thus, there is a need to create a more suitable option formula on calendar spreads for storable commodities that takes account of all three factors: opportunity cost of interest, storage cost, and convenience yield.

The objectives of this study are to determine an analytical solution of calendar spread option for storable commodities that accounts for the lower bound on calendar spreads due to imposing no arbitrage opportunities, to determine the empirical distribution of calendar spreads and convenience yield using historical corn data, and to determine the accuracy of alternative calendar spread option pricing models in the payoff distribution of calendar spread options simulated using historical market data.

To do this, a two factor model is derived where nearby futures prices follow a geometric Brownian motion and convenience yield is an arithmetic Brownian motion. The call option valuation problem is like an option bear spread where a long call option is combined with a short call option with a strike price of zero. It is possible to test hypotheses about distributional properties of futures spread and convenience yield since spread is observable and convenience yield can be estimated.

Daily Chicago Board of Trade (CBOT) corn futures prices are used for the empirical tests of assumptions and models. The sample period is for the last 100 calendar days to expiration. The prime rate is used for interest rate and storage costs are estimated using historical data on commercial storage rates between 1975 and 2012.

Based on the theory of storage, implicit convenience yield is equal to adding spread, interest costs, and physical storage cost. We perform the distributional tests to examine the distribution of calendar spread and convenience yield. As expected, we reject the hypothesis of normal distribution for both the calendar spread and convenience yield. Nevertheless, this finding partially supports our assumption of truncated convenience yield at zero. Arbitrage should prevent the price difference between two futures from exceeding full carry and thus convenience yield should not be negative. The observed instances of negative convenience yield can be explained by market participants having varying interest cost or physical storage costs or possibly lack of an incentive to take risks without some return. It could also be measurement error.

Monte Carlo simulation is used to obtain option payoffs for the new model as well as Gibson and Schwartz model. Payoffs for Poitras and Hinz and Fehr model are calculated, using analytical formulas. Bias and RMSE are used to compare the performance of the four models. The new model outperforms three other models and negative bias of the new model suggests underestimated payoffs of puts due to the restriction of truncated convenience yield at zero.

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[2] The price difference between (futures) contracts with different maturity is prevented from exceeding the full cost of carrying the commodity. Carrying costs include interest, insurance and storage.

We also perform significance tests to see whether error mean and variance are significantly different with the four models. The results of the significance tests for forecast error mean show that (1) the null of no difference in two means is rejected in most cases, which implies that predicted error is biased and (2) the mean coefficients are positive in most cases, but put mean error of the new model is negative. The findings for error variance imply that (1) there is no difference between Hinz and Fehr, Poitras, and the new model for the variance and (2) Gibson and Schwartz has higher variances than the new model.

Gibson and Schwartz (1990) develop a two-factor model taking account of stochastic convenience yield in order to price oil contingent claims. They assume a mean reverting convenience yield since Gibson and Schwartz's (1989) research reports strong evidence of mean reversion in convenience yield of crude oil. Schwartz (1997) extends this model to a three-factor model including a stochastic interest rate and analyzes futures prices of copper, oil, and gold. He finds that copper and oil have strong mean reversion while gold has weak mean reversion. Note that almost all gold is stored, while long-term storage of copper and oil is less frequent. Shimko (1994) derives a closed form approximation to the futures spread option model, based on the framework of Gibson and Schwartz (1990).

Hinz and Fehr (2010) propose a commodity option pricing model under no physical arbitrage where the calendar spread cannot exceed the storage returns. They derive an upper bound observed in the situation of contango limit by using an analogy between commodity and money markets. Their work represents an important theoretical contribution, however, their empirical work is based on using a shifted lognormal distribution and the Black-Scholes pricing formula. Their model does satisfy the no arbitrage condition created by the contango limit, but it does not reflect the economic reality that spreads can stay near the contango limit for long periods of time.

The Theory of Storage

The theory of storage predicts the spread between futures and spot prices will be a function of the interest costs, $S(t)R(t,T)$, the storage cost, $W(t,T)$, and the convenience yield, $C(t,T)$:

(1) $$F(t,T) - S(t) = S(t)R(t,T) + W(t,T) - C(t,T)$$

where $F(t,T)$ is the futures price at time t for delivery at time T and $S(t)$ denotes the spot price at t. Some studies argue that the commodity spot price is not readily observable and use the futures contract closest to maturity as a proxy for the spot price in empirical analysis for this reason (Brennan 1958; Gibson and Schwartz 1990; Schwartz 1997; Hinz and Fehr 2010). This is a tenuous argument since daily commodity spot prices are readily available. There are good reasons for using the nearby as a proxy for spot prices, but it is not because spot prices do not exist. Futures prices reflect the cheapest-to-deliver commodity and thus the spot price represented by futures contracts can change over time. Also, as Irwin et al. (2011) discuss, grain futures markets require the delivery of warehouse receipts or shipping certificates rather than the physical delivery of grain.

During much of 2008-2011, the price of deliverable warehouse receipts (or shipping certificates) exceeded the spot price of grain and thus futures and spot prices diverged.

Inverse carrying charges have been observed in not only futures and spot prices but also prices of distant and nearby futures. In this point, we extend the relationship in the theory of storage from the futures and spot prices to two futures prices. Nearby futures $F_1(t, T_1)$ with maturity T_1 is treated as the spot $S(T_1)$ at time T_1 and the periods for the interest rate, storage cost, and convenience yield are the difference between deferred time T_2 and near time T_1. Equation (1) is rewritten as:

(2) $$F_2(t, T_2) - F_1(t, T_1) = F_1(t, T_1)R(t, T_2 - T_1) + W(t, T_2 - T_1) - C(t, T_2 - T_1)$$

where $F_2(t, T_2)$ denotes the distant futures price at time *t* for delivery at T_2 and $F_1(t, T_1)$ is the nearby futures price. $R(t, T_2 - T_1)$, $W(t, T_2 - T_1)$, and $C(t, T_2 - T_1)$ denote the interest rate, the storage cost, and the convenience yield for the period $T_2 - T_1$ at time *t*, respectively.

Convenience yield is a benefit from holding physical commodities. It may be regarded as a negative storage price in which it reflects the benefits rather than the cost of holding inventory. Zulauf et al. (2006) argue that convenience yield is at least partly explained by the benefits from being able to take advantage of temporary increases in cash prices. As a result, convenience yield tends to be highest when stocks are low and cash price are variable. In the theory of storage, convenience yield approaches zero as calendar spread goes near full carry. Below full carry, convenience yield can be zero or positive. The explanation of this finding is that interest and storage costs would be misspecified and convenience yield may have measurement error. To explain this phenomenon, we assume that convenience yield is truncated at full carry. The truncated convenience yield can be represented as:

(3) $$C^*(t, T_2 - T_1) = \begin{cases} C(t, T_2 - T_1) & \text{if } C(t, T_2 - T_1) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

We propose a new model that takes account of the convenience yield being truncated at full carry and derive a formula for options on calendar spreads.

The calendar spread option is an option on the price difference between two futures prices of the same commodity with different maturities. When a calendar spread call option is exercised at expiration, the buyer receives a long position in the nearby futures and a short position in the distant futures. Consider a European calendar spread call option. The call option expires at time $T \le T_1$, that is, the option expires prior to the delivery time of the nearby futures contract. The value of the calendar spread call option with exercise price *K* at maturity *T* is

(4) $$\max(F_1(T, T_1) - F_2(T, T_2) - K, 0), \qquad T \le T_1 \le T_2$$

As seen, the payoff of the call option is affected by the price difference between nearby and distant futures prices. The theory of storage shows the spread between two futures is equal to interest costs plus storage costs minus convenience yield. We simplify the model by assuming that both interest rate and storage costs are constant. Thus, a two-factor model with stochastic nearby futures price and convenience yield is used to derive the price of the first call option.

Convenience Yield Follows Arithmetic Brownian Motion: Calendar Spread Call Option Solution

We initially consider the case where convenience yield follows arithmetic Brownian motion with constant drift. The effects of the truncation created at full carry are considered in later sections. The nearby futures price F_1 is assumed to follow geometric Brownian motion with drift μ and volatility σ_1. The convenience yield C^* follows arithmetic Brownian motion. The drift of convenience yield is given by δ and its volatility is given by σ_2. The two standard Brownian motions have constant correlation ρ. The two stochastic factors can be expressed as

(5) $$dF_1(t) = \mu F_1(t)dt + \sigma_1 F_1(t)dZ_1(t)$$

(6) $$dC^*(t) = \delta dt + \sigma_2 dZ_2(t)$$

where $dZ_1(t)$ and $dZ_2(t)$ are standard Wiener processes and $dZ_1(t)dZ_2(t) = \rho dt$. The stochastic volatility model of Heston (1993) is one of the most popular option pricing models. Our model does not consider stochastic volatility but the approach to derive the call option follows steps similar to Heston's work.

The first call price is

(7)
$$\begin{aligned}\mathbb{C}(F_{1,t}, C_t^*, t, T) &= e^{-r\tau}E\left[\max\left(F_{1,t} - K\right)\right] \\ &= F_{1,t}P_1(x_t, C_t^*, \tau) - Ke^{-r(T-t)}P_2(x_t, C_t^*, \tau) \\ &= e^{x_t}P_1(x_t, C_t^*, \tau) - e^{-r\tau}KP_2(x_t, C_t^*, \tau)\end{aligned}$$

where $P_1(x_t, C_t^*, \tau)$ and $P_2(x_t, C_t^*, \tau)$ denote the probabilities of the call that expires in the money, conditional on $x_t = \ln F_1$of the nearby futures and on convenience yield of C_t^*, $\tau = T - t$ denotes the time to expiration.

Since the call price $\mathbb{C}(F_{1,t}, C_t^*, t, T)$ is an option, it must satisfy the partial differential equation (PDE) under no arbitrage condition

(8)
$$\frac{1}{2}{\sigma_1}^2 {F_1}^2 \frac{\partial^2 \mathbb{C}}{\partial {F_1}^2} + \rho\sigma_1\sigma_2 F_1 \frac{\partial^2 \mathbb{C}}{\partial F_1 \partial C^*} + \frac{1}{2}{\sigma_2}^2 \frac{\partial^2 \mathbb{C}}{\partial C^{*2}} + rF_1 \frac{\partial \mathbb{C}}{\partial F_1} + \delta \frac{\partial \mathbb{C}}{\partial C^*} - r\mathbb{C} + \frac{\partial \mathbb{C}}{\partial \tau} = 0.$$

Once we know two probability P_1 and P_2, we can calculate the call option price of equation (7). To obtain the probability P_1 and P_2, we should first compute two characteristic functions f_1 and f_2 which correspond to the in the money probabilities P_1 and P_2 and then can obtain two probabilities P_1 and P_2.

The characteristic functions f_1 and f_2 are assumed for j = 1, 2

(9)
$$f_1(\emptyset; x, C^*) = \exp\left\{r\emptyset i\tau + \delta\frac{1}{2}\sigma_2(b_1 - \rho\sigma_1\emptyset i \pm d)\tau + \frac{1}{2}\sigma_2(b_1 - \rho\sigma_1\emptyset i \pm d) + i\emptyset x\right\}$$
$$f_2(\emptyset; x, C^*) = \exp\left\{r\emptyset i\tau + \delta\frac{1}{2}\sigma_2(b_2 - \rho\sigma_1\emptyset i \pm d)\tau + \frac{1}{2}\sigma_2(b_2 - \rho\sigma_1\emptyset i \pm d) + i\emptyset x\right\}$$

where $b_1 = -\rho\sigma_2,\ b_2 = 0, d = \sqrt{(\rho\sigma_1\emptyset i - b_j)^2 - \sigma_1^2(2u_j\emptyset i - \emptyset^2)},\ u_1 = \frac{1}{2},\ u_2 = -\frac{1}{2}.$

The probabilities P_1 and P_2corresponding to the characteristic functions f_1 and f_2 are

(10)
$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re[\frac{e^{-i\emptyset lnK} f_j(\emptyset; x, C^*)}{i\emptyset}]d\emptyset, \qquad j = 1, 2.$$

A slight difficulty is to evaluate integrals in equation (10) for two probabilities P_1 and P_2. Those integrals cannot be evaluated directly, but can be approximated using numerical integration. The above specification should work well when there is no truncation due to the arbitrage bound at full carry. But, when the arbitrage bound is introduced, constant drift is no longer sufficient as the drift will need to vary with the amount of truncation in order to impose market efficiency on the underlying calendar spread market.

The Case of General $\delta(t)$: Calendar Spread Call Option Solution

In this section, we generalize the previous derivation so that the drift is no longer a constant. The stochastic process for nearby futures prices remains the same. The convenience yield C follows an arithmetic Brownian motion truncated at zero.

(5)' $$dF_1(t) = \mu F_1(t)dt + \sigma_1 F_1(t)dZ_1(t)$$

(11) $$dC(t) = \delta(t)dt + \sigma_2 dZ_2(t)$$

where $\delta(t)$ is a function of lagged convenience yield, volatility of truncated convenience yield, and time to expiration, i.e. ($\delta(t) = \delta(C(t-1), \sigma_2, \tau)$ and σ_2 is volatility of truncated convenience yield. Constant δ does not reflect calendar spread market efficiency. Market efficiency on calendar spread implies that the expectation of calendar spread at time t equals the expectation of calendar spread at time T

(12) $$E(Spread(t)) = E(Spread(T)).$$

To impose market efficiency on calendar spread, we suggest general $\delta(t)$ rather than constant δ in which $\delta(t)$ should be negative due to truncation at zero. The expected change in convenience yield will vary with it being near zero when away from the bound. Truncation at zero leads to the increase in mean and the increased mean affects put payoff more than call payoff. That is, put payoffs are underestimated due to truncation at zero. Negative $\delta(t)$ would be a solution to fix underestimated put payoffs. We propose that $\delta(t)$ would be related by put option of convenience yield which has strike price of zero since the put option is a function of lagged convenience yield, volatility, and time to expiration.

As convenience yield increases, $\delta(t)$ goes up i.e. $\frac{\partial\delta}{\partial C(t)} > 0$. As volatility decreases, $\delta(t)$ goes up i.e. $\frac{\partial\delta}{\partial\sigma_2} < 0$. The relationship between $\delta(t)$ and time to expiration is not determined. Dec-Mar, May-Jul and Dec-Jul convenience yields against time to expiration are going down and then up like a U-shape while Mar-May and Jul-Dec are going down as time to expiration close to zero. The sign of the derivative of $\delta(t)$ with respect to time to expiration is not determined.

The call price is

(13) $$\mathbb{C}\big(F_{1,t}, C_t(\delta(t)), t, T\big) = e^{x_t}P_1(x_t, C_t, \tau) - e^{-r\tau}KP_2(x_t, C_t, \tau)$$

The characteristic functions f_1 and f_2 are assumed for j = 1, 2

(14) $$f_1(\emptyset; x, C) = \exp\left\{r\emptyset i\tau + \delta(t)\frac{1}{2}\sigma_2(b_1 - \rho\sigma_1\emptyset i \pm d)\tau + \frac{1}{2}\sigma_2(b_1 - \rho\sigma_1\emptyset i \pm d) + i\emptyset x\right\}$$

$$f_2(\emptyset; x, C) = \exp\left\{r\emptyset i\tau + \delta(t)\frac{1}{2}\sigma_2(b_2 - \rho\sigma_1\emptyset i \pm d)\tau + \frac{1}{2}\sigma_2(b_2 - \rho\sigma_1\emptyset i \pm d) + i\emptyset x\right\}$$

where $b_1 = -\rho\sigma_2,\ b_2 = 0, d = \sqrt{(\rho\sigma_1\emptyset i - b_j)^2 - \sigma_1^2(2u_j\emptyset i - \emptyset^2)},\ u_1 = \frac{1}{2},\ u_2 = -\frac{1}{2}.$

The probabilities P_1 and P_2 corresponding to the characteristic functions f_1 and f_2 are

(15) $$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\emptyset lnK}f_j(\emptyset; x, C)}{i\emptyset}\right]d\emptyset, \qquad j = 1, 2.$$

Short Put Option Solution

To adjust for convenience yield being truncated at zero, we propose a second option for convenience yield which follows arithmetic Brownian motion (Bachelier, 1990) and has a strike price of zero. We specify the convenience yield $C_B(t)$ satisfying at time t

(16) $$dC_B(t) = \sigma_B dW(t), \qquad t \le T$$

where $dW(t)$ denotes standard Brownian motion, subscript B stands for Bacheiler, and σ_B represents the volatility.

The value of a European put option p_B at maturity T is

(17) $$p_B(T) = \max(K - C(T), 0)$$

where K is the exercise price. Following the Bachelier framework, the convenience yield is normally distributed with mean zero and variance $\sigma_B^2\tau$. The put option at time t is

(18) $$p_B(t) = e^{-r\tau}\left[\big(K - C(t)\big)\Phi\left(\frac{K - C(t)}{\sigma_B\sqrt{\tau}}\right) + \sigma_B\sqrt{\tau}\phi\left(\frac{K - C(t)}{\sigma_B\sqrt{\tau}}\right)\right], \quad \tau = T - t$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ is the standard normal density function. Our interest is a put option with a strike price of zero. Substituting the strike price of zero yields

(19) $$p_B(t)_{K=0} = e^{-r\tau}\left[-C(t)\Phi\left(\frac{-C(t)}{\sigma_B\sqrt{\tau}}\right) + \sigma_B\sqrt{\tau}\phi\left(\frac{-C(t)}{\sigma_B\sqrt{\tau}}\right)\right]$$

where $p_B(t)_{K=0}$ represents the put option with a strike price of zero.

A Solution to Final Calendar Spread Option

Truncation at zero does affect less call option on calendar spread rather than put option on calendar spread. The final calendar spread call option is

(13) $$c_{cs} = \mathbb{C}\big(F_{1,t}, C(\delta(t))_t, t, T\big) = e^{x_t} P_1(x_t, C_t, \tau) - e^{-r\tau} K P_2(x_t, C_t, \tau).$$

And the final calendar spread put option is

(14) $$\boldsymbol{p}_{cs} = \boldsymbol{p}\big(F_{1,t}, C(\delta(t))_t, t, T\big) = e^{-r\tau} K P_2(x_t, C_t, \tau) - e^{x_t} P_1(x_t, C_t, \tau).$$

where $\mathbb{C}\big(F_{1,t}, C(\delta(t))_t, t, T\big)$ and $\boldsymbol{p}\big(F_{1,t}, C(\delta(t))_t, t, T\big)$ are derived assuming truncated convenience yield at zero. There is one thing to note put call parity at the money. The put call parity implies that put option value is equal to call option value at the money.

(15) $$\begin{aligned} c_{cs} &= \boldsymbol{p}_{cs} \\ \mathbb{C}\big(F_{1,t}, C(\delta(t))_t, t, T\big) &= \boldsymbol{p}\big(F_{1,t}, C(\delta(t))_t, t, T\big) \\ &= \boldsymbol{p}\big(F_{1,t}, C_t^*, t, T\big) - p_B(t)_{K=0} \\ &= [e^{x_t} P_1(x_t, C_t^*, \tau) - e^{-r\tau} K P_2(x_t, C_t^*, \tau)] \\ &\quad - e^{-r\tau}\left[-C_t \Phi\left(\frac{-C_t}{\sigma_B\sqrt{\tau}}\right) + \sigma_B\sqrt{\tau}\phi\left(\frac{-C_t}{\sigma_B\sqrt{\tau}}\right)\right] \end{aligned}$$

where C_t^* is convenience yield assuming no truncation at zero, C_t is convenience yield with truncation at zero, c_{cs} is the calendar spread call option price and $\boldsymbol{p}_{cs}$ is the calendar spread put option which equals put value of untruncated convenience yield at zero less put of convenience yield with zero strike price. While this is an analytical solution, the solution is still one that has to be solved numerically. Future research may want to consider using Malliavin calculus or characteristic functions as approaches that might yield a solution that can be directly solved.

Data

The data used to test the assumptions and models consists of daily futures prices, daily prime interest rate, and annual physical storage costs. Corn futures prices are from the Chicago Board of Trade between 1975 and 2012. The interest costs are calculated by nearby futures prices times the prime interest rates.

Five calendar spreads are used since the CME group offers five sets of calendar spread options: Dec-Mar, Mar-May, May-Jul, Jul-Dec, and Dec-Jul. The calendar spreads of Dec-Mar, Mar-May, May-Jul, Dec-Jul crop years use two same crop year whereas Jul-Dec spread is a combination of old-new crop year. For intra-year spreads, the difference between two futures would be mostly negative and convenience yield may be small or close to zero because inventories are plentiful after harvest. The Jul-Dec calendar spread would have positive spreads and large convenience yields. The movements of five corn futures spreads are described in figures 1 through 5.

Nonparametric regression is used to determine both the trend and the sample period of calendar spreads. The whole sample period of calendar spread ranges between 800 and 1300 calendar days to expiration and calendar spreads vary over time as shown in figure 6. All figures show positive intercepts although average spreads are negative. The reason is that the plots are only partial predictions and so the negative intercept term is not included. All five spreads decrease as maturity approaches. This downward trend might reflect a risk premium. Since historical corn spreads exhibit a downward trend over 100 calendar days to maturity, the sample period is selected by the last 100 calendar dates to expiration. Until 100 calendar days before expiration, full carry is hardly ever hit. The markets is always at less than full carry since there is a positive probability that the market will move away from full carry.

Table 2 presents the summary statistics for nearby and distant futures, calendar spread, the change in calendar spread, convenience yield, the change in convenience yield, interest rates, and storage cost according to different calendar spreads. The difference between nearby and distant futures prices has a negative value except the Jul-Dec spread. The average spread is between -20.11 and 5.22; the huge difference between Dec-Jul futures prices (-20.11) may be explained by the longer time between the two maturity dates. The average convenience yield is between 1.24 and 25.75. As expected, the Jul-Dec convenience yield is positive and large. It may reflect the scarce inventories since the Jul-Dec spread contains the corn harvest period. The mean of the prime rate (8.3%) is higher than that of three-month Treasury Bills (5.2%).

The convenience yield, displayed in figures 12 through 16, is plotted against the trading days to expiration. It shows that convenience yield has positive value mostly but negative convenience yield is often found. The convenience yield would be negative when the spread is greater than the cost of storage. Thus, the negative convenience yield could occur from underestimating storage costs.

Methods

Testing the Truth of Assumptions

One way of testing models is to test the truth of their assumptions. Poitras (1998) proposes that futures prices have a joint normal distribution so calendar spread would be normally distributed and Hinz and Fehr (2010) suggest that the distribution of calendar spread is a shifted lognormal distribution. We test the assumptions of normality using skewness ($\sqrt{\beta_1}$), kurtosis (β_2), and an omnibus test (K^2).

With Poitras (1998) assumption, calendar spread follows arithmetic Brownian motion with mean $\mu_p\tau$ and variance $\sigma_p^2\tau$ which is proportional to time to expiration τ

(16) $$d(F_1 - F_2) \sim N(\mu_p\tau, \sigma_p^2\tau).$$

Hinz and Fehr (2010) suppose that the distribution of calendar spread can be approximated by a shifted lognormal distribution where the shift parameter is estimated by a maximum value of the difference between two futures prices. We partially test for Hinz and Fehr's distributional assumption in which the logarithm of the ratio Z is normally distributed. The ratio Z is $\frac{F_1+\kappa-F_2}{F_2}$. The ratio implies no arbitrage in that calendar spread cannot exceed the costs of storage so that the ratio should be above zero. The lognormal distribution of the ratio Z with mean $\Pi_{HF}\tau$ and variance $\Phi_{HF}^2\tau$ is

(17) $$dlnZ = dln(\frac{F_1+\kappa-F_2}{F_2}) \sim N(\Pi_{HF}\tau, \Phi_{HF}^2\tau)$$

where $\kappa = \max(F_2 - F_1)$ is upper bound created by the contango limit.

Gibson and Schwartz (1990) assume that convenience yield is mean reverting. We test for the presence of mean reversion in convenience yield as well as calendar spread using panel unit root tests. Im, Pesaran and Shin (IPS) panel unit root test is used since it allows for heterogeneous coefficients (ρ_i) whereas Levin-Lin-Chu test imposes a restriction of homogeneous coefficients (ρ) across cross section *i*. Thus, Im, Pesaran and Shin test employs a unit root test for each cross section *i*. The IPS test uses a *t*-statistic which is the average of the individual unit root tests. The Dicky Fuller regression is

(18) $$\Delta y_{it} = \alpha_i + \rho_i y_{i,t-1} + \varepsilon_{it}, \qquad i = 1, \dots, N, \quad t = 1, \dots, T.$$

The equation (18) can be expressed as the augmented Dickey Fuller (ADF) regression

(19) $$\Delta y_{it} = \alpha_i + \rho_i y_{i,t-1} + \sum_{j=1}^{P_i} \theta_{ij}\,\Delta y_{it-j} + \varepsilon_{it}$$

where y_{it} is the observation on the *i*th cross-section unit at time *t*, α_i is the intercept, and ε_{it} is the error term. In our model, y_{it} is calendar spread or convenience yield, the cross section *i* is year, and the time *t* is trading days. The null hypothesis that all individuals *i* have a unit root is

(20) $$H_0: \rho_i = 0, \qquad \text{for all } i.$$

The alternative hypothesis is

(21) H_1: $\rho_i < 0$, for all i

A unit root implies that the calendar spread or convenience yield is not stationary. Since IPS test is applied only for a balanced panel data alternatively, Fisher-type test is also performed. Fisher-type test uses the p-values from unit root tests for each cross section i. The null and alternative hypotheses are the same as those of IPS test.

The Stata commands for the IPS and Fisher-type test are

xtunitroot ips spread (or convenience yield)

xtunitroot fisher spread (or convenience yield), dfuller lags(#).

We also test whether convenience yield follows arithmetic Brownian motion truncated at zero employing historical data. For that purpose, we first estimate convenience yield according to the theory of storage since convenience yield is not obtainable directly. In equation (2), we substitute the spot price into the nearby futures price and use the CBOT definition of calendar spread is the nearby futures minus distant futures. In contango, calendar spread is negative whereas it is positive in backwardation. Equation (2) is multiplied by negative one to match the CBOT definition of calendar spreads:

(22) $F_1(t,T_1) - F_2(t,T_2) = -F_1(t,T_1)R(t,T_2-T_1) - W(t,T_2-T_1) + C(t,T_2-T_1)$.

To obtain convenience yield, rearrange equation (22) as

(23) $C(t,T_2-T_1) = \left(F_1(t,T_1) - F_2(t,T_2)\right) + F_1(t,T_1)R(t,T_2-T_1) + W(t,T_2-T_1)$.

Equation (23) provides a formula to compute implicit convenience yield. Namely, implicit convenience yield is estimated by adding calendar spread, interest costs, and storage cost.

Two normality tests are conducted for the assumption of convenience yield; first is to test truncation at zero using implicit convenience yield, second is to test arithmetic Brownian motion employing residuals of convenience yield. Means are allowed to vary by year so the data used are residuals from a regression of the change in convenience yield against year dummies.

A New Model for Calendar Spread Options

The option payoffs are obtained by Monte Carlo simulation in that payoffs are calculated by a number of calendar spreads created by two stochastic processes, various interest costs and constant physical storage costs. Parameter for the nearby futures drift μ, the nearby futures volatility σ_1, the convenience yield drift δ, and the convenience yield volatility σ_2 are estimated using historical data. The nearby futures price follows geometric Brownian motion by equation (5)

(5)' $dF_1(t) = \mu F_1(t)dt + \sigma_1 F_1(t)dZ_1(t)$.

Applying Ito's lemma

(24) $dlnF_1(t) = \left(\mu - \frac{1}{2}\sigma_1^2\right)dt + \sigma_1 dZ_1(t)$.

The distribution of the nearby futures return is

(25) $\Delta lnF_1(t) \sim N[\left(\mu - \frac{1}{2}\sigma_1^2\right)\tau, \sigma_1\sqrt{\tau}]$

where τ is time to expiration. Since the futures market is assumed to be efficient, μ is regarded as zero for an initial value. We examine whether there exists autocorrelation in residuals using the first order autoregression. The first order autoregressive model for the nearby futures returns is

(26) $\ln\left(\frac{F_{1,t}}{F_{1,t-1}}\right) = p + q\ln\left(\frac{F_{1,t-1}}{F_{1,t-2}}\right) + \varepsilon_t$.

If $\hat{q} = 0$, it supports that the model is not misspecified. Thus, the assumption of independence of nearby futures returns is supported.[3] The nearby futures volatility of σ_1 is estimated by the daily

[3] The result is shown in table 5. As expected, the results are consistent with the no autocorrelation assumption of nearby futures price.

standard deviation of $dln\frac{F_{1,t}}{F_{1,t-1}}$ over a past 20 trading day period covering the period 1976 to 2011[4].

We collected 41 years of daily convenience yield covering the period 1971 to 2011. Rewrite the stochastic convenience yield process in equation (6) as

(6)' $$dC(t) = \delta dt + \sigma_2 dZ_2(t)$$

We assume that the calendar spread market is efficient so that δ is assumed to be zero[5]. To reflect the mean differences across year (group) over time, we include fixed effects (crop year dummy variables as a group). We regress changes in convenience yield of ΔC_{it} on crop year dummies of D_{ji}

(27) $$\Delta C_{it} = \sum_{j=1}^{40} s_j D_{ji} + \varepsilon_{it}^{C}.$$

Obtain residuals of $\widehat{\varepsilon_{it}^{C}}$ and then compute the standard deviation of residuals for a past 5 year period to estimate the volatility of convenience yield σ_2. For example, we can compute a standard deviation of Dec-Mar convenience yield of σ_2 for year 1976 covering the period 1971 to 1975. The standard deviation of convenience yield for year 1976 is

(28) $$\sigma_2^{Year\ 1976} = \sqrt{\frac{1}{N-1}\sum_{i=1971}^{1975}\sum_{t=1}^{69}(\widehat{\varepsilon_{it}^{C}} - \overline{\varepsilon_{it}^{C}})^2 * dum_{it}^{C}}$$

where $N = \sum_{i=1971}^{1975}\sum_{t=1}^{69} dum_{it}$, $\overline{\varepsilon_{it}^{C}} = \frac{1}{N}\sum_{i=1971}^{1975}\sum_{t=1}^{69}\varepsilon_{it}^{C} * dum_{it}^{C}$. $\overline{\varepsilon_{it}^{C}}$ is a mean of residual $\widehat{\varepsilon_{it}^{C}}$, $\sigma_2^{Year\ 1976}$ is a standard deviation of convenience yield for year 1976, *i* is year, *t* is trading days, and dum_{it} is dummy variables to deal with truncation at zero if ΔC_{it} is greater than zero then dum_{it} is one, otherwise dum_{it} is zero. Thus, the dummies allow negative convenience yields to become zero since we assume convenience yield only has zero or positive value.

Although we consider correlation between nearby futures and convenience yield in the theoretical model, we assume no correlation between the two processes in the empirical analysis. Correlation tests are conducted using the change in logarithm of nearby futures prices and residuals of convenience yield across all calendar spreads. The overall correlation is 0.034 and significant; correlations are significant for Dec-Mar and Dec-Jul spreads, but not for other spreads. Correlation reduces payoffs around 3% in Dec-Mar and Dec-Jul and around 1% in the three others. The effect of assuming correlation is not as important as that of truncation at zero in estimating payoffs. Hence, we simulate the model with two stochastic processes assuming no correlation. Discounting of the payoff function is not needed since all payoffs would be discounted at the same rate.

We run Monte Carlo simulation to obtain option payoffs since there is no closed form solution for distribution of calendar spread based on our assumptions of log-normally distributed change in nearby futures and truncated convenience yield at zero.

A simulation procedure is as follows:

1) Create 10,000 replications of size *M* = the total number of trading days –trading days for each year *i* from stochastic processes to equation (23) and (8).
2) Impose the restriction of truncated convenience yield at zero
   $$C = \max(C^*, \min(0, C))$$
   where C is the observed convenience yield and C^* is the latent convenience yield.
   Compute calendar spread using the theory of storage of equation (4)
   $$\text{Calendar Spread} = F_1 - F_2 = -F_1 * R - W + C.$$

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[4] The implied volatility is also considered as an estimate for σ_1. In the study, however, historical volatility is used since implied volatility is not obtainable for the entire sample period. The comparison of payoffs which are obtained using historical and implied volatility shows that it is not different. The reason for this is that the movement in the interest cost component is small relative to the movement in convenience yield.

[5] Due to truncation, the condition for efficiency is not actually zero.

3) Calculate the payoffs of option resulting in M values $P_{11}, \dots P_{iM}$ for each year i, and then take the average of option payoffs

$$\bar{P} = \frac{1}{i*M}\sum_{z=1}^{i}\sum_{v=1}^{M} P_{vz}.$$

Gibson and Schwartz Model: Stochastic Convenience Yield

Gibson and Schwartz (1990) introduce a two-factor model with mean-reverting convenience yield and spot price following geometric Brownian motion. We simulate Gibson and Schwartz's model to obtain payoffs for calendar spread options; draw 10,000 replication of size M from below two stochastic process of equation (29), obtain the calendar spread using equation (4), calculate the payoffs, and then take the average of payoffs.

The spot price process is replaced by the nearby futures price process. The two joint stochastic processes are

(29)
$$\frac{dF_1}{F_1} = \mu dt + \sigma_1 dz_1$$
$$dC = k(\alpha - C)dt + \sigma_2 dz_2$$
$$dz_1 dz_2 = \rho$$

The nearby futures drift of μ and the volatility of σ_1 are estimated according to the same method in the new model section. k is the speed of adjustment and α is the long run mean of convenience yield. The Dickey-Fuller regression is used to estimate parameters of k and α over the past 5 year period

(30)
$$\Delta C_t = C_t - C_{t-1} = \beta_0 + \beta_1 C_{t-1} + \epsilon_t = \alpha k + kC_{t-1} + \epsilon_t$$

Let $k = -\beta_1$ and $\alpha = -\frac{\beta_0}{\beta_1}$[6]. The parameter of ρ is estimated by the correlation between $\Delta lnF_{1,t}$ and ϵ_t from the Dickey-Fuller regression over the past 1 year period.

Poitras: Bachelier Model of Calendar Spread Options

Poitras (1998) proposes a calendar spread option pricing formula in which individual futures prices follow arithmetic Brownian motion. Since spread can be negative or positive, the assumption of a normal distribution on spread is more realistic than that of a log-normal distribution for two futures. The calendar futures spread stochastic process is

(31)
$$d(F_1 - F_2) = \sigma_P dz_F$$

where $\sigma_P^2 = \sigma_{F_1}^2 - 2\sigma_{F_1,F_2} + \sigma_{F_2}^2$ is the variance of the joint process, $2\sigma_{F_1,F_2}$is covariance between two futures prices. The solution to Bacheiler calendar spread call option is

(32)
$$C_P = (F_1 - F_2 - X)N(u) + \sigma_P\sqrt{\tau}\, n(u)$$

where $u = \frac{F_1 - F_2 - X}{\sigma_P\sqrt{\tau}}$. C_P is the price of the Bachelier calendar spread option, τ is trading days to expiration, $N(u)$ is the cumulative normal probability function, and $n(u)$ is the normal density function. Two futures prices of F_1 and F_2 are used for last 100 calendar days to expiration. Call option volatility of σ_P is estimated by two futures prices over the past 20 days.

Hinz and Fehr Model: A Shifted Lognormal Distribution Model for Calendar Spread Options

Hinz and Fehr (2010) develop an option pricing model for calendar spread options, which imposes a physical arbitrage condition that calendar spread cannot exceed storage cost returns. They propose a shifted lognormal distribution for calendar spread and derive the call option price on calendar spread based on the Black-Scholes formula. The payoff on expiration date T is

[6] Table 6 reports estimates of the parameters k and α.

(33) $C_H = \max(F_1(T_1) + \kappa - (1 + X)F_2(T_2), 0), \qquad t < T < T_1 < T_2$
given $\kappa = \max(F_2(T_2) - F_1(T_1))$.

κ is an upper bound created by the contango limit. T is date to expiration for calendar spread option, and T_1 and T_2 are the maturity date for nearby and distant futures.

The price of calendar spread option is

(34) $C_H = F_2\{ZN(d_1) - XN(d_2)\}, \qquad \tau = T - t$

where $Z = \frac{F_1 + k - F_2}{F_2}$, $Z > 0$, $d_1 = \frac{1}{\upsilon\sqrt{\tau}}\left\{\log\left(\frac{Z}{X}\right) + \frac{1}{2}\upsilon^2\tau\right\}$, and $d_2 = d_1 - \upsilon\sqrt{\tau}$.

Z is the ratio and should be greater than zero due to the arbitrage condition that calendar spread cannot exceed full carry. υ is the standard deviation of the ratio of Z. X is the exercise price.

The storage cost parameter of κ is estimated from the past 5 year period. The ratio Z is calculated by a formula of equation (34) over the past 20 day period. Hinz and Fehr obtain the full carry parameter κ as a maximum value of calendar spread for the whole sample period. In order to make our results out of sample, we estimate κ as a maximum value of calendar spreads over the previous 5 year period for each year so that we can have 36 different values of κ. The full carry parameter of κ can be less than the calendar spread, which results in the negative ratio of Z. Any negative Z is deleted to satisfy the constraint, which leads to missing values in calculation of volatility. The volatility of υ is computed by the standard deviation of the ratio over the past 20 trading days.

Prediction Tests

The bias and root mean squared error (RMSE) are computed to analyze the degree of accuracy provided by the models for specific trading days and these statistics are provided with respect to moneyness (at the money, in the money, and out of money).

Three different exercise prices in cents are used for the prediction tests; the exercise price of at the money is given by the calendar spread of the first trading day for each year (*X*=*Spread*), the exercise price for in the money is designed to the spread of the first trading day of each year minus three (*X*=*Spread*-3), the exercise price for out of money is specified as the spread of the first trading day of each year plus three (*X*=*Spread*+3). Also, the exercise price given in the first trading day is used to compute option payoffs over the subsequent trading days. Four trading days are trading day 1, 15, 30, and 50 in the sample chosen.

The bias in cents for a given year is calculated

(35) $Bias = \frac{1}{N}\sum_{i=1}^{N}[\widehat{P_i} - (P_i|\text{trading days} = g)], \qquad g = 1, 15, 30, 50$

where *N* denotes the number of the calendar spreads, *N* =1, …, 5, $\widehat{P_i}$ denotes actual payoffs, and $P_i|\text{trading days} = g$ denotes payoffs from each model given the specific trading days.

The RMSE in cents is computed

(36) $RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[\widehat{P_i} - (P_i|\text{trading days} = g)]^2}, \qquad g = 1, 15, 30, 50.$

Significance tests using a regression are performed to compare two means between forecast and actual errors as well as two variances between another model and the new model. We include two interactions of option and method and of method and trading days since option, method, and trading days may affect forecast errors, but it is also possible that the effect of option will vary by the method. We regress errors on dummy variables of trading day, spread, option, and method, exercise price as fixed effects and two interactions. Regression is given by

(37) $$error_{gnlht} = a_0 + \sum_{i=1}^{3} a_i D_{ig} + \sum_{i=1}^{4} b_i D_{in} + c_l D_l + \sum_{i=1}^{3} d_i D_{ih} + \sum_{i=1}^{2} e_i D_{iu} + \sum_{i=1}^{3} f_i D_l * D_{ih} + \sum_{j=1}^{3}\sum_{i=1}^{3} s_{ij} D_{ig} * D_{jh} + \varepsilon_{gnlht}$$

where $error_{gnlht}$ denotes errors between actual and expected payoffs for each trading day g, each calendar spread n, option type *l*, each method *h*, and year *t*, D_{ig}, D_{in}, D_l , and D_{ih}, and D_{iu}

represent dummy variables for trading day, spread, option, method, and exercise price, $D_l * D_{ih}$ and $D_{ig} * D_{ih}$ are interactions of option and method, and of trading day and method, and $\varepsilon_{gnlht} \sim N(0, \sigma^2_{gmlht})$.

We test for the bias hypotheses that the mean of forecast errors has the same value as the mean of actual errors, that is, the forecast errors have zero means. The null hypothesis of testing a difference of means is

(38) $H_0: \quad \mu_h = 0, \qquad h = 1,2,3,4$

where μ_h represents the average of means of forecast errors in terms of model h; the new model is regarded as a base. To obtain forecast error means of one model, take expectation of equation (37) on both sides. We test unbiasedness for each model by testing whether the expected forecast error for each model is zero. The average of means of forecast error in terms of model h as a linear combination of model parameters is

(39) $$H_0: \ a_0 + 0.25\sum_{i=1}^{3} a_i + 0.2\sum_{i=1}^{4} b_i + c_l + d_h + 0.33\sum_{i=1}^{2} e_i + D_l * f_h + 0.25\sum_{i=1}^{3} s_{ih} = 0, \qquad h = 1,2,3,4, \ l = 1,2.$$

Since there are 4 trading days in the trading day group, each trading day gets a weight of 1/4. In the same fashion, the weights of spread and exercise prices are 0.2 and 0.33.

Error variance equation allowing for heteroskedasticity is

(40) $$\sigma^2_{gnlht} = \exp\left[\zeta_0 + \sum_{i=1}^{3}\zeta_i D_{ig} + \sum_{i=1}^{4}\vartheta_i D_{in} + \xi_l D_l + \sum_{i=1}^{3}\varpi_i D_{ih}\right].$$

We are interested in the error variance difference between one model and the new model. For simplicity, the error variance difference in model h and the new model is

(41) $$\zeta_0 + \sum_{i=1}^{3}\zeta_i D_{ig} + \sum_{i=1}^{4}\vartheta_i D_{in} + \xi_l D_l + \varpi_h, \qquad h = 1, 2, 3.$$

The null hypothesis is

(42) $$H_0: \ \zeta_0 + \sum_{i=1}^{3}\zeta_i D_{ig} + \sum_{i=1}^{4}\vartheta_i D_{in} + \xi_l D_l + \varpi_h = 0, \qquad h = 1, 2, 3.$$

If coefficient of variance difference is significant and positive, the model h has larger variance than the new model. Otherwise, the variance of the model h is not different from the variance of the new model.

Results

The distributional tests for the calendar spread, convenience yield and nearby futures are reported in Table 3. In all cases, the changes in spread (Poitras) and logarithm of ratio (Hinz and Fehr) are not normally distributed at the 5 %. The cases of the changes in spread and convenience yield present that skewness of Mar-May is similar to that of a normal distribution and kurtosis of Jul-Dec is very high, which implies a distinct peak near the mean. The change of logarithm of the ratio has negative skewness except Dec-Mar and kurtosis is very large. In all cases the normality of convenience yield is rejected and skewness is positive except the change in logarithm of ratio. The log-normality of the nearby futures is rejected but the Dec-Mar and Dec-Jul futures returns are similar to a normal distribution.

Figures 7 through 11 present histograms for the calendar spread, convenience yield, and nearby futures. The histograms of convenience yield[7] show a long right tail and skewness to the right. Jul-Dec histogram of convenience yield is especially skewed to the right. The right skewness is consistent with the assumption of convenience yield being truncated at zero. Jul-Dec likely has the least truncation since it is often in backwardation.

Although the normality of convenience yield is rejected, the shape of the distribution provides modest support for assuming truncation at zero once values less than zero are regarded as measurement noise. Convenience yield has measurement noise due to estimating storage and

[7] The convenience yield is computed by the prime rate times nearby futures prices plus storage costs.

interest costs. All of the models rely on normality assumptions and the normality assumption is rejected. Rejection of the normality assumption is often not as critical as rejection of other assumptions.

We use a panel unit root test to examine Gibson and Schwartz's assumption that convenience yield follows mean reversion. Table 4 presents t-statistics and p-values for calendar spreads and convenience yield. We cannot reject the hypotheses that calendar spread and convenience yield have a unit root. The t-statistics in the Augmented Dickey-Fuller regression for the calendar spread and convenience yield range between -2.74 and 1.69. This is not significant at the 5% level, which means that calendar spread and convenience yield are non-stationary. This also indicates that the new model's assumptions cannot be rejected in favor of the Gibson and Schwartz alternative.

Table 8 reports payoffs of the five different maturities given exercise price equal to spread at the first trading day, which indicates at the money. In all cases, actual payoffs of put options are higher than actual payoffs of calls. At the first trading day, put-call parity holds for Poitras and Hinz and Fehr models whereas it is not satisfied for Gibson and Schwartz and the new model. The Gibson and Schwartz model does not impose put-call parity, but it has no built-in biases. The lack of parity means that the convenience yield at 100 calendar days from expiration differs from the historical mean convenience yield. This could occur if there is a trend in convenience yield or seasonality. The new model does have a builit-in bias due to truncating convenience yield at zero and not imposing efficiency on the spread market. This bias causes the model to overestimate call payoffs relative to puts. The payoffs of Dec-Mar and Dec-Jul spread, nevertheless, satisfy put-call parity, which can be explained by these maturities usually having positive convenience yield (not at full carry) and thus there is little truncation at zero.

The payoffs of in the money and out of money are presented in tables 9 and 10. Poitras and Hinz and Fehr models overestimate option payoffs. The problem with Poitras option pricing formula is that it assumes no cointegration of the two futures processes. If they are cointegrated then the Poitras model will overestimate the volatility of the spread. Table 7 reports panel cointegration tests for two futures. As expected, the hypothesis of no cointegration is rejected. The problem with Hinz and Fehr's model is that when the spread is almost equal to the lower bound of full carry, the simple ratio can approach infinity (very large), which inflates the volatility. Adding an arbitrary constant to keep the ratio away from zero might improve the Hinz and Fehr model.

Figures 17 to 21 depict nonparametric regressions of calendar spread and convenience yield against trading days to expiration for the last 100 calendar days. Dec-Mar, May-Jul, and Dec-Jul spread and convenience yield have a smile pattern while Mar-May spread and convenience yield decrease as maturity approaches and Jul-Dec spread and convenience yield declines after the last 20 trading days to maturity.

We report the bias (mean error) and the root mean squared error (RMSE) to compare the relative performance of the four models: alternative calendar spread option pricing models. As shown in tables 11 and 12, the new model clearly outperforms the three other models. For instance, at the first trading day of table 12, the RMSEs of call and put are 0.69 and 1.57 for the new model and the RMSEs of call are 4.1, 6.77, and 8.15 for Gibson and Schwartz, Poitras, and Hinz and Fehr model, respectively. The bias of the new model is negative for puts, which suggests underestimated put payoffs. This is possible because of imposing the restriction of truncation at zero for convenience yield. We also compute the prediction errors for significance tests with respect to the mean and variance. Table 13 indicates that three methods overestimate actual payoffs and the new model underestimates actual put payoffs. For example, the mean coefficients of put payoffs of the three models are positive and significant, 1.94, 0.83, and, 0.37, but that of put payoff of the new model is -0.05. The new model has the least bias, which again favors the new model over the other three models.

Table 14 presents the results of the significance test for the variance difference in models. The null hypothesis is no difference in error variance between the new model and the other models. The variance of Hinz and Fehr model is positive but insignificant (0.27), which implies that the variance of Hinz and Fehr is not different from the variance of the new model. Similarly, the variance of Poitras model of -0.13 is negative, but not significantly different from zero, which indicates no difference in error variance compared to the new model. The variance of Gibson and Schwartz is higher than that of the new model. In summary, there is no difference in Hinz and Fehr, Poitras, and the new model for the variance. The variance for Gibson and Schwartz is higher than that for the new model.

Summary and Conclusion

The theory of storage says that calendar spreads on a storable commodity are the sum of the opportunity cost of interest, the physical cost of storage, and convenience yield. We develop a new calendar spread option pricing model in which convenience yield follows arithmetic Brownian motion that is truncated at zero, nearby futures follows geometric Brownian motion, and interest rates and the physical cost of storage are held constant. A call option premium of the two-factor model is obtained using steps similar to that used to derive the Heston stochastic volatility model although our model does not assume stochastic volatility. The premium of a call option on a calendar spread is then obtained as the sum of the premium of the two-factor model minus the premium of call option on the convenience yield that has a strike price of zero.

We compute the implicit convenience yield based on the theory of storage since convenience yield is not obtainable directly. We perform the distributional tests for the calendar spreads and convenience yield to examine whether the models' assumptions are true. In all cases the null hypothesis of normality is rejected for both calendar spread and convenience yield. The histogram of the change in calendar spread, however, is somewhat similar to a normal distribution for Dec-Mar, May-Jul, and Dec-Jul spreads.

The distribution of convenience yield is strongly skewed to the right which supports the assumption that full carry is acting as a lower bound. Jul-Dec convenience yield is well fitted by a normal distribution since Jul-Dec calendar spreads are in backwardation where convenience yield is positive mostly. The variance of observed convenience yield does not go to zero and convenience yield usually stops a little short of full carry. This result may reflect market participants that have varying physical cost of storage and varying interest rates. Most commercial elevators are likely net borrowers, but some producers may be net lenders. It is also another possible reason that convenience yield may have measurement error.

We conduct a panel unit root test for five calendar spreads and convenience yield. The null hypothesis of a unit root cannot be rejected and thus the results support our assumption of Brownian motion over the Gibson and Schwartz (1990) assumption of mean reverting convenience yield. This also suggests that we cannot reject the new model's assumptions in favor of the Gibson and Schwartz alternative.

Monte Carlo simulation is used to obtain option payoffs for the new model as well as the Gibson and Schwartz model. The option payoffs for Poitras model as well as Hinz and Fehr model are calculated using analytical formulas. To compare the relative performance of the four models, we compute bias and RMSE. The findings imply that the new model outperforms the three other models. The negative bias of the new model suggests underestimated payoffs of puts, which is due to the restriction of truncated convenience yield at zero.

The significance tests for mean and volatility of the prediction error imply that the models overestimated payoffs in most cases, no difference in error variance for Hinz and Fehr and Poitras as well as the new model, and the least variance of the new model compared to Gibson and Schwartz.

Future study will need to improve the estimation of the drift of convenience yield δ in the new model. With efficient markets, the drift of convenience yield should be negative rather than zero although this study assumes zero drift of convenience yield. The performance of the Hinz and Fehr model is disappointing. It could perhaps be improved by adding a constant to expected full carry to assure that relative spreads were always sufficiently away from zero to avoid the creation of outliers when spread is near full carry.

The new calendar spread option pricing model developed here has the potential to allow traders to lower bid-ask spreads, which ultimately could increase volume in these markets much like has occurred with traders use of the Black-Scholes model. The new model is clearly more accurate than the three alternative models considered.

References

Bachelier, L. 1900. “ThCorie de la spkculation.” *Annales de 1’Ecole Superieure*, 17:21-86. Translated in Cootner, P. H., ed. 1964. *The Random Character of Stock Market Prices*. Cambridge MA: MIT Press, pp.17-78.

Black, F., and Scholes, M. 1973. “The Pricing of Options and Corporate Liabilities.” *Journal of Political Economy* 81: 637–654.

Brennan, M. J. 1958. “The Supply of Storage.” *The American Economic Review* 48: 50–72.

Bresson G. 2002. “Nonstationary Panels: Panel Unit Root Tests and Panel Cointegration.” Paper presented at EuroLab Courses High-Level Scientific Conferences, Nice, 17-29 June.

Chicago Mercantile Exchange (CME) Group. “Grain and Oilseed Calendar Spread Options.” accessible at http://www.cmegroup.com/trading/agricultural/files/AC-357_CSO_Sell_Sheet-FC_3.pdf.

Commandeur, J.J., and Koopman, S.J. 2007. *An Introduction to State Space Time Series Analysis*. New York: Oxford University Press

Fama, E.F., and K.R. French. 1987. “Commodity Futures Prices: Some Evidence on Forecast Power, Premiums, and the Theory of Storage.” *The Journal of Business* 60: 55–73.

Franken, J.R.V., P. Garcia, and S.H. Irwin. 2009. “Is Storage at a Loss Merely an Illusion of Spatial Aggregation?” *Journal of Agribusiness* 27:65-84.

Gibson, R., and Schwartz, E. 1990. “Stochastic Convenience Yield and the Pricing of Oil Contingent Claims.” *Journal of Finance* 45: 959–976.

de Goeij, W.J. 2008. “Modeling Forward Curves for Seasonal Commodities with an Application to Calendar Spread Options.” PhD dissertation, Free University of Amsterdam.
Heston, S.L. 1993. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies* 6: 327–343.

Heath, D., Jarrow, R. A., and Morton, A.J. 1992. “Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation.” *Econometrica* 60: 77–105.

Hinz, J., and Fehr, M. 2010. “Storage Costs in Commodity Option Pricing.” *SIAM Journal on Financial Mathematics* 1: 729-751.

Irwin, S.H., Garcia, P., Good, D.L., and Kunda, L.E. 2011. “Spreads and Non-convergence in Chicago Board of Trade Corn, Soybean, and Wheat Futures: Are Index Funds to Blame?” *Applied Economic Perspectives and Policy* 33: 116–142.

Nakajima K., and Maeda A. 2007. “Pricing Commodity Spread Options with Stochastic Term Structure of Convenience Yields and Interest Rates.” *Asia-Pacific Financial Markets* 14: 157-184.

Murphy, J.A. 1990. “A Modification and Re-Examination of the Bachelier Option Pricing Model.” *American Economist* 34: 34–41.

Poitras, G. 1990. “The Distribution of Gold Futures Spreads.” *The Journal of Futures Markets* 10: 643–659.

____. 1998. “Spread Options, Exchange Options, and Arithmetic Brownian Motion.” *Journal of Futures Markets* 18: 487-517.

Schachermayer, W., and Teichmann J. 2008. “How Close Are the Option Pricing Formulas of Bachelier and Black-Merton-Scholes?” *Mathematical Finance* 18:155-170.

Schwartz, E.S. 1997. "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging." *Journal of Finance* 52: 923-973.

Shimko, D. 1994. “Options on Futures Spreads: Hedging, Speculation and Valuation.” *The Journal of Futures Markets* 14: 183–213.

Thompson, S. 1986. “Returns to Storage in Coffee and Cocoa Futures Markets.” *The Journal of Futures Markets* 6: 541-564.

Working, H. 1949. “The Theory of Price of Storage.” *American Economic Review* 39: 1254-1262.

Zulauf, C.R., H. Zhou, and M.C. Roberts. 2006. “Updating the Estimation of the Supply of Storage.” *Journal of Futures Markets* 26: 657-676.

Table 1. The volume of Chicago Board of Trade calendar spread options and futures

Type	Name	VOLUME Y.T.D 2013[a]	VOLUME Y.T.D 2012
Futures	Corn	61,283,591	69,830,860
Futures	Soybean	42,666,223	48,159,958
Futures	Soybean Meal	18,603,518	16,952,919
Futures	Soybean Oil	21,913,169	25,457,886
Futures	Wheat	23,609,816	25,939,886
	SUM	**168,076,317**	**186,341,509**
Options	Corn	22,575,455	25,083,104
Options	Soybean	13,946,834	17,315,067
Options	Soybean Meal	1,906,741	1,547,393
Options	Soybean Oil	1,367,277	2,084,934
Options	Wheat	4,241,180	4,939,028
	SUM	**44,037,487**	**50,969,526**
CSOs	Consecutive Corn	156,590	68,017
CSOs	Consecutive Soybean	9,426	735
CSOs	Consecutive Soybean Meal	115	0
CSOs	Consecutive Soybean Oil	1,724	3,236
CSOs	Consecutive Wheat	30,703	43,206
CSOs	Corn Jul-Dec	92,000	102,292
CSOs	Corn Dec-Dec	2,239	4,933
CSOs	Corn Dec-Jul	562	2,476
CSOs	Soybean Jul-Nov	59,152	95,692
CSOs	Soybean Aug-Nov	363	1,831
CSOs	Soybean Nov-Jul	50	1,106
CSOs	Soybean Nov-Nov	587	52
CSOs	Soybean May-Nov	80	0
CSOs	Soybean Jan-May	0	270
CSOs	Soy Meal Jul-Dec	0	0
CSOs	Soy Oil Jul-Dec	460	50
CSOs	Wheat Dec-Jul	0	2,278
CSOs	Wheat Jul-Jul	675	2,539
CSOs	Wheat Jul-Dec	6,871	5,750
	SUM	**361,597**	**334,463**

a The data are from Dec 02, 2013.

Table 2. Summary statistics

Variable	Sample Period	Mean	Standard Deviation	Minimum	Maximum
Dec-Mar CBOT Corn					
Dec Futures (¢/bu)	8/12/1976-11/16/2011	279.18	102.10	161.50	775.25
Mar Futures (¢/bu)	8/12/1976-11/16/2011	289.33	103.08	173.00	787.25
ΔlnDec (¢/bu)	8/12/1976-11/16/2011	0.00	0.01	-0.07	0.09
Dec-Mar Spread (¢/bu)	8/12/1976-11/16/2011	-10.15	3.74	-19.50	3.75
ΔDec-Mar Spread (¢/bu)	8/12/1976-11/16/2011	0.00	0.50	-2.50	3.25
Dec-Mar Implicit Convenience Yield	8/12/1976-11/16/2011	1.52	4.08	-8.30	19.42
ΔDec-Mar Implicit Convenience Yield	8/12/1976-11/16/2011	0.00	0.52	-2.61	3.20
Three-month TB (%)	8/12/1976-11/16/2011	5.24	3.27	0.00	15.85
Prime rate(%)	8/12/1976-11/16/2011	8.29	3.33	3.25	20.50
Storage Costs (¢/bu)	8/12/1976-11/16/2011	6.16	1.00	4.50	9.00
Mar-May CBOT Corn					
Mar Futures (¢/bu)	11/12/1975-2/17/2012	287.65	102.01	142.75	712.75
May Futures (¢/bu)	11/12/1975-2/17/2012	294.36	102.71	150.75	723.00
ΔlnDec (¢/bu)	11/12/1975-2/17/2012	0.00	0.01	-0.08	0.07
Mar-May Spread (¢/bu)	11/12/1975-2/17/2012	-6.71	2.92	-13.75	2.50
ΔMar-May Spread (¢/bu)	11/12/1975-2/17/2012	-0.02	0.41	-2.25	2.25
Mar-May Implicit Convenience Yield	11/12/1975-2/17/2012	1.24	3.11	-5.17	12.57
ΔMar-May Implicit Convenience Yield	11/12/1975-2/17/2012	0.00	0.50	-2.46	9.02
Three-month TB (%)	11/12/1975-2/17/2012	5.23	3.42	0.00	17.14
Prime rate(%)	11/12/1975-2/17/2012	8.25	3.53	3.25	21.50
Storage Costs (¢/bu)	11/12/1975-2/17/2012	4.12	0.70	3.00	6.00
May-Jul CBOT Corn					
May Futures (¢/bu)	1/12/1976-4/20/2012	300.03	114.01	150.75	776.00
Jul Futures (¢/bu)	1/12/1976-4/20/2012	304.96	113.99	155.25	781.25
ΔlnDec (¢/bu)	1/12/1976-4/20/2012	0.00	0.01	-0.07	0.07
May-Jul Spread (¢/bu)	1/12/1976-4/20/2012	-4.93	4.43	-14.75	19.75
ΔMay-Jul Spread (¢/bu)	1/12/1976-4/20/2012	0.01	0.50	-2.50	4.50
May-Jul Implicit Convenience Yield	1/12/1976-4/20/2012	3.17	4.75	-5.77	30.06
ΔMay-Jul Implicit Convenience Yield	1/12/1976-4/20/2012	0.00	0.50	-2.79	4.37
Three-month TB (%)	1/12/1976-4/20/2012	5.28	3.56	0.02	16.00
Prime rate(%)	1/12/1976-4/20/2012	8.22	3.62	3.25	20.00
Storage Costs (¢/bu)	1/12/1976-4/20/2012	4.15	0.73	3.00	6.00

Note: Implicit convenience yield is computed by the prime rate times nearby futures prices plus storage costs.

Table 2. Summary statistics (continued)

Variable	Sample Period	Mean	Standard Deviation	Minimum	Maximum
Jul-Dec CBOT Corn					
Jul Futures (¢/bu)	3/12/1976-6/20/2012	307.23	119.69	160.75	787.00
Dec Futures (¢/bu)	3/12/1976-6/20/2012	302.00	105.29	171.75	780.00
ΔlnJul (¢/bu)	3/12/1976-6/20/2012	0.00	0.01	-0.06	0.06
Jul-Dec Spread (¢/bu)	3/12/1976-6/20/2012	5.22	30.49	-34.25	159.25
ΔJul-Dec Spread (¢/bu)	3/12/1976-6/20/2012	-0.04	2.55	-21.75	29.75
Jul-Dec Implicit Convenience Yield	3/12/1976-6/20/2012	25.75	32.29	-9.13	186.58
ΔJul-Dec Implicit Convenience Yield	3/12/1976-6/20/2012	0.00	2.54	-22.48	30.67
Three-month TB (%)	3/12/1976-6/20/2012	5.21	3.47	0.02	17.01
Prime rate(%)	3/12/1976-6/20/2012	8.25	3.66	3.25	20.50
Storage Costs (¢/bu)	3/12/1976-6/20/2012	10.39	1.81	7.50	15.00
Dec-Jul CBOT Corn					
Dec Futures (¢/bu)	8/12/1976-11/16/2011	279.21	102.11	161.50	775.25
Jul Futures (¢/bu)	8/12/1976-11/16/2011	299.33	103.62	182.00	794.00
ΔlnDec (¢/bu)	8/12/1976-11/16/2011	0.00	0.01	-0.07	0.09
Dec-Jul Spread (¢/bu)	8/12/1976-11/16/2011	-20.11	8.88	-42.00	11.00
ΔDec-Jul Spread (¢/bu)	8/12/1976-11/16/2011	-0.01	0.96	-3.75	5.00
Dec-Jul Implicit Convenience Yield	8/12/1976-11/16/2011	7.23	10.39	-18.84	47.77
ΔDec-Jul Implicit Convenience Yield	8/12/1976-11/16/2011	0.00	1.02	-4.12	5.40
Three-month TB (%)	8/12/1976-11/16/2011	5.24	3.27	0.00	15.85
Prime rate(%)	8/12/1976-11/16/2011	8.29	3.33	3.25	20.50
Storage Costs (¢/bu)	8/12/1976-11/16/2011	14.38	2.33	10.50	21.00

Note: Implicit convenience yield is computed by the prime rate times nearby futures prices plus storage costs.

Table 3. Distribution tests for corn futures spread and convenience yield

	Obs.	Skewness	Kurtosis	Kolmogorov-Smirnov	Cramer-von Mises	Anderson-Darling
Dec-Mar						
ΔDec-Mar Spread (c/bu)	2553	0.6	4.3	0.01*	0.005*	0.005*
ΔlnZ	2224	-0.8	19.1	0.01*	0.005*	0.005*
ΔDec-Mar Implicit Convenience Yield	2553	0.5	3.8	0.01*	0.005*	0.005*
Dec-Mar Implicit Convenience Yield	2553	0.9	1.5	0.01*	0.005*	0.005*
ΔlnDec	2553	0.1	3.1	0.01*	0.005*	0.005*
Mar-May						
ΔMar-May Spread (¢/bu)	2501	0.0	2.2	0.01*	0.005*	0.005*
ΔlnZ	2080	0.0	10.1	0.01*	0.005*	0.005*
ΔMar-May Implicit Convenience Yield	2501	4.4	76.9	0.01*	0.005*	0.005*
Mar-May Implicit Convenience Yield	2501	1.0	0.9	0.01*	0.005*	0.005*
ΔlnMar	2501	-0.3	5.8	0.01*	0.005*	0.005*
May-Jul						
ΔMay-Jul Spread (¢/bu)	2582	1.0	8.1	0.01*	0.005*	0.005*
ΔlnZ	2274	-0.2	10.0	0.01*	0.005*	0.005*
ΔMay-Jul Implicit Convenience Yield	2582	0.7	7.2	0.01*	0.005*	0.005*
May-Jul Implicit Convenience Yield	2582	1.4	**3.2**	0.01*	0.005*	0.005*
ΔlnMay	2582	0.0	4.0	0.01*	0.005*	0.005*
Jul-Dec						
ΔJul-Dec Spread (¢/bu)	2588	0.2	25.8	0.01*	0.005*	0.005*
ΔlnZ	2144	-1.6	36.8	0.01*	0.005*	0.005*
ΔJul-Dec Implicit Convenience Yield	2588	0.2	25.5	0.01*	0.005*	0.005*
Jul-Dec Implicit Convenience Yield	2588	2.1	4.7	0.01*	0.005*	0.005*
ΔlnJul	2588	0.1	1.9	0.01*	0.005*	0.005*
Dec-Jul						
ΔDec-Jul Spread (¢/bu)	2552	0.4	2.4	0.01*	0.005*	0.005*
ΔlnZ	2181	0.2	20.9	0.01*	0.005*	0.005*
ΔDec-Jul Implicit Convenience Yield	2552	0.3	2.2	0.01*	0.005*	0.005*
Dec-Jul Implicit Convenience Yield	2552	0.8	1.0	0.01*	0.005*	0.005*
ΔlnDec	2552	0.1	3.1	0.01*	0.005*	0.005*

Note: Implicit convenience yield is computed as the spread minus the prime rate times nearby futures price and also minus storage costs. * indicates rejection of the null hypothesis of normality at the 5% level.

Table 4. Panel unit root tests in corn futures spread and convenience yield, (1975-2012)

Variable	Spread	Convenience Yield
Dec-Mar		
Im-Pesaran-Shin Test	-1.40 (0.08)	-1.17 (0.12)
Fisher-type unit-root test	0.39 (0.35)	-0.37 (0.64)
Mar-May Futures Spread		
Im-Pesaran-Shin Test	-2.53 (0.01)	-2.74 (0.00)
Fisher-type unit-root test	0.30 (0.38)	0.10 (0.46)
Mar-July Futures Spread		
Im-Pesaran-Shin Test	-0.54 (0.29)	-0.28 (0.39)
Fisher-type unit-root test	-0.97 (0.16)	-0.01 (0.50)
July-Dec Futures Spread		
Im-Pesaran-Shin Test	-0.28 (0.39)	-0.34 (0.37)
Fisher-type unit-root test	1.69 (0.05)	1.41 (0.08)
Dec-July Futures Spread		
Im-Pesaran-Shin Test	0.35 (0.64)	1.12 (0.87)
Fisher-type unit-root test	-0.39 (0.65)	-1.31 (0.90)

Note: The null hypothesis is that panels contain a unit root and thus the null hypothesis is not rejected using any of the tests. Numbers in parentheses indicate p-values.

Table 5. The results of the first order auto regression for nearby futures prices

	$\hat{q}$	t-statistic	p-value
Dec-Mar spread	0.031	1.22	0.22
Mar-May spread	0.020	0.99	0.32
May-Jul spread	0.017	0.70	0.48
Jul-Dec spread	-0.005	-0.22	0.83
Dec-Jul spread	0.018	0.79	0.43

Table 6. Estimation of the parameters k and α of the Gibson and Schwartz model

	Dec-Mar		Mar-May		May-Jul		Jul-Dec		Dec-Jul	
Year	k	α	k	α	k	α	k	α	k	α
1976	0.040	3.117	0.027	4.421	0.023	5.359	-0.005	4.510	0.029	10.045
1977	0.038	2.952	0.026	4.040	0.021	4.882	-0.003	-21.664	0.031	10.483
1978	0.034	2.660	0.028	4.285	0.020	4.746	-0.003	-21.001	0.029	10.262
1979	0.029	1.806	0.056	2.527	0.021	5.161	-0.005	5.721	0.020	8.736
1980	0.041	0.446	0.015	0.618	0.015	4.754	-0.006	12.694	0.020	5.180
1981	0.016	0.407	0.029	0.190	0.000	-3.810	-0.001	12.597	0.003	4.768
1982	0.017	-0.641	0.002	-2.805	0.019	3.873	0.017	13.115	0.003	-8.177
1983	0.017	-0.042	0.011	-1.385	0.025	4.453	0.018	16.835	0.004	-2.685
1984	0.002	-1.159	0.012	-1.820	0.030	4.068	-0.007	14.079	-0.002	16.450
1985	0.005	6.245	0.012	-1.551	0.008	2.372	0.001	184.934	0.000	78.801
1986	0.003	12.253	0.010	-2.566	0.008	5.073	0.002	85.777	0.000	228.998
1987	0.004	18.117	0.018	-0.413	-0.003	-1.543	0.004	94.236	0.001	75.202
1988	0.004	15.357	0.012	0.237	0.000	402.146	0.003	123.198	0.001	76.274
1989	0.014	4.495	0.012	0.926	-0.005	3.028	-0.001	-14.017	0.018	12.088
1990	0.015	4.805	0.016	2.096	-0.007	4.118	-0.003	6.075	0.020	12.050
1991	0.016	3.932	0.015	1.639	-0.008	3.448	-0.003	0.100	0.018	10.030
1992	0.019	3.759	0.016	2.211	0.005	6.053	0.002	21.369	0.017	8.676
1993	0.013	2.145	0.033	1.222	0.000	5.006	-0.002	54.283	0.012	3.843
1994	0.012	2.480	0.039	1.186	0.001	25.565	0.002	1.761	0.006	3.720
1995	0.038	1.016	0.025	0.373	0.014	3.750	0.007	9.522	0.015	2.390
1996	0.011	1.123	0.014	0.000	0.030	3.060	0.010	6.589	-0.002	10.799
1997	0.001	22.348	0.023	-0.424	-0.007	4.636	-0.003	23.862	-0.002	-11.098
1998	0.004	7.492	0.023	1.201	-0.006	5.341	-0.002	-0.092	0.001	60.744
1999	0.000	27.513	0.019	2.876	-0.008	8.311	-0.003	17.421	-0.001	-24.375
2000	0.001	12.248	0.019	4.535	-0.007	5.778	-0.003	-4.283	-0.002	4.770
2001	0.000	-25.364	0.018	4.042	-0.007	5.171	-0.003	-6.194	0.004	1.609
2002	0.012	-1.489	0.018	4.702	0.006	0.410	0.003	-3.923	0.009	-3.903
2003	0.004	-1.763	0.004	3.770	0.037	1.575	0.023	0.627	0.006	-5.353
2004	0.004	0.962	0.003	0.352	0.003	3.099	0.000	183.699	0.006	-1.335
2005	0.001	-4.762	0.063	-0.433	0.003	-0.777	0.008	-1.214	0.003	-6.826
2006	-0.002	10.727	0.015	0.338	0.003	-2.645	0.008	-3.572	0.000	-622.796
2007	0.008	-1.053	0.016	0.128	-0.002	13.843	0.006	-9.253	0.005	4.090
2008	0.023	-2.484	0.013	-0.528	0.000	425.854	0.022	2.592	0.009	-0.638
2009	0.034	-3.954	0.007	-2.702	0.014	-2.977	0.022	-2.483	0.008	-7.228
2010	0.031	-3.801	0.031	0.069	0.024	-1.179	0.029	-2.145	0.007	-5.472
2011	0.020	-2.595	0.044	-0.795	0.025	-1.204	0.030	-0.994	0.003	17.739
2012	0.005	-1.364	0.053	-1.928	0.008	-1.998	0.006	-11.571	0.003	0.945
Average	0.014	3.187	0.022	0.828	0.008	25.914	0.005	21.438	0.008	-0.573

Table 7. Panel cointegration tests for corn futures

Futures	Pt - statistic	p-value	Pa - statistic	p-value
Dec-Mar	-6.60	0.001	-2.52	0.001
Mar-May	-7.93	0.000	-3.53	0.000
May-Jul	-5.63	0.015	-2.43	0.002
Jul-Dec	-11.55	0.000	0.00	0.000
Dec-Jul	-6.23	0.00	-2.19	0.008

Table 8. Payoffs of calendar spread options at the money

X=Spread: At the money

	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
Payoffs of Dec-Mar Calendar Spread Options										
Actual Payoff at Maturity					call:	1.33	put:	1.43		
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	1.76	1.75	1.36	0.61	1.18	1.49	2.03	2.03	4.11	4.11
15	1.43	1.89	1.11	0.91	1.00	1.57	1.40	1.85	3.12	3.62
30	1.23	1.85	1.01	1.05	0.89	1.52	1.27	1.86	2.21	2.84
50	1.33	1.58	1.26	1.15	0.97	1.36	2.07	2.21	2.00	2.03
Payoffs of Mar-May Calendar Spread Options										
Actual Payoff at Maturity					call:	0.30	put:	1.90		
	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	1.40	1.38	1.41	0.53	0.94	1.47	1.62	1.62	2.82	2.82
15	1.25	1.43	1.26	0.62	0.89	1.36	1.50	1.89	2.58	2.50
30	0.95	1.45	0.97	0.82	0.73	1.40	1.08	1.79	1.77	2.12
50	0.64	1.58	0.66	1.21	0.50	1.40	0.83	1.98	1.96	1.15
Payoffs of May-Jul Calendar Spread Options										
Actual Payoff at Maturity					call:	1.46	put:	1.61		
	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	1.13	1.13	1.13	0.96	8.43	0.00	2.13	2.13	4.28	4.28
15	1.15	1.09	1.11	1.06	8.32	0.00	1.75	1.72	2.94	3.08
30	0.90	1.39	0.90	1.23	7.93	0.01	1.51	2.09	2.12	2.49
50	1.03	1.54	1.57	1.64	7.86	0.01	1.26	1.72	1.83	1.37
Payoffs of Jul-Dec Calendar Spread Options										
Actual Payoff at Maturity					call:	4.07	put:	6.12		
	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	5.74	1.68	3.85	2.79	11.94	15.48	6.15	6.15	17.39	17.39
15	5.50	2.04	3.62	3.16	10.57	13.76	7.27	6.31	11.81	11.00
30	7.07	2.68	5.35	3.96	9.02	11.75	8.02	8.10	10.33	11.40
50	5.67	3.88	4.17	5.37	7.50	9.13	7.78	6.33	8.28	5.48
Payoffs of Dec-Jul Calendar Spread Options										
Actual Payoff at Maturity					call:	2.40	put:	2.85		
	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	2.61	2.61	2.72	1.84	3.31	3.83	16.73	16.73	9.52	9.52
15	2.46	2.65	2.55	1.94	2.89	3.86	4.02	4.67	6.23	7.39
30	1.95	3.20	2.05	2.66	2.55	3.81	3.31	4.39	3.86	5.62
50	2.23	3.18	2.29	2.86	2.42	3.42	2.48	3.65	2.65	3.83

Table 9. Payoffs of calendar spread options in the money

X=Spread-3: In the money								
Payoffs of Dec-Mar Calendar Spread Options								
Actual Payoff at Maturity			Call:	3.35	Put:	0.46		
	New model		Gibson & Schwartz		Poitras		Hinz	
Trading days to expiration	Call	Put	Call	Put	Call	Put	Call	Put
68	3.60	0.28	3.17	0.48	3.94	0.94	5.60	2.60
54	3.18	0.33	2.94	0.50	3.29	0.74	4.17	1.89
39	3.01	0.33	2.80	0.43	3.12	0.74	3.11	1.06
19	3.39	0.34	2.97	0.37	3.57	0.82	3.50	0.55
Payoffs of Mar-May Calendar Spread Options								
Actual Payoff at Maturity			Call:	1.76	Put:	0.36		
	New model		Gibson & Schwartz		Poitras		Hinz	
Trading days to expiration	Call	Put	Call	Put	Call	Put	Call	Put
61	3.91	0.04	2.90	0.43	3.62	0.62	4.75	1.75
47	3.71	0.07	2.95	0.42	3.40	0.78	4.71	1.27
32	3.23	0.08	2.72	0.39	2.94	0.65	3.48	0.80
12	2.69	0.25	2.45	0.35	2.50	0.64	3.34	0.61
Payoffs of May-Jul Calendar Spread Options								
Actual Payoff at Maturity			Call:	3.30	Put:	0.45		
	New model		Gibson & Schwartz		Poitras		Hinz	
Trading days to expiration	Call	Put	Call	Put	Call	Put	Call	Put
64	3.30	0.13	2.48	0.63	4.09	1.09	6.40	3.40
50	3.26	0.20	2.48	0.56	3.74	0.71	4.50	1.68
35	2.91	0.24	2.16	0.46	3.34	0.92	3.90	1.18
15	2.89	0.30	2.32	0.35	3.09	0.55	3.43	0.63
Payoffs of Jul-Dec Calendar Spread Options								
Actual Payoff at Maturity			Call:	5.45	Put:	4.50		
	New model		Gibson & Schwartz		Poitras		Hinz	
Trading days to expiration	Call	Put	Call	Put	Call	Put	Call	Put
63	5.74	1.68	13.43	13.97	7.84	4.84	19.57	16.57
49	6.42	1.89	12.02	12.21	8.98	5.03	12.98	9.18
34	7.07	2.68	10.51	10.24	9.70	6.78	12.03	10.25
14	7.04	3.66	9.05	7.68	9.56	5.10	10.48	4.63
Payoffs of Dec-Jul Calendar Spread Options								
Actual Payoff at Maturity			Call:	4.19	Put:	1.65		
	New model		Gibson & Schwartz		Poitras		Hinz	
Trading days to expiration	Call	Put	Call	Put	Call	Put	Call	Put
66	4.70	0.82	4.94	2.46	18.40	15.40	11.26	8.26
52	4.13	1.01	4.48	2.45	5.67	3.32	7.33	5.67
37	3.73	1.34	4.08	2.34	4.91	2.98	5.02	3.97
17	4.02	1.59	4.06	2.06	4.01	2.19	4.05	2.28

Table 10. Payoffs of calendar spread options out of money

X=Spread+3: Out of money

Payoffs of Dec-Mar Calendar Spread Options

Actual Payoff at Maturity call: 0.44 put: 3.55

Trading days to expiration	New model Call	New model Put	Gibson & Schwartz Call	Gibson & Schwartz Put	Poitras Call	Poitras Put	Hinz Call	Hinz Put
68	0.38	2.48	0.25	3.56	0.94	3.94	3.96	6.96
54	0.29	2.93	0.18	3.75	0.47	3.92	2.85	6.37
39	0.23	3.12	0.15	3.78	0.47	4.09	1.69	5.34
19	0.33	3.05	0.15	3.54	0.80	4.05	1.09	4.14

Payoffs of Mar-May Calendar Spread Options

Actual Payoff at Maturity call: 0.03 put: 4.63

Trading days to expiration	New model Call	New model Put	Gibson & Schwartz Call	Gibson & Schwartz Put	Poitras Call	Poitras Put	Hinz Call	Hinz Put
61	0.43	2.56	0.18	3.71	0.62	3.62	2.62	5.62
47	0.33	2.68	0.14	3.61	0.60	3.98	2.31	5.25
32	0.22	3.06	0.09	3.76	0.33	4.04	1.36	4.71
12	0.15	3.71	0.05	3.96	0.23	4.38	0.83	3.27

Payoffs of May-Jul Calendar Spread Options

Actual Payoff at Maturity call: 0.69 put: 3.84

Trading days to expiration	New model Call	New model Put	Gibson & Schwartz Call	Gibson & Schwartz Put	Poitras Call	Poitras Put	Hinz Call	Hinz Put
64	0.23	3.06	0.13	4.28	1.09	4.09	4.15	7.15
50	0.23	3.18	0.16	4.24	0.73	3.69	2.62	5.76
35	0.17	3.50	0.04	4.34	0.66	4.23	1.63	5.03
15	0.32	3.74	0.07	4.10	0.49	3.95	0.89	4.31

Payoffs of Jul-Dec Calendar Spread Options

Actual Payoff at Maturity call: 3.16 put: 8.22

Trading days to expiration	New model Call	New model Put	Gibson & Schwartz Call	Gibson & Schwartz Put	Poitras Call	Poitras Put	Hinz Call	Hinz Put
63	2.57	4.52	10.68	17.22	4.84	7.84	16.99	19.99
49	3.20	4.68	9.33	15.52	5.88	7.93	11.12	13.31
34	4.10	5.71	7.76	13.50	6.69	9.77	9.49	13.54
14	4.10	6.72	6.25	10.89	6.37	7.92	6.95	7.11

Payoffs of Dec-Jul Calendar Spread Options

Actual Payoff at Maturity call: 1.13 put: 4.59

Trading days to expiration	New model Call	New model Put	Gibson & Schwartz Call	Gibson & Schwartz Put	Poitras Call	Poitras Put	Hinz Call	Hinz Put
66	1.41	3.54	2.12	5.63	15.40	18.40	9.27	12.27
52	1.12	3.99	1.75	5.72	2.81	6.46	5.82	9.98
37	0.98	4.59	1.49	5.75	2.23	6.31	3.30	8.03
17	1.17	4.74	1.29	5.29	1.53	5.71	1.84	6.07

Table 11. Bias and RMSE of calendar spread options by exercise price

Bias and RMSE of Calendar Spread Options

X=Spread: At the money

	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
	Bias		Bias		Bias		Bias		Bias	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.6	-1.1	0.2	-1.4	3.3	1.7	3.8	3.0	5.7	4.9
15	0.5	-1.0	0.0	-1.2	2.8	1.4	1.3	0.5	3.4	2.8
30	0.5	-0.7	0.1	-0.8	2.3	0.9	1.1	0.9	2.1	2.1
50	0.3	-0.4	0.1	-0.3	1.9	0.3	1.0	0.4	1.4	0.0
	RMSE		RMSE		RMSE		RMSE		RMSE	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.9	2.0	0.5	1.7	4.7	4.3	6.5	6.2	7.1	6.1
15	0.8	1.9	0.5	1.5	4.2	3.5	1.7	0.8	4.1	3.2
30	1.4	1.6	0.7	1.1	3.7	2.6	1.8	1.2	3.0	2.8
50	0.8	1.0	0.2	0.5	3.3	1.5	1.7	0.5	2.1	0.7

X=Spread-3: In the money

	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
	Bias		Bias		Bias		Bias		Bias	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.5	-0.5	0.7	-0.9	2.0	2.1	5.1	3.1	7.0	5.0
15	0.3	-0.5	0.6	-0.8	1.6	1.7	2.5	0.6	4.2	2.5
30	0.2	-0.3	0.4	-0.6	1.0	1.3	2.3	0.9	3.0	2.0
50	0.0	-0.2	0.4	-0.3	0.7	0.7	2.0	0.4	2.4	0.3
	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
	RMSE		RMSE		RMSE		RMSE		RMSE	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.8	0.8	1.0	1.3	4.1	4.2	7.3	6.2	9.6	6.4
15	0.7	0.7	1.0	1.2	3.4	3.5	4.1	0.8	6.2	2.9
30	0.9	0.5	1.0	0.9	2.8	2.6	4.4	1.2	5.5	2.8
50	0.4	0.3	0.8	0.4	2.1	1.4	4.3	0.4	4.7	0.3

X=Spread+3: Out of money

	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
	Bias		Bias		Bias		Bias		Bias	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.0	-0.9	-0.1	-1.7	1.6	1.9	3.5	2.6	6.3	5.4
15	-0.2	-0.8	-0.1	-1.4	1.2	1.6	1.0	0.2	3.9	3.2
30	0.1	-0.3	0.0	-0.9	0.8	1.3	1.0	0.7	2.4	2.4
50	0.0	-0.1	0.1	-0.5	0.5	0.6	0.8	0.2	1.2	0.0
	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
	RMSE		RMSE		RMSE		RMSE		RMSE	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.4	1.3	0.4	2.0	3.4	4.1	6.4	6.2	7.6	6.7
15	0.5	1.2	0.3	1.9	2.8	3.3	1.5	0.9	4.5	3.7
30	0.5	0.9	0.5	1.3	2.1	2.5	1.7	1.1	3.1	3.0
50	0.2	0.4	0.5	0.8	1.4	1.3	1.5	0.6	1.8	1.1

Table 12. Overall Bias and RMSE of calendar spread options according by exercise price

Overall Bias and RMSE

Bias of Calendar Spread Options

	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
	Bias		Bias		Bias		Bias		Bias	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.35	-0.83	0.26	-1.35	2.26	1.91	4.12	2.90	6.34	5.11
15	0.19	-0.73	0.17	-1.16	1.87	1.57	1.59	0.47	3.83	2.80
30	0.27	-0.44	0.20	-0.77	1.39	1.17	1.46	0.85	2.51	2.16
50	0.07	-0.24	0.20	-0.37	1.05	0.53	1.26	0.35	1.70	0.10

RMSE of Calendar Spread Options

	Untruncation		Constant δ		Gibson & Schwartz		Poitras		Hintz	
	Bias		Bias		RMSE		RMSE		RMSE	
Trading days	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
1	0.74	1.47	0.71	1.72	4.10	4.20	6.77	6.19	8.15	6.39
15	0.65	1.35	0.65	1.56	3.54	3.44	2.72	0.86	5.00	3.28
30	1.02	1.10	0.78	1.11	2.91	2.57	2.91	1.17	4.02	2.86
50	0.51	0.67	0.57	0.59	2.38	1.42	2.80	0.51	3.16	0.76

Table 13. Significance tests for error means

Least Squares Means						
Method	Option	Estimate	Standard Error	DF	t Value	Pr > |t|
Hinz & Fehr	Call	1.95	0.15	454	12.95	<.0001
	Put	1.94	0.12	454	15.85	<.0001
Poitras	Call	0.70	0.13	454	5.23	<.0001
	Put	0.83	0.11	454	7.39	<.0001
Gibson & Schwartz	Call	0.32	0.17	454	1.89	0.06
	Put	0.37	0.14	454	2.69	0.01
New model	Call	**0.20**	0.10	454	1.92	**0.06**
	Put	**-0.05**	0.09	454	-0.55	**0.58**

Table 14. Significance tests for error variances

Variance Parameter Estimates			
Cov Parm	Estimate	Standard Error	Pr Z
Trading days 1	1.29	0.16	<.0001
Trading days 2	-0.09	0.12	0.45
Trading days 3	-0.36	0.12	0.00
Dec-Mar spread	0.94	0.22	<.0001
Mar-May spread	-2.32	0.18	<.0001
May-Jul spread	2.33	0.14	<.0001
Jul-Dec spread	-0.40	0.17	0.02
Call option	0.39	0.09	<.0001
Exercise: ATM	0.25	0.12	0.03
Exercise: ITM	0.03	0.12	0.79
Hinz & Fehr	**0.27**	0.14	**0.05**
Poitras	**-0.02**	0.17	**0.88**
Gibson & Schwartz	**0.59**	0.20	**0.00**
Residual	1.59	0.11	<.0001

Note: Exponential function is used to estimate variance for the prediction error. The variance equation is specified as $\sigma^2_{gnlht} = \exp\left[a_0^v + \sum_{i=1}^{3} a_i^v D_{ig} + \sum_{i=1}^{4} b_i^v D_{in} + c_l^v D_l + \sum_{i=1}^{3} d_i^v D_{ih}\right]$.

Figure 1. Plots of CBOT Dec-Mar corn spread

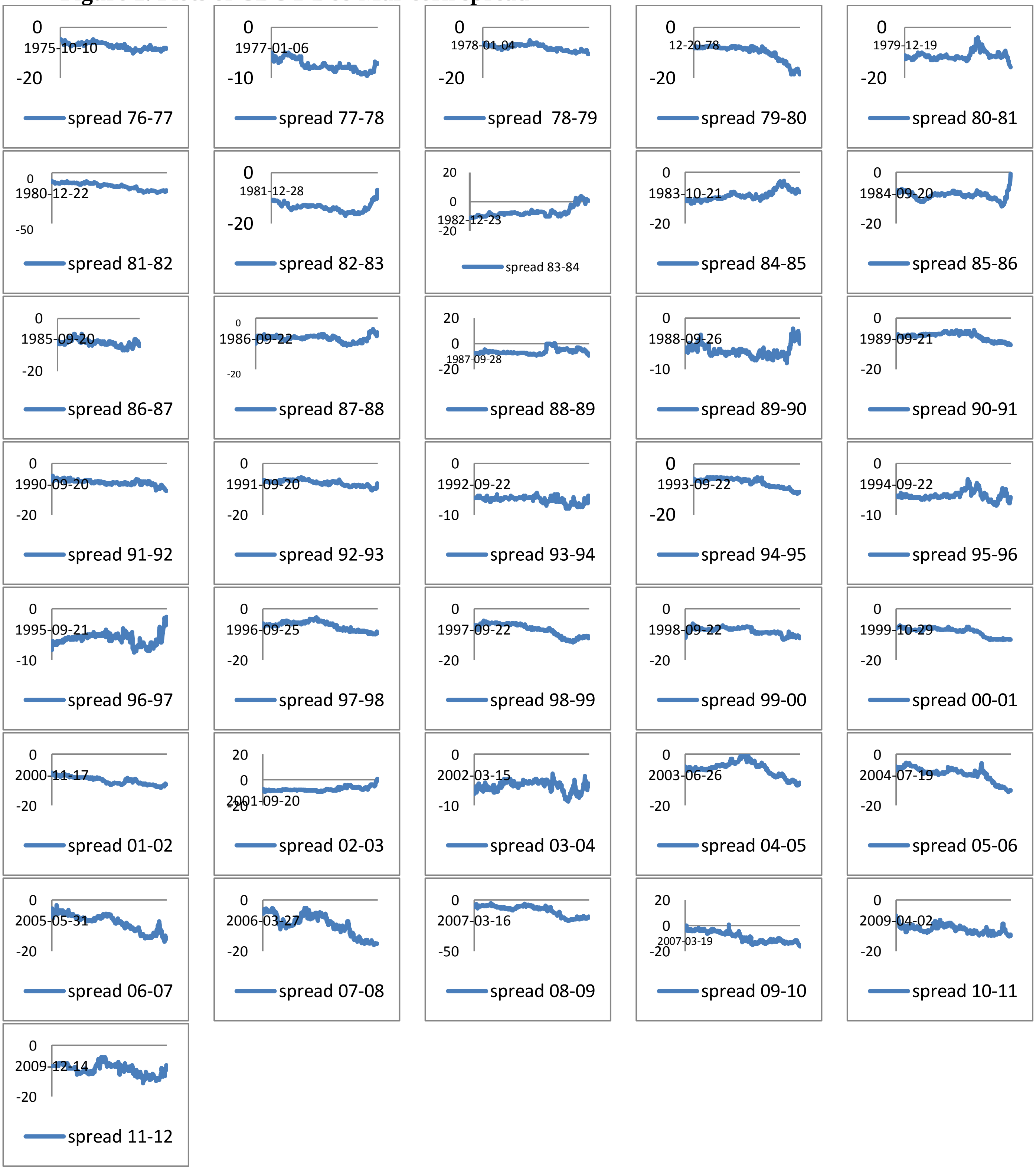


Figure 2. Plots of CBOT Mar-May corn spread

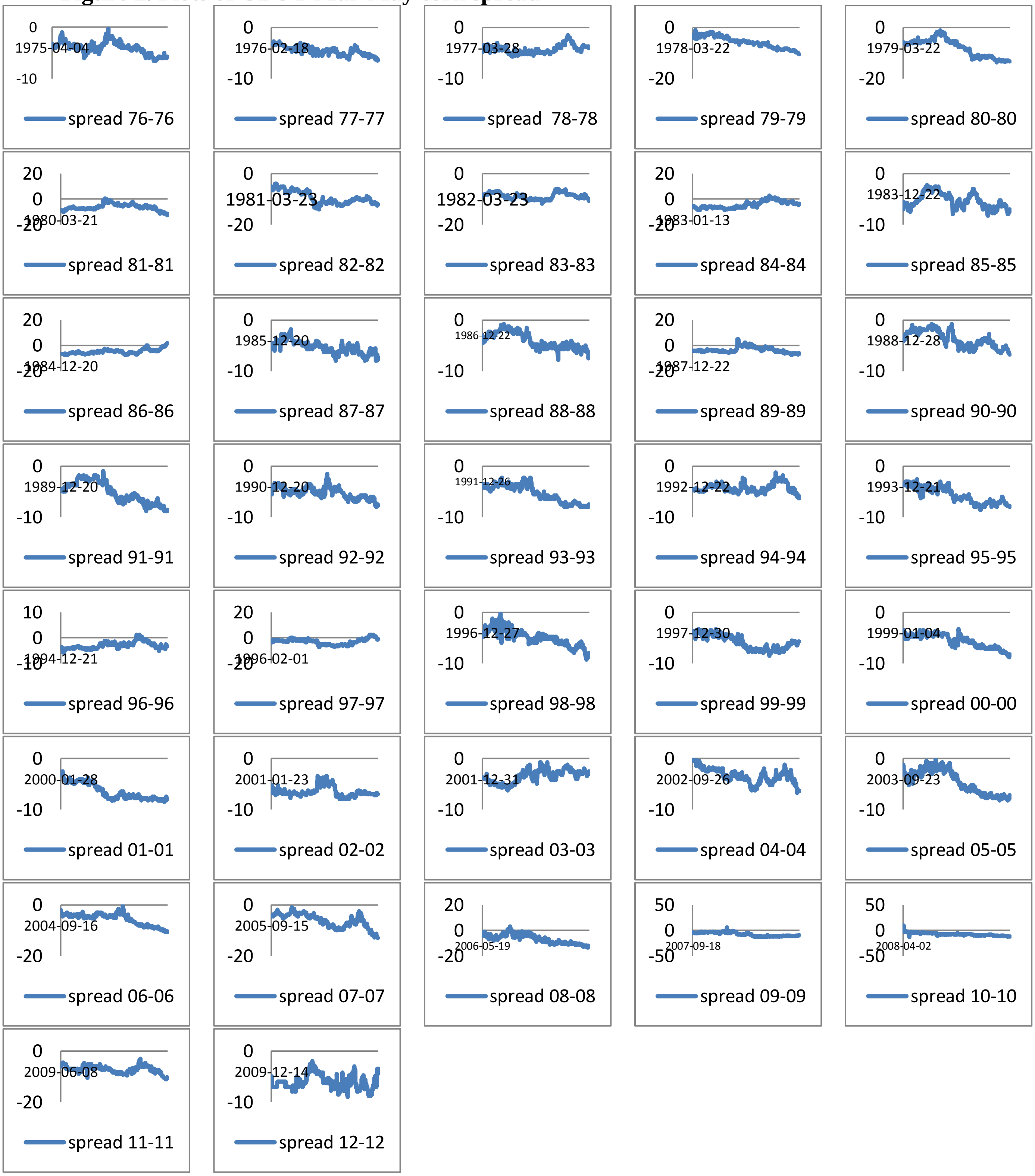


Figure 3. Plots of CBOT May-Jul corn spread

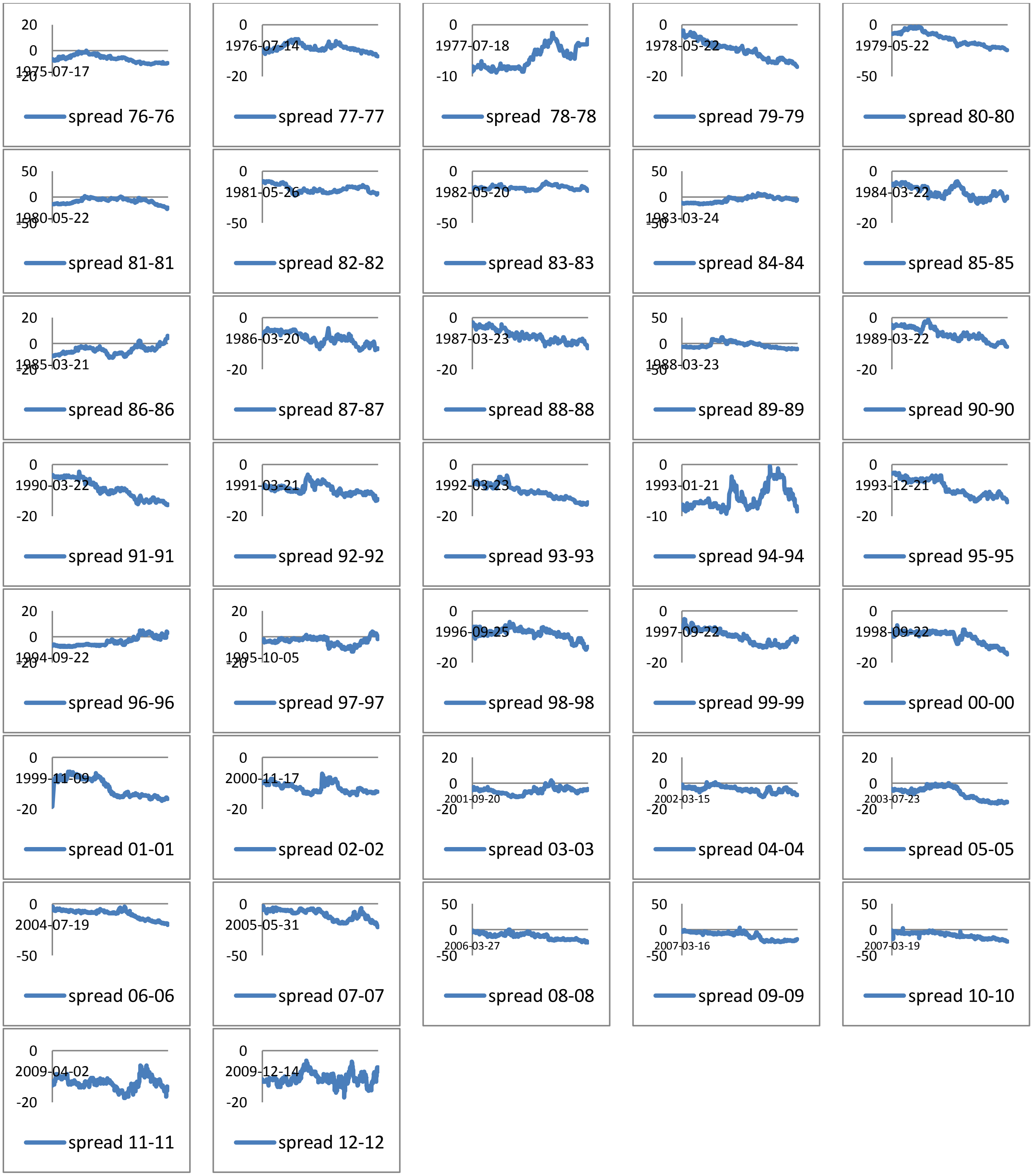


Figure 4. Plots of CBOT Jul-Dec corn spread

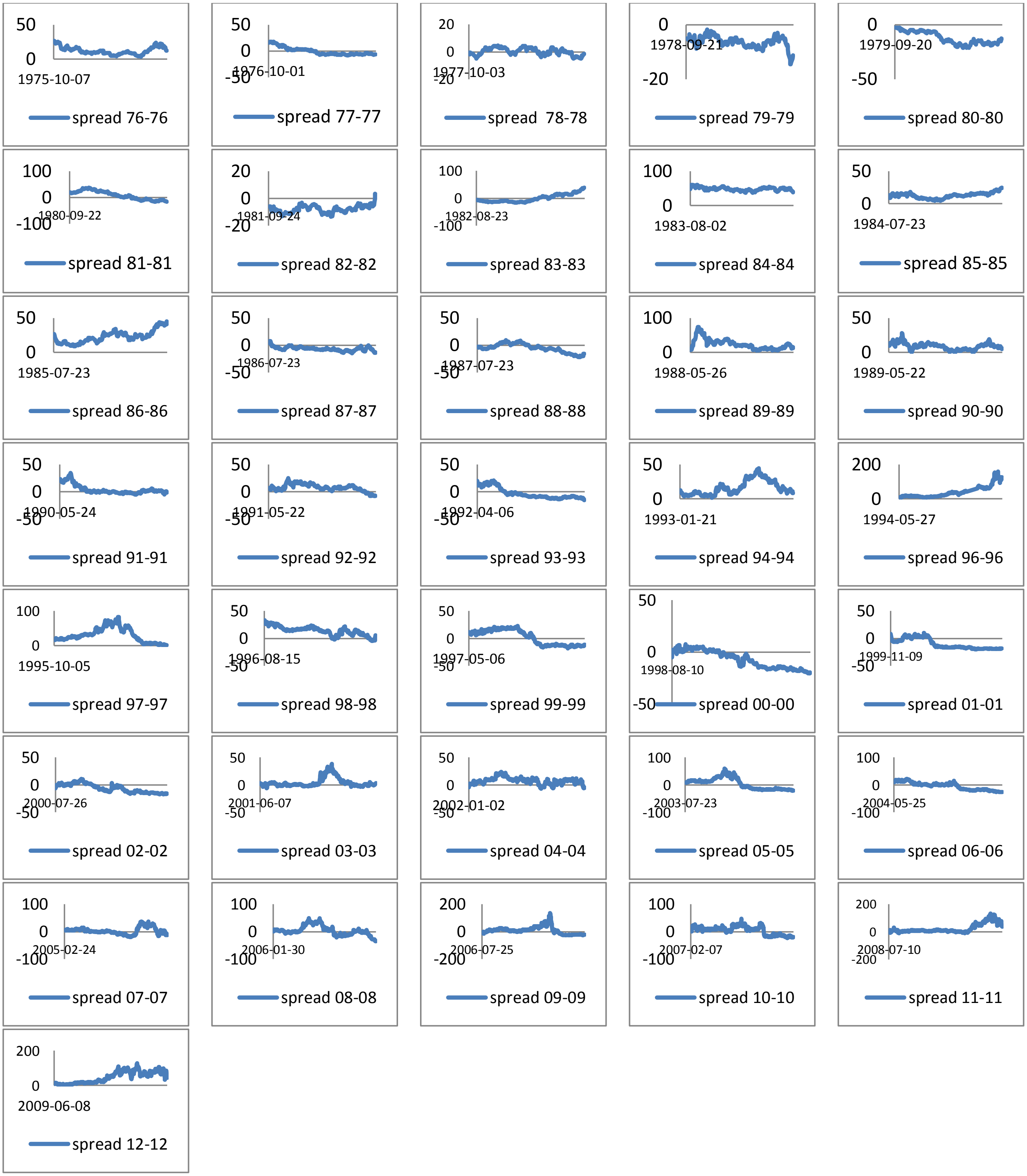


Figure 5. Plots of CBOT Dec-Jul corn spread

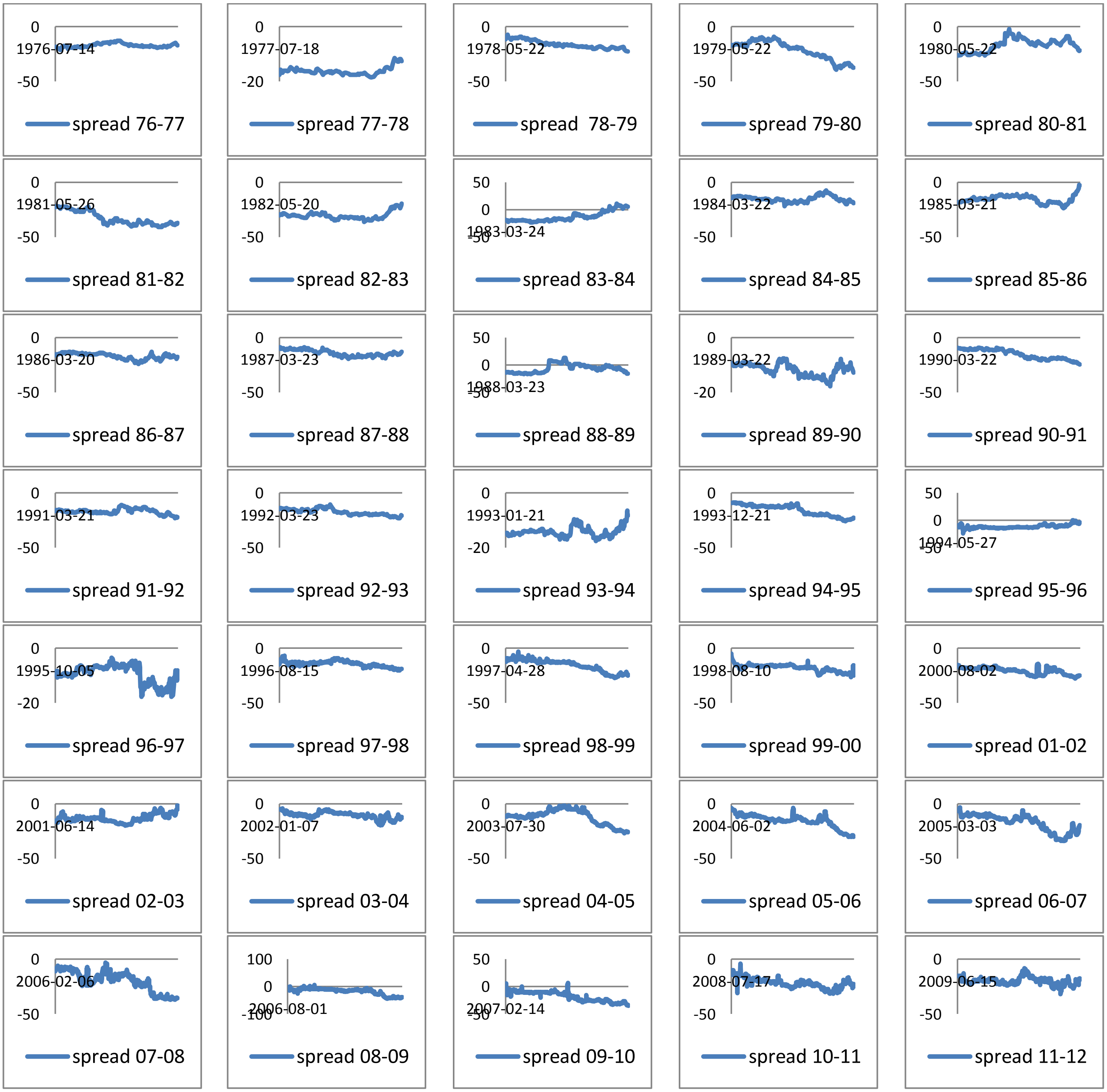
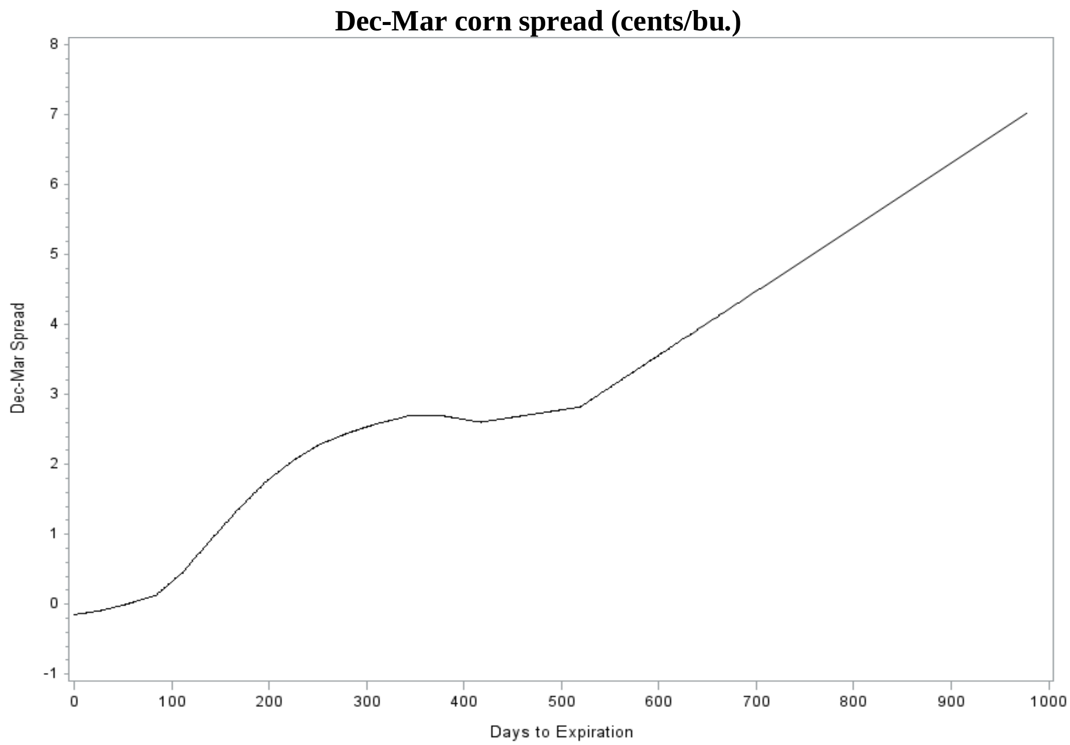


Figure 6. Nonparametric regression of corn spread versus days to expiration



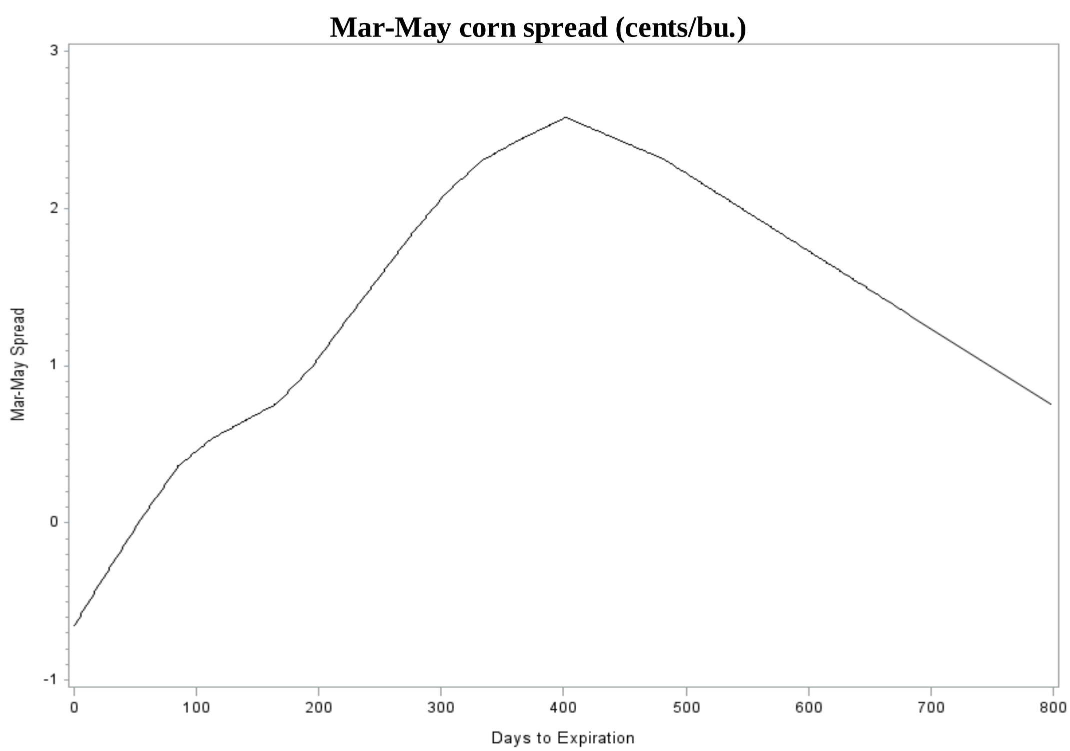
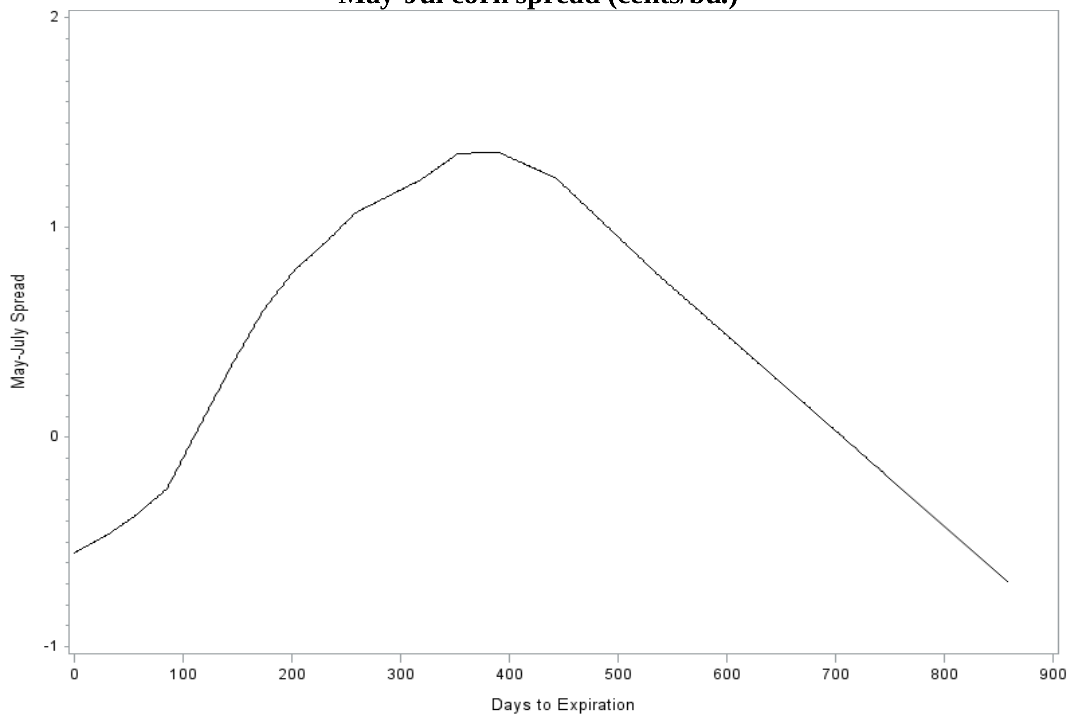


Figure 6. Nonparametric regression of corn spread versus days to expiration (continued)

May-Jul corn spread (cents/bu.)



Jul-Dec corn spread (cents/bu.)

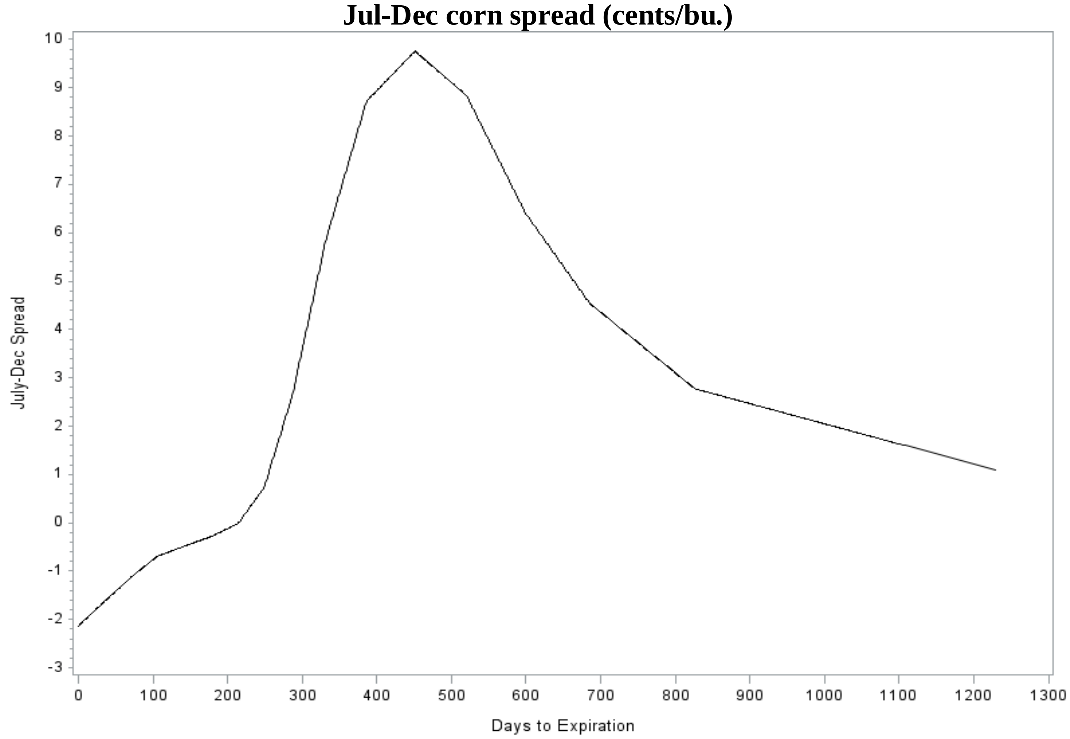


Figure 6. Nonparametric regression of corn spread versus days to expiration (continued)

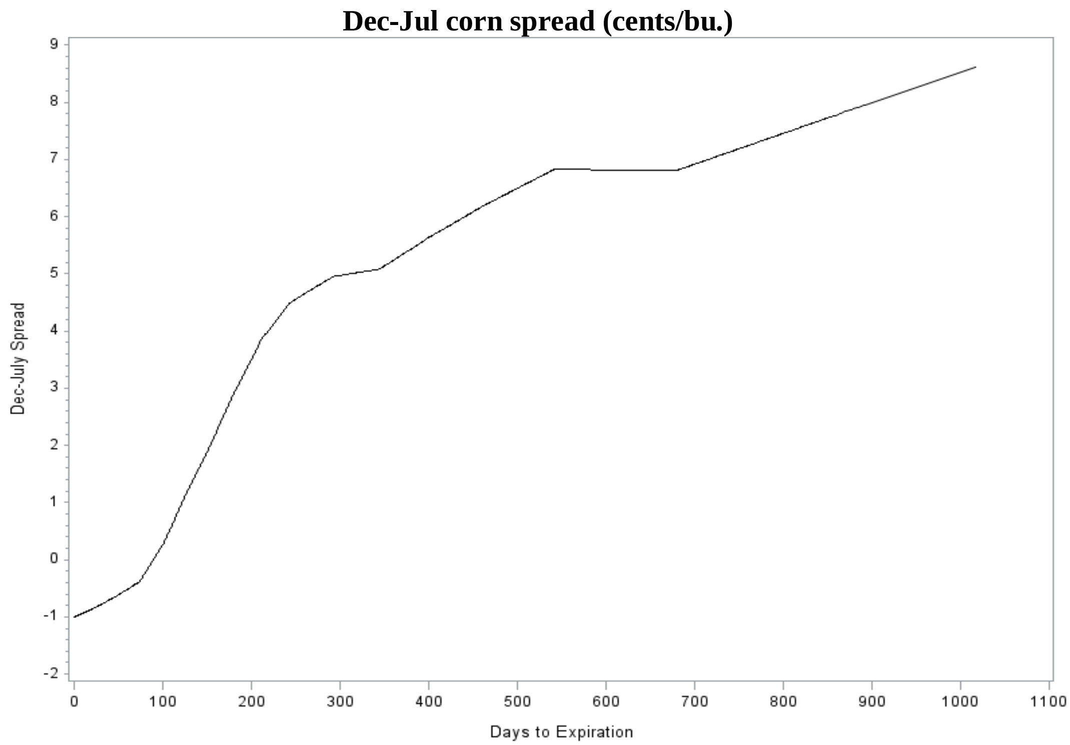
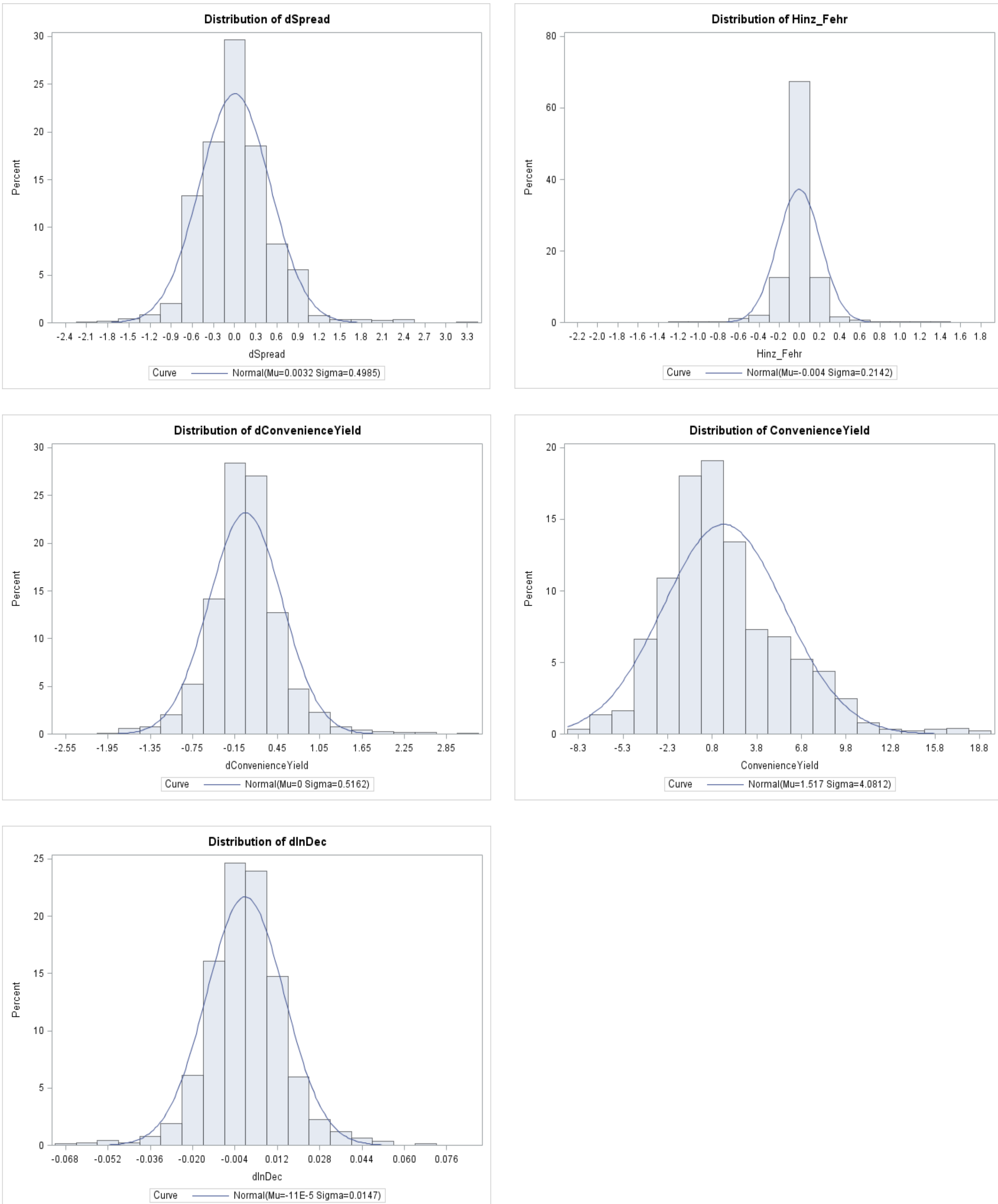
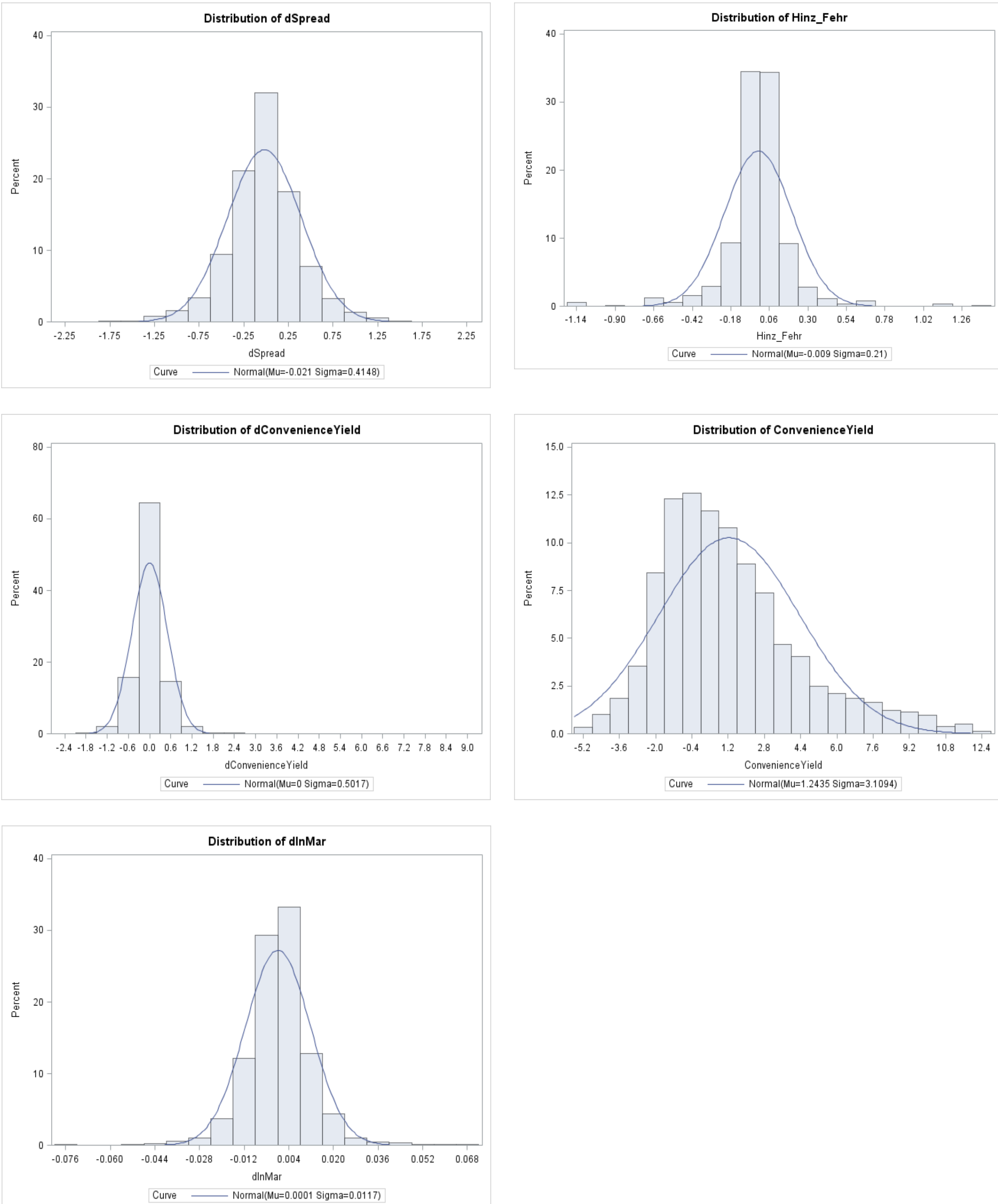


Figure 7. Histograms of Dec-Mar spread and convenience yield, (1976-2011)



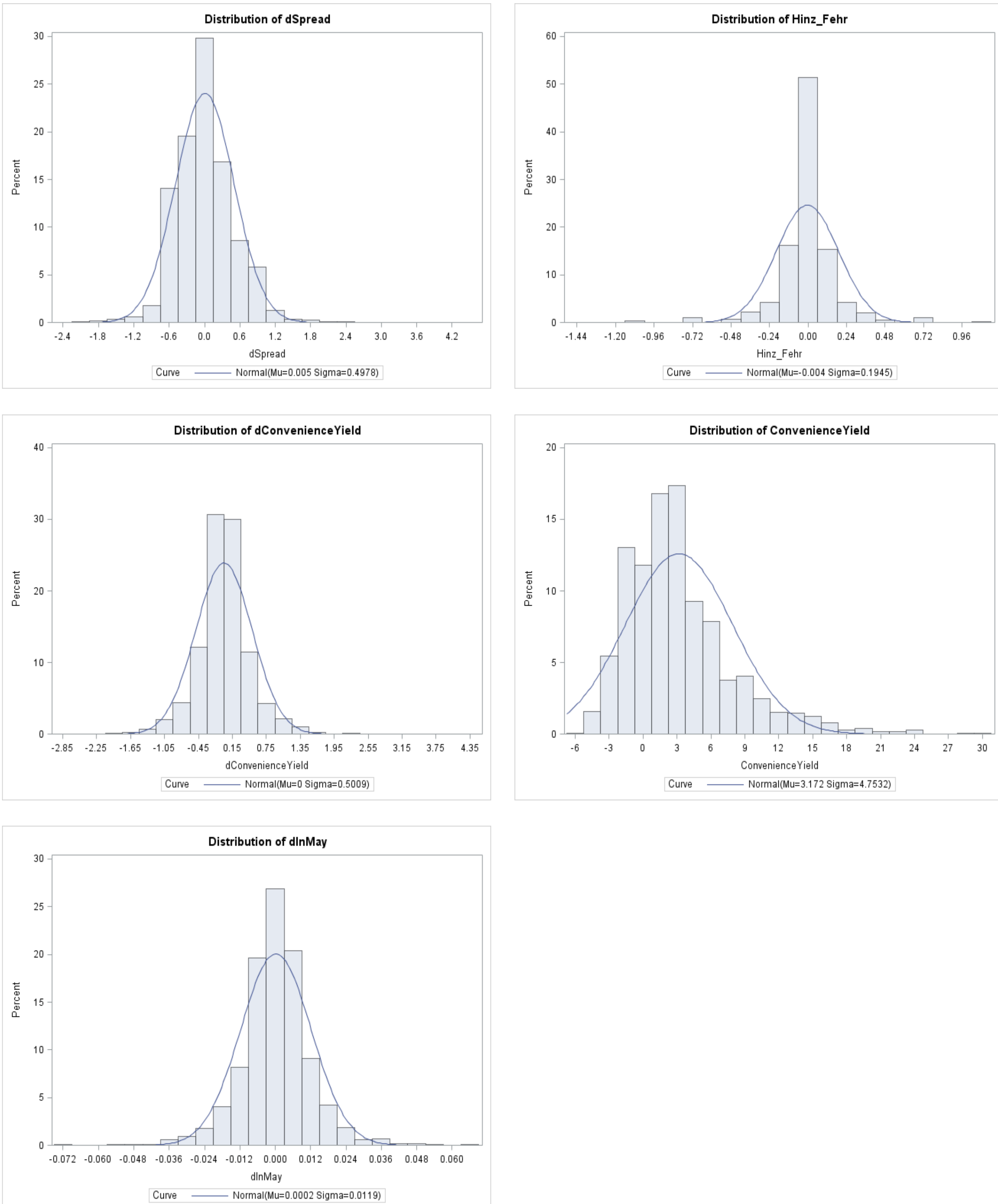
Note: Convenience yield is computed as the spread minus the prime rate times nearby futures price and also minus storage costs.

Figure 8. Histograms of Mar-May spread and convenience yield, (1975-2012)



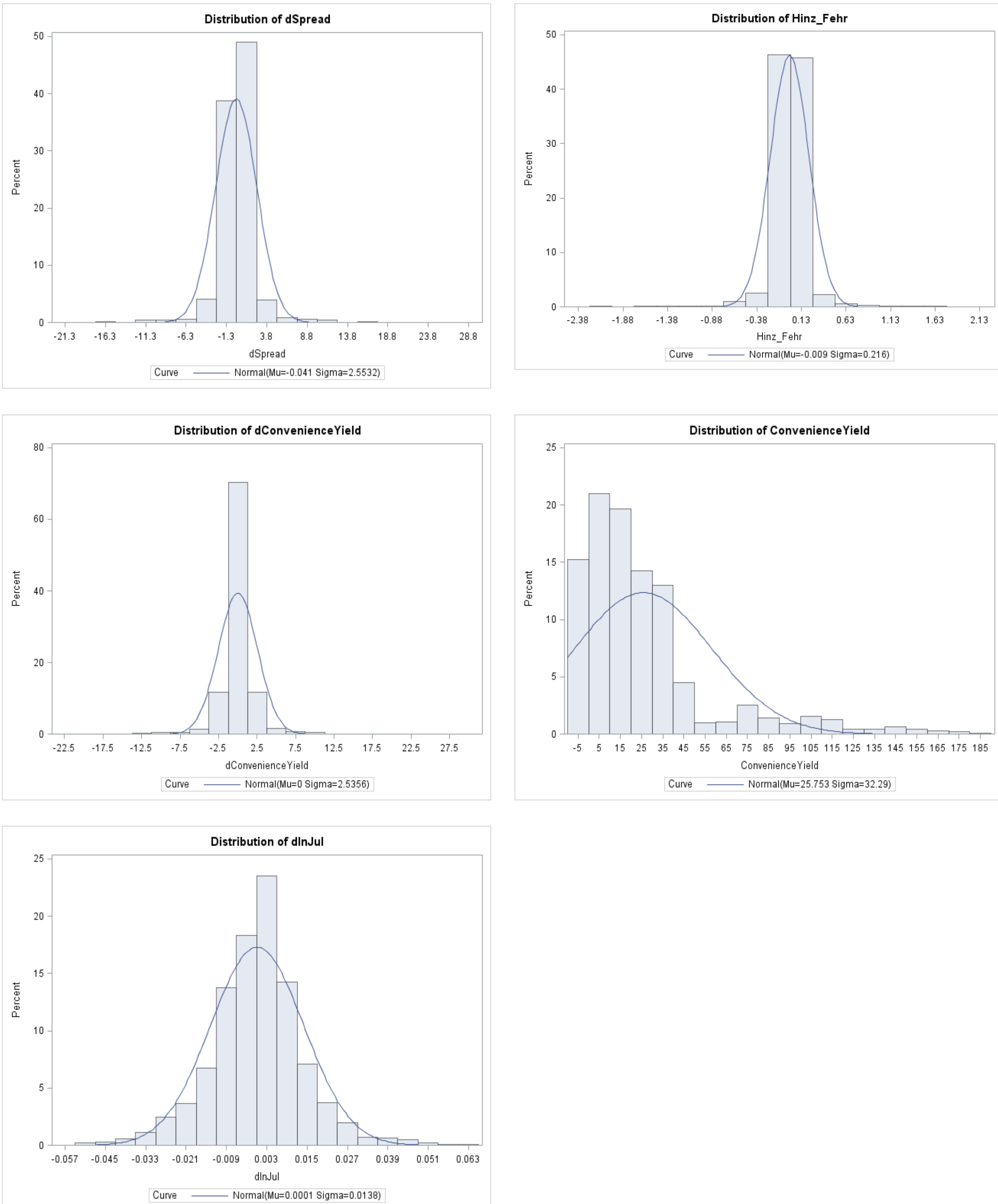
Note: Convenience yield is computed as the spread minus the prime rate times nearby futures price and also minus storage costs.

Figure 9. Histograms of May-Jul spread and convenience yield, (1975-2012)



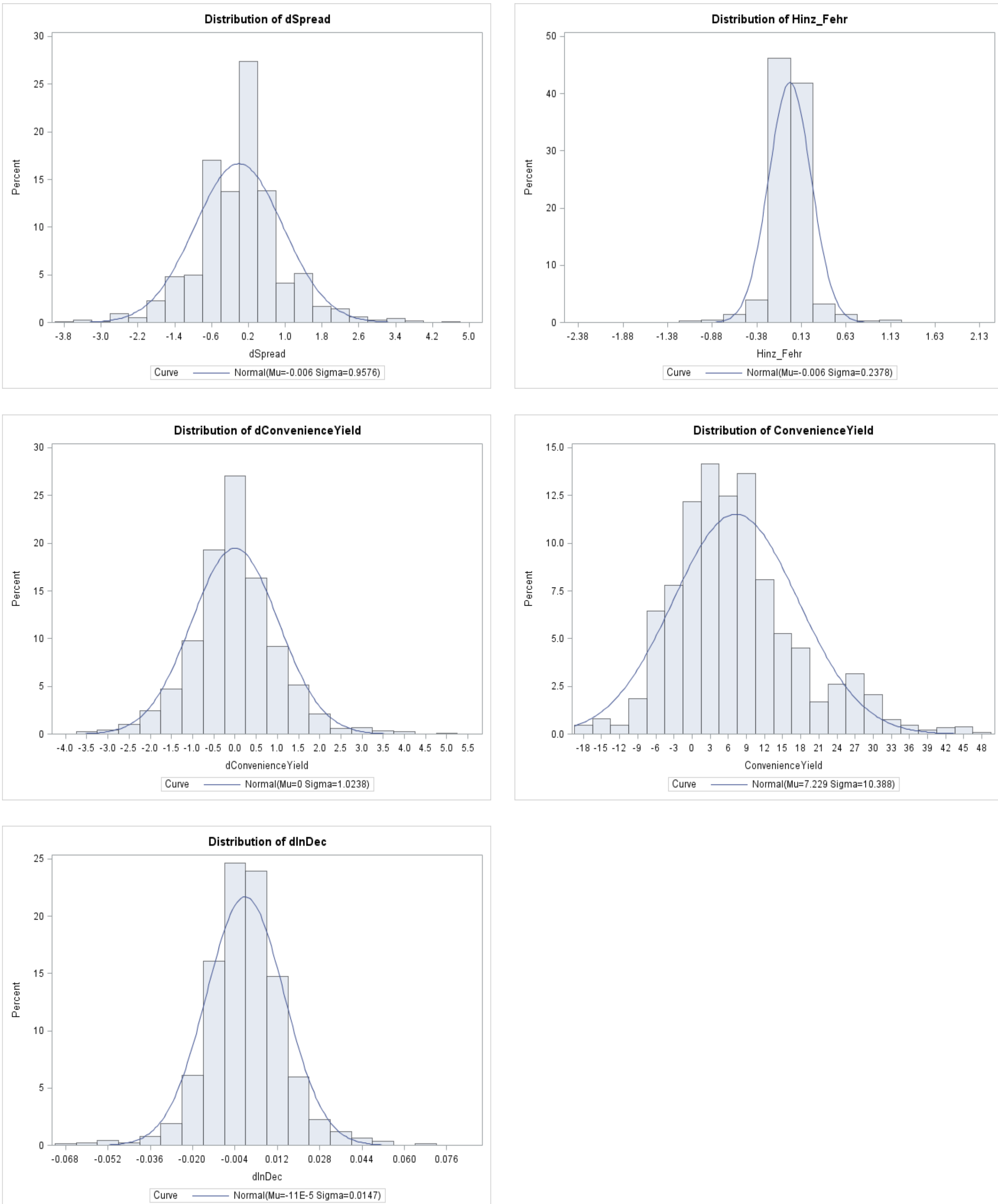
Note: Convenience yield is computed as the spread minus the Prime rate times nearby futures price and also minus storage costs.

Figure 10. Histograms of Jul-Dec spread and convenience yield, (1976-2012)



Note: Convenience yield is computed as the spread minus the Prime rate times nearby futures price and also minus storage costs.

Figure 11. Histograms of Dec-Jul spread and convenience yield, (1976-2011)



Note: Convenience yield is computed as the spread minus the Prime rate times nearby futures price and also minus storage costs.

Figure 12. Dec-Mar convenience yield plots by year, (1976-2011)

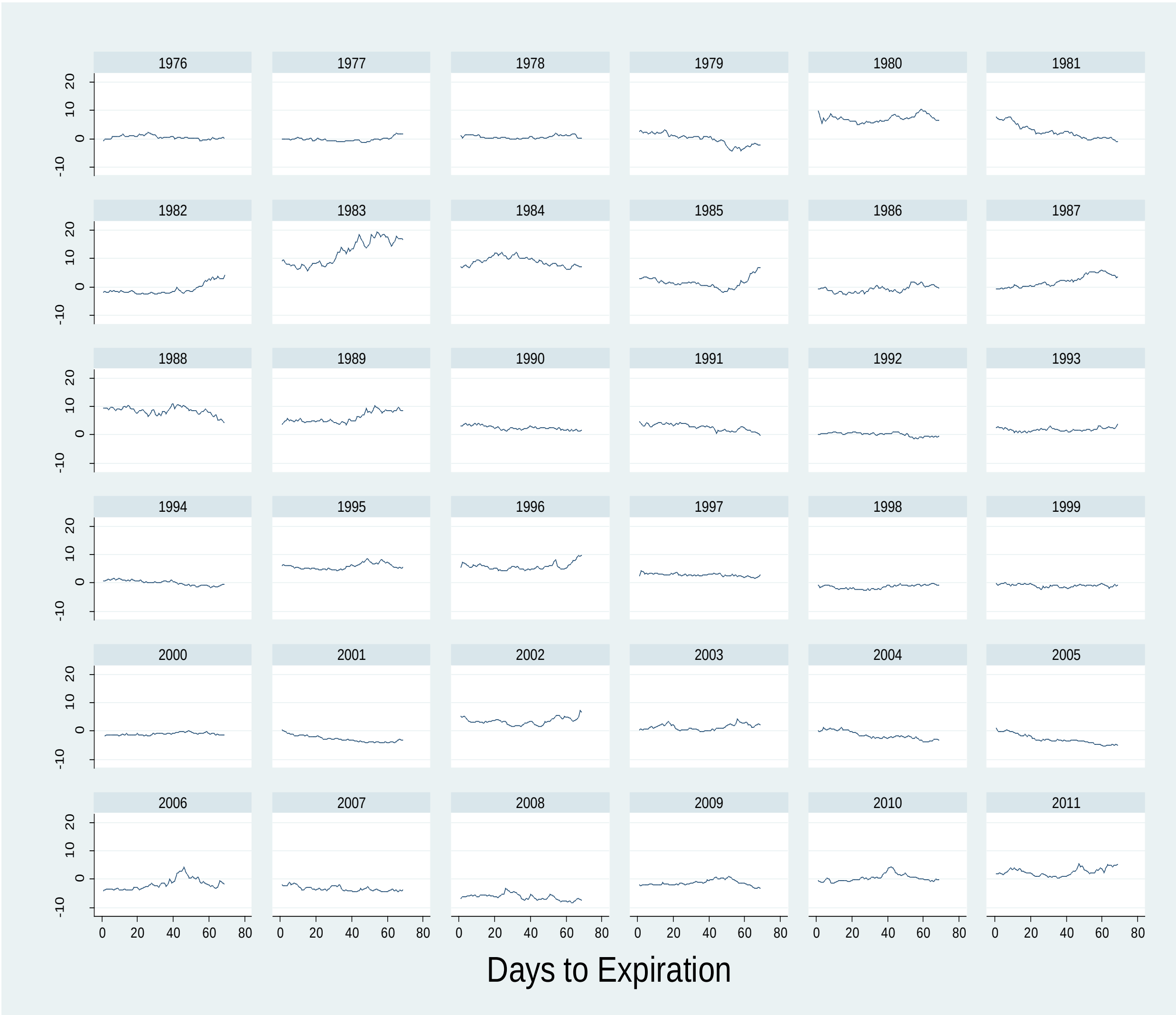


Figure 13. Mar-May convenience yield plots by year, (1975-2012)

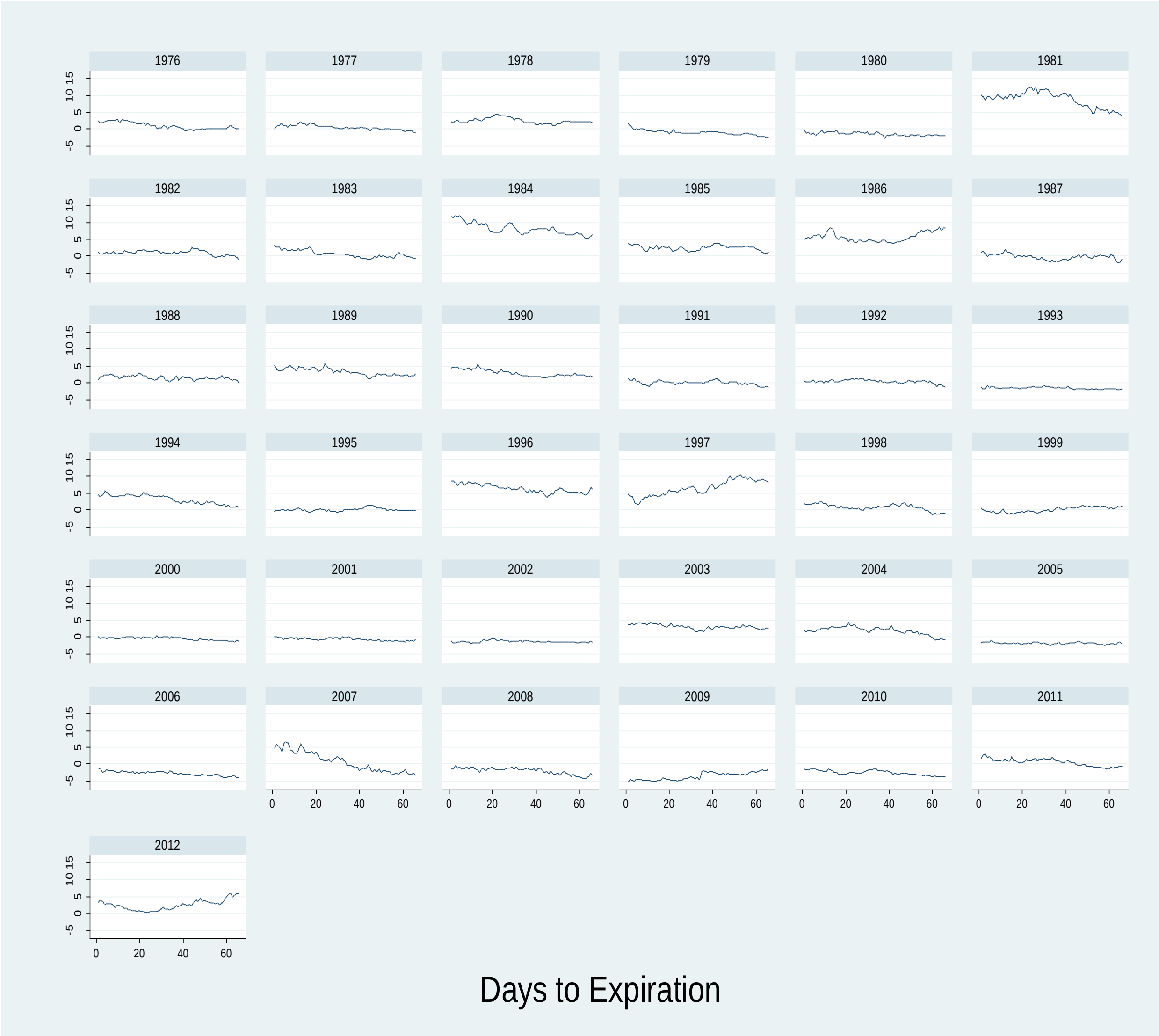


Figure 14. May-Jul convenience yield plots by year, (1976-2012)

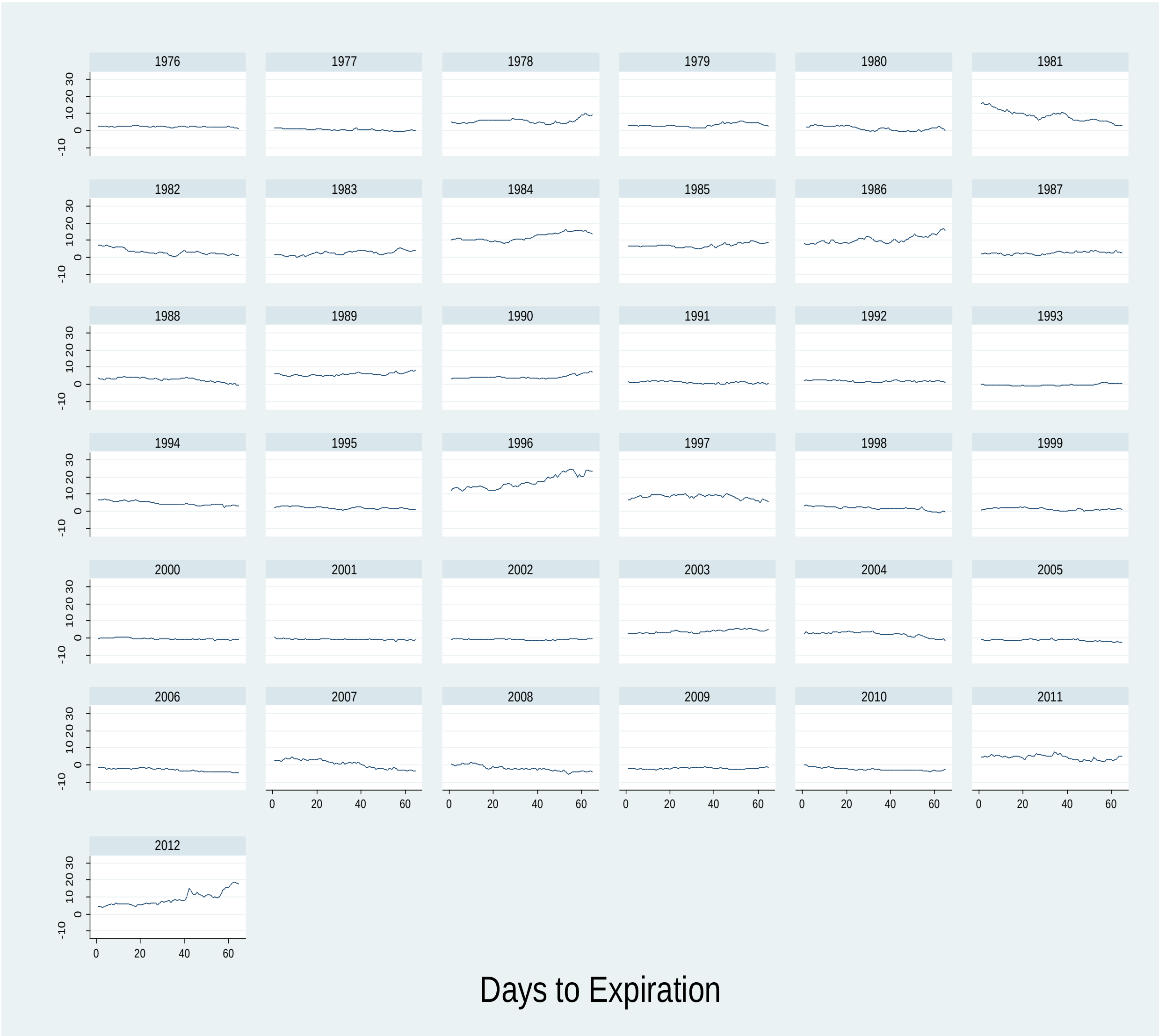


Figure 15. Jul-Dec convenience yield plots by year, (1976-2012)

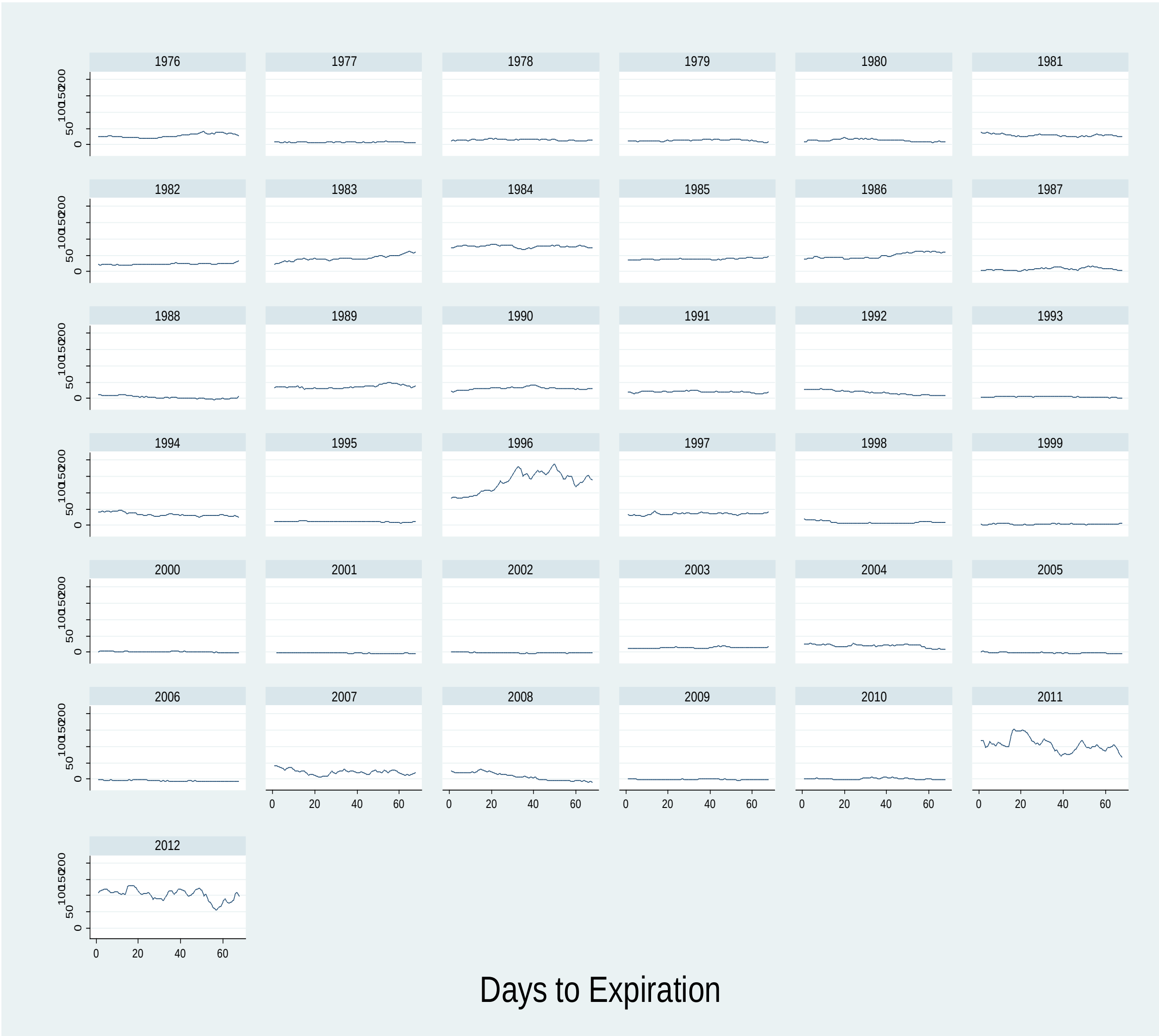


Figure 16. Dec-Jul convenience yield plots by year, (1976-2011)

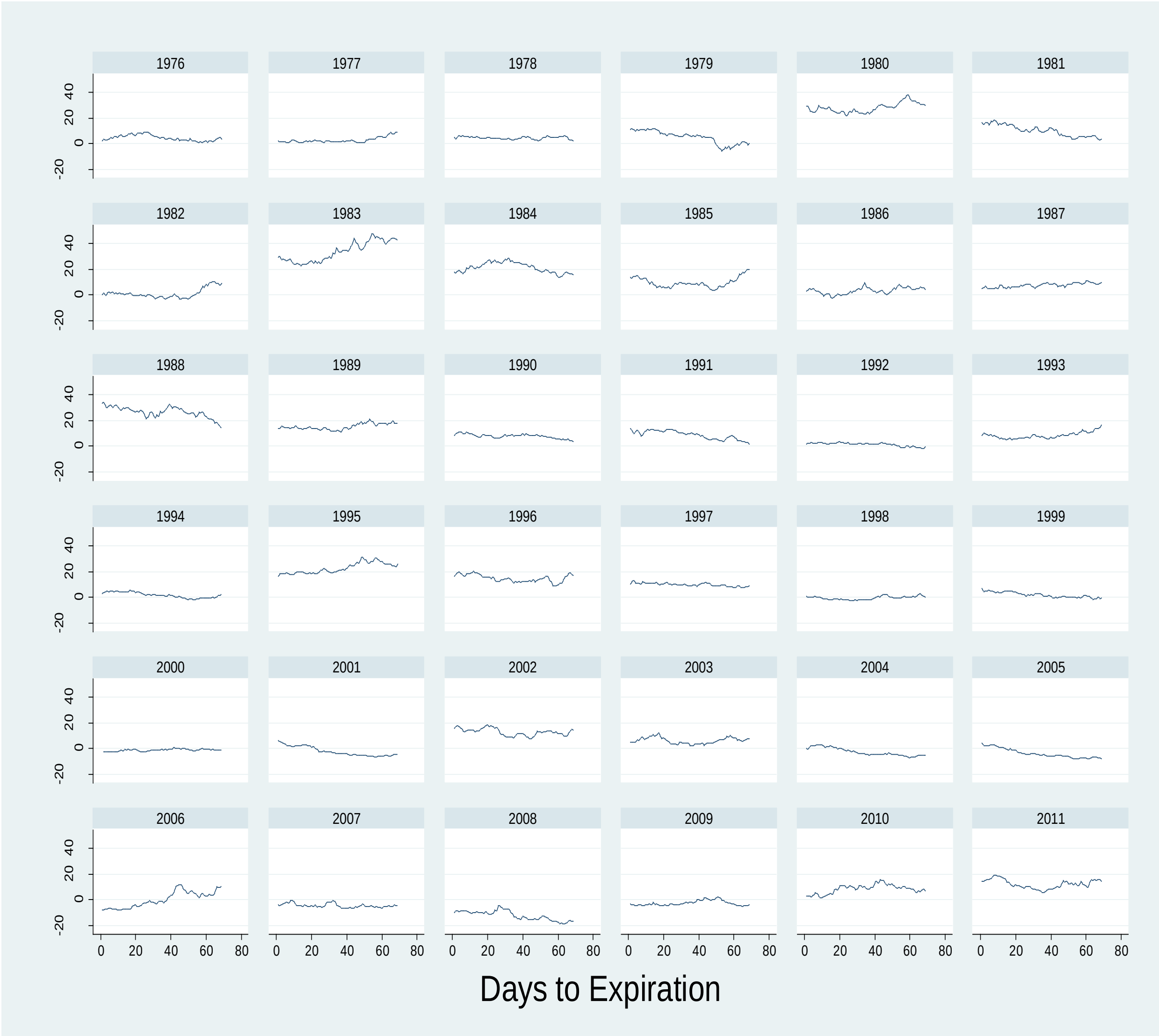
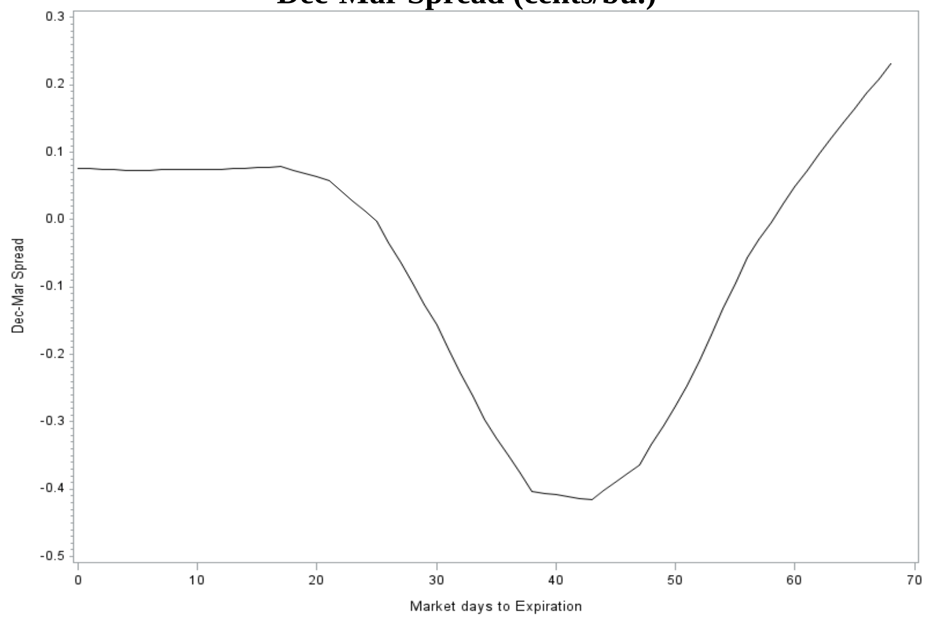


Figure 17. Nonparametric regressions of Dec-Mar calendar spread and convenience yield against trading days to expiration for the sample period

Dec-Mar Spread (cents/bu.)



Dec-Mar Convenience yield (cents/bu.)

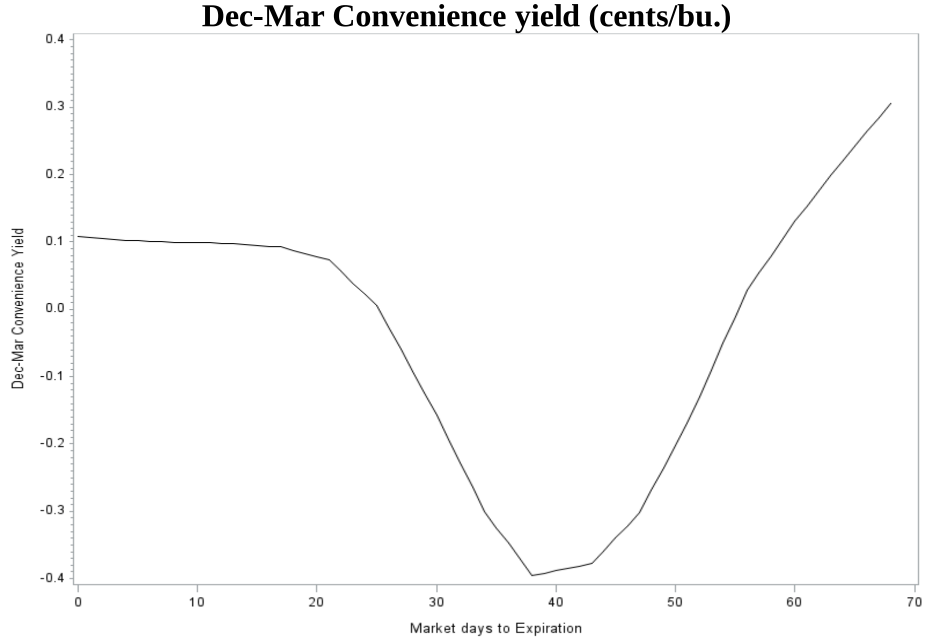
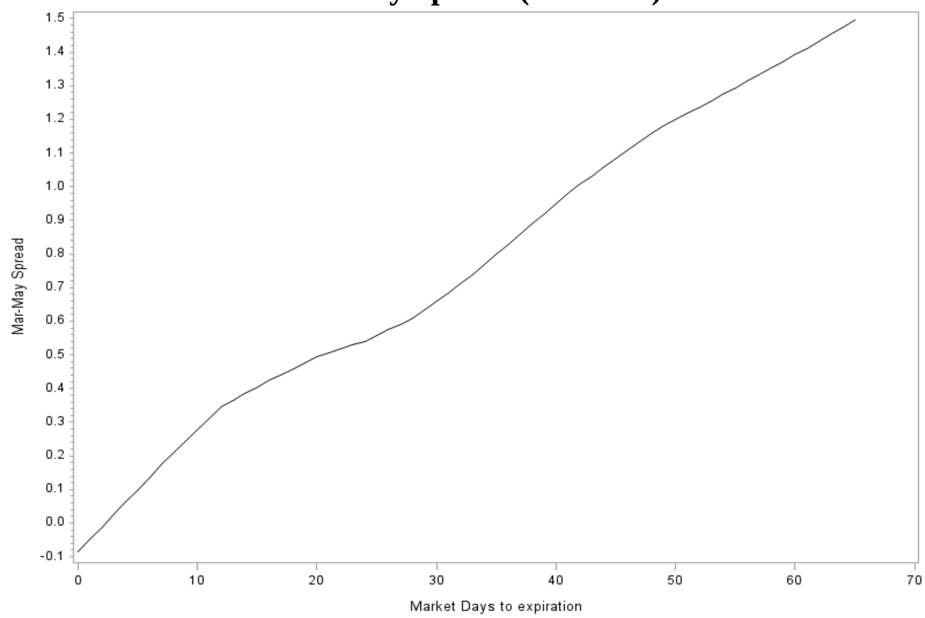


Figure 18. Nonparametric regressions of Mar-May calendar spread and convenience yield against trading days to expiration for the sample period (continued)

Mar-May Spread (cents/bu.)



Mar-May Convenience Yield (cents/bu.)

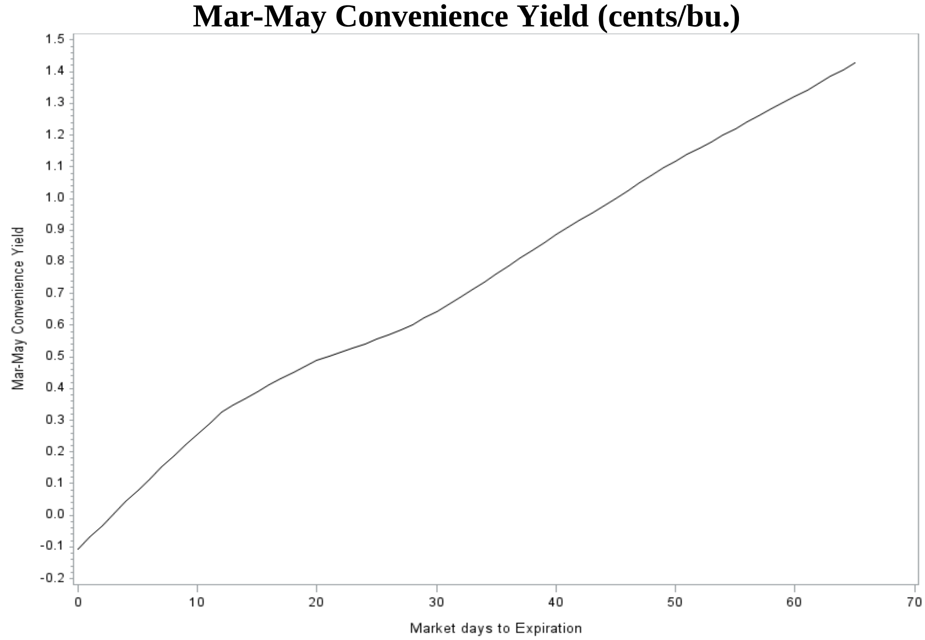
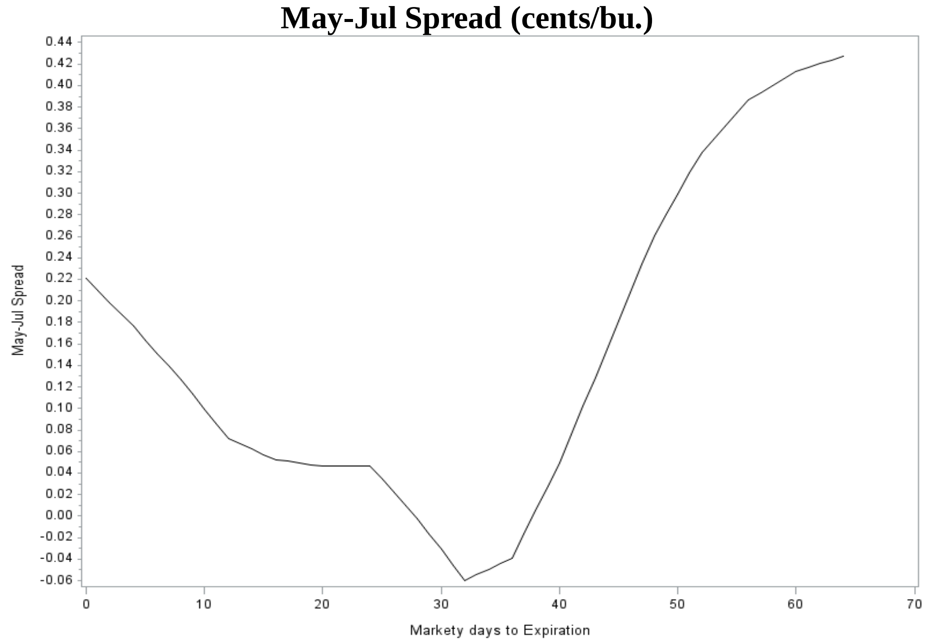


Figure 19. Nonparametric regressions of May-Jul calendar spread and convenience yield against trading days to expiration for the sample period (continued)



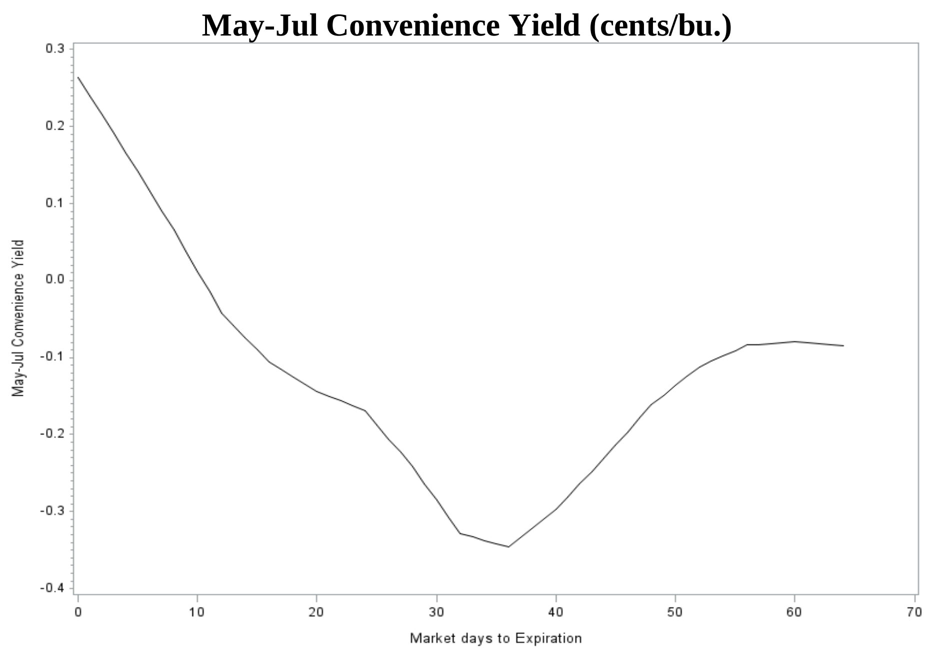
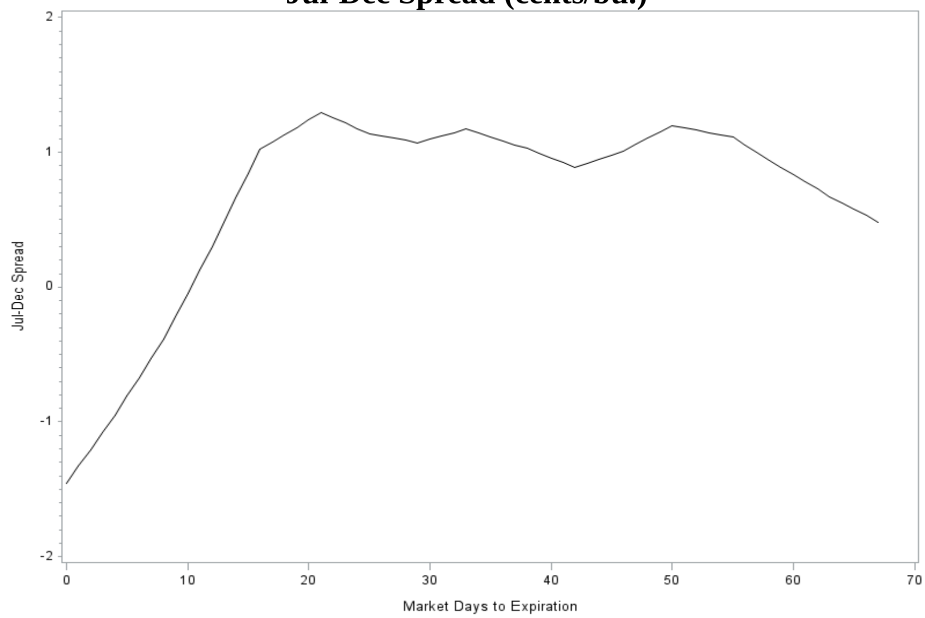


Figure 20. Nonparametric regressions of Jul-Dec calendar spread and convenience yield against trading days to expiration for the sample period (continued)

Jul-Dec Spread (cents/bu.)



Jul-Dec Convenience Yield (cents/bu.)

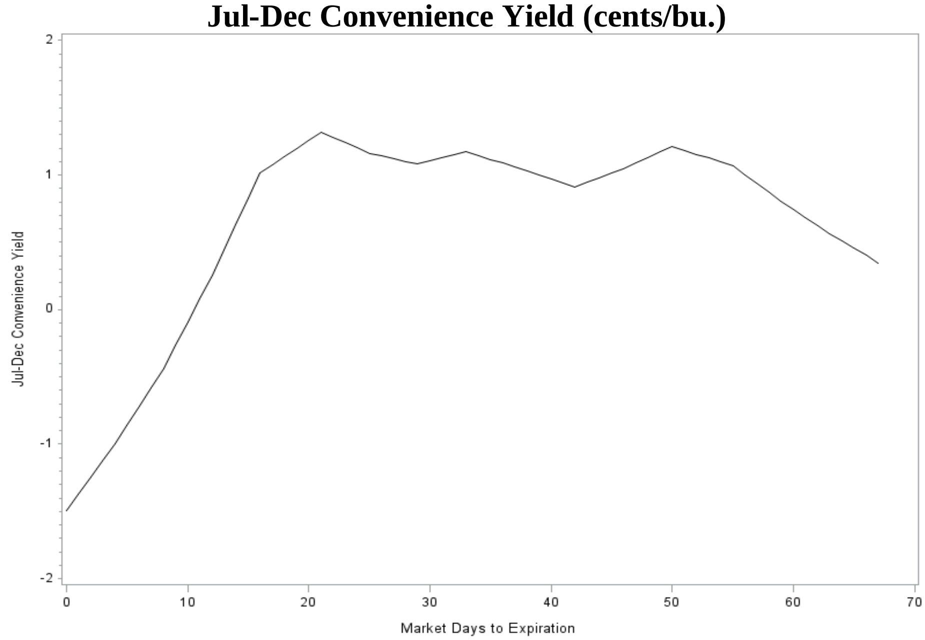
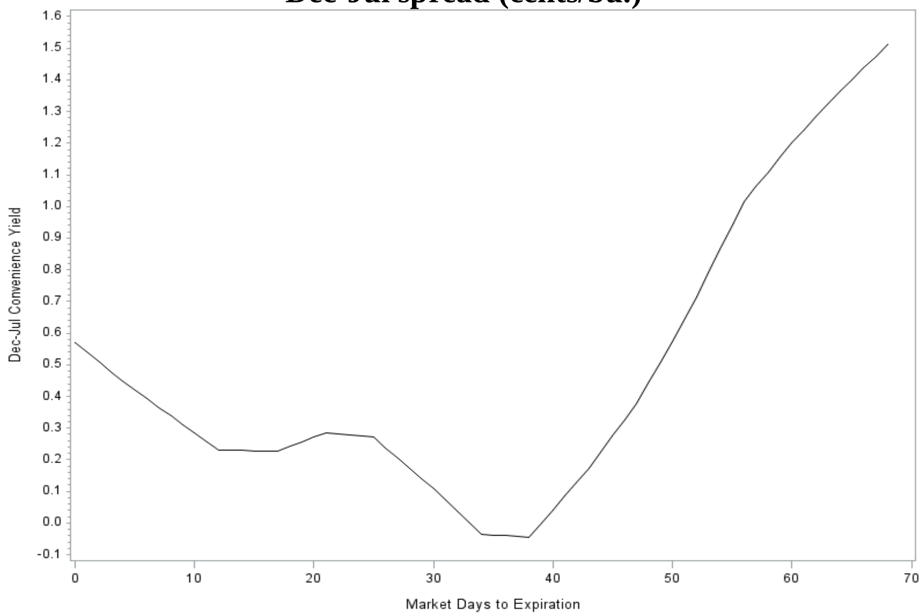


Figure 21. Nonparametric regressions of Dec-Jul calendar spread and convenience yield against trading days to expiration for the sample period (continued)

Dec-Jul spread (cents/bu.)



Dec-Jul convenience yield (cents/bu.)