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Rui Luo and T. Randall Fortenbery

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Rui Luo and T. Randall Fortenbery*

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* Rui Luo, Ph.D Student at School of Economic Sciences, Washington State University, Hullbert Hall 123B, Washington State University, Pullman, WA 99164. Contact: rui.luo2@wsu.edu.
T. Randall Fortenbery, Professor and Grain Commission Endowed Chair, School of Economic Sciences, Washington State University.

Corporate Hedging In Incomplete Markets: A Solution Under Price Transmission

This paper provides dynamic minimum-variance hedges for firms in incomplete markets. Our hedges accounts for price transmission between the input and output prices, and thereby enable firms to minimize both input and output price fluctuations through tradable securities.

Specifically, the model conditions on the direction and magnitude of price transaction between raw materials and products, as well as on the availability of futures contracts. A two-factor diffusion model is assumed for the underlying asset. The optimal hedges are the weighted average of the classic direct hedging and cross hedging ratios. We apply our results to the problem of a hypothetical jet fuel producer. Empirical results demonstrate the hedging effectiveness of this model.

Keywords: Hedging, Price Transmission, Commodity Futures

1. Introduction

A firm has input and output price risk exposures, but it is not often that all these price risks could be eliminated through exchange traded futures contracts as in incomplete markets, futures contracts are only written on a limited number of assets. Though forward contracts are alternatives, they may be too expensive. If an appropriate related futures contract does not exist for the input/output side, the firm may remain unhedged to that side.

However, vertical transmission of shocks among various levels of the market renders price transmission (PT, hereafter) along the supply chain, making one-side hedge more risky – profit volatility may actually be higher than remaining both sides exposed. For example, COFCO TUNHE (600737.SH), a Chinese company producing sugar from sugar beets, had a CN¥ 308 million loss from hedging its sugar sales in 2010. This occurred because sugar prices in China climbs nearly 66 percent and sugar beet prices also rise. Therefore, the fluctuation of its net profit from selling sugar increased.

In this paper, we attempt to provide a dynamic incomplete-market hedging strategy for firms to reduce both input and output risk. Toward that, we consider a firm that is concerned with eliminating the volatility of its profits. The market is incomplete in that the firm cannot take exact offsetting positions to both input and output payoffs, as appropriate futures contracts exist for only one side. The traditional minimum-variance criterion is employed since it is reasonable for a firm to aim to avoid costly financial distress resulted from volatile profits (Fok, Carroll, and Chiou 1997). By taking PT into account, we obtain the minimum-variance hedging policy for the firm, which retains the intuitive elements of classic minimum-variance hedges.

Hedging has been an active topic in derivatives and risk management research for decades. In complete markets without frictions, the classic approach is to employ static minimum-variance

hedges (e.g., Duffie 1989, Stulz 2003, Cvitanic and Zapatero 2004, McDonald 2006, and Hull 2008). Though useful for real-life risk management applications, these hedges are suboptimal when hedging in multiple periods (e.g., Brandt 2003, Choudhry 2003) and do not completely eliminate risks in dynamically complete settings (Basak and Chabakauri 2012). The alternative is to use dynamic minimum-variance strategies that take into account the time-varying joint distribution of underlies (e.g. Alizadeh and Nomikos 2008, Bertus, Godbey, and Hilliard 2009, Schwartz 1997) or “Greeks” hedges (e.g. Bakshi, Cao, and Chen 1997, He et al. 2006).

However, when applied in incomplete markets, complete-market hedges are not necessarily optimal in the sense that they do not optimally consider market incompleteness (Basak and Chabakauri 2012). One standard solution is cross hedges, which use a related futures contract to offset price risk that cannot be hedged by dynamically trading in available securities. For example, Ederington (1979) suggests a minimum-variance static hedge when no futures contracts’ maturity matches the hedger’s time horizon. Wilson (1989) extends the approach of Ederington (1979) and uses soybeans or soybean oil futures to hedge sunflowers positions that do not have futures markets. Berths, Godbey, and Hilliard (2009) highlight the spread risk between the asset underlying the contract and the specific source of risk and provide a dynamic minimum-variance cross hedging strategy.

In a general incomplete-market setting, Basak and Chabakauri (2012) analyze minimum-variance hedging by incorporating a new parameter for market incompleteness into the standard “Greeks” model. Despite the usefulness of this strategy in replication and hedging of financial derivatives, it may not be optimal for hedging commodity risk in the sense that commodities do not satisfy the standard no-arbitrage condition for traded assets (Schwartz 1997). Gibson and Schwartz (1990), Schwartz (1997) and Schwartz and Smith (2000), among others, suggest that models allowing for stochastic, mean-reverting convenience yields, are necessary to capture the dynamics of commodity prices. Adjusting for instantaneous convenience yield has been also shown to improve risk reduction when hedging commodity risk (e.g. Godbey and Hilliard 2007). In this paper, we derive the dynamically optimal hedging policy assuming a two-factor diffusion model for the underlying asset with a stochastic, mean-reverting convenience yield.

Using the specific case of a hypothetical jet fuel producer that uses light sweet crude oil to produce jet fuel as motivation, we compare performance between the one-sided and two-sided hedging. Markets are incomplete in the sense that only light sweet crude oil futures contracts exist. Simulations and empirical results show that the two-side model outperforms the one-side strategy.

The contribution of this paper consists of devising a dynamic hedging ratio for firms to jointly offset input and output risk in incomplete markets by incorporating PT mechanism into the

traditional complete-market minimizing hedging model. As PT is an important characteristic describing the overall operation of the market (Goodwin and Holt 1999), this strategy may be practical for firms in multiple industries.

The article proceeds as follows: in section 2, we develop and discuss the optimal hedging policy in incomplete markets; section 3 compares hedging effectiveness of different hedging strategies and section 4 concludes.

2. The Hedging Model

2.1 Minimum-variance hedging in complete markets

[Figure 1 is about here]

Consider a firm that uses q units of input to produce the output under current technology utilized. In complete markets, the hedging strategy could be depicted as in Figure 1. At time 0, the firm buys h_I units of input futures contracts at price f_0^I , and sells at price f_1^I at $t=1$. The final cost, H_I , for producing 1 unit output is:

$$H_I = I_1 q - (f_1^I - f_0^I) h_I, \quad (1)$$

where I_1 is the spot input price at time 1. At the same time, the firm shorts h_O output futures contracts at price f_0^O . The final income after selling output and its futures contracts at time 2, H_O , is then:

$$H_O = O_2 - (f_1^O - f_0^O) h_O, \quad (2)$$

where f_2^O and O_2 are the futures and spot prices of output at time 2, respectively. The expected profit in the presence of transaction costs, Π , is then

$$\Pi = H_O - H_I - m(|h_I| + |h_O|), \quad (3)$$

where $m(|h_I| + |h_O|)$ is the proportional brokerage fee, which is assumed to be m for each position transacted. The variance-minimizing hedging ratio in complete market is

$$(h_I, h_O)' = \arg \{ \min \text{Var}(\Pi) \}. \quad (4)$$

2. 2 Minimum-variance hedging in incomplete markets

The market in this economy is incomplete in that futures contracts are limited in kinds, thereby making the common approach in complete market impossible. If futures contracts are not available for the asset to be hedged, the firm may remain unhedged and thus exposed to changes in the spot price of the asset. Alternatively, the firm may wish to eliminate this exposure through another hedging vehicle. In this article, we attempt to help the firm improve the quality of hedging in incomplete markets by accomplishing the price transmission (PT) mechanism between the producer and the consumer prices in the sector of the firm in the traditional variance-minimizing hedge.

In different industries, the PT mechanism is expected to vary in directions and magnitude (e.g. Goodwin and Holt 1999, von Cramon-Taubadel 1998). According to the direction of PT and the availability of futures contracts, four subcases are considered: (CO) cost-driving PT in which supply forces lead to equilibrium between input and output prices with output futures contracts; (CI) cost-driving PT with input futures contracts; (DO) demand-driving PT with output futures contracts; (DI) demand-driving PT with input futures contracts. In all cases, the firm may either only hedge cash positions with futures contracts (one-sided hedge) or jointly hedge input and output price risks (two-sided hedge).

Without input futures contract, a CO firm use only output futures to hedge input and output price exposures. The resulting cash flow is

$$\Pi = (O_2 - (f_2^O - f_0^O)h_O^c) - I_1q - m|h_O^c| \quad (5)$$

where O_t ($t = 0, 1, 2$) and I_t ($t = 0, 1$) are spot output and input prices, respectively; f_t^O ($t = 0, 1, 2$) is output futures price, and h_O^c is transactions in output futures market. If h_O^c is positive (negative), the firm shorts (longs). Profits Π equal to total income $(O_2 - (f_2^O - f_0^O)h_O^c)$ minus expenses for inputs I_1q and transaction costs $m|h_O^c|$.

In a widely-applied framework, the PT mechanism for a CO firm could be modeled linearly as

$$O_2 = \theta_0 + \sum_{i=1}^p \theta_i O_{2-i} + \sum_{j=1}^q b_j I_{2-j} \quad , \quad (6)$$

where θ_i and b_j are marginal effects of lagged output and input prices on O_2 , respectively.

This paper employs the traditional variance-minimizing criterion for the hedger whose problem in incomplete market is

$$\min_h \text{Var}(\Pi) \quad . \quad (7)$$

The effectiveness of two-sided hedge through PT, Eff_{two} , is calculated as

$Eff_{two} = -\left(\text{var}(\Pi_{two}) - \text{var}(\Pi_{unhedged})\right)$ and the effectiveness of traditional one-sided hedge is

$Eff_{one} = -\left(\text{var}(\Pi_{one}) - \text{var}(\Pi_{unhedged})\right)$, where Π_{one} , Π_{two} , and $\Pi_{unhedged}$ stands for profits when applying a traditional one-sided hedge, a two-sided PT hedge, and profits of unhedged positions, respectively. The quality-improvement of using the two-sided hedge policy is:

$$G = Eff_{two} - Eff_{one} \quad (8)$$

The optimal hedge ratio for a CO firm, h_o^c is then

$$h_o^c = \theta_1 \beta_1 - (q - b_1) \beta_2 \quad , \quad (9)$$

where $\beta_1 = \frac{\text{cov}(O, f^o)}{\text{var}(f^o)}$ is the minimizing hedge ratio using output futures to hedge output

exposures; and $\beta_2 = \frac{\text{cov}(I, f^o)}{\text{var}(f^o)}$ is the optimal hedge ratio when cross-hedging input price risk

through output futures.

The hedge policy given by (9) suggests that since output prices are driven by input price dynamics, the firm may eliminate more risk by a strategy adjusted to PT. The adjusted policy –

$\theta_1 \beta_1 - (q - b_1) \beta_2$ – is the weighted average of direct hedge policy β_1 and cross hedge policy

β_2 . Specifically, the firm sells $\theta_1 \beta_1$ for output exposures and buys $(q - b_1) \beta_2$ for input price

risks. For each output cash position, it is optimal to sell β_1 output futures without considering for the input price risks. When jointly hedge input and output risks in a PT framework, the short position is adjusted to $\theta_1\beta_1$ because product price at time 2 is affected by its price at time 1. θ_1 gauges the magnitude of this lagged-price effect. The greater is θ_1 , the stronger is the autocorrelation in output price series, and the higher is the weight for β_1 to hedge output price exposures.

As for input price exposures, the firm uses $-(q-b_1)\beta_2$ (i.e. longs $(q-b_1)\beta_2$) output futures positions. More specifically, the optimal cross hedge for each input cash position is β_2 . When not accounting for cost-driving PT, the firm could use $-q\beta_2$ output futures positions to cross hedge input price exposure since the firm has a constant input-output ratio of q . However, cost driving indicates that product prices are affected by lagged prices of raw materials, this mechanism therefore provides “natural hedge” to output price fluctuations, ending in a deduction of $b_1\beta_2$ in long positions. b_1 is the magnitude of PT and measures the marginal output price effect of lagged input price. The greater is b_1 , the less the firm has to short output futures positions.

Similarly, for CI firms with cost-driving PT and input futures market, the problem is that

$$\begin{aligned} & \min_{h_I^c} \{Var(\Pi)\} \\ s.t. \quad & O_2 = \theta_0 + \sum_{i=1}^p \theta_i O_{2-i} + \sum_{j=1}^q b_j I_{2-j} \\ & \Pi = O_2 - (I_1 q - (f_1^I - f_0^I) h_I^c) - m |h_I^c| \end{aligned} \quad (10)$$

where f_1^I is input futures prices. h_I^c is input futures positions and positive (negative) h_I^c indicates buying (selling) input futures. Solving (10) yields

$$h_I^c = -\theta_1\beta_2 + (q-b_1)\beta_1 \quad (11)$$

where $\beta_1 = \frac{\text{cov}(I, f^I)}{\text{var}(f^I)}$ is the minimizing β -ratio between futures and the underlying or the

optimal one-side input hedge ratio; and $\beta_2 = \frac{\text{cov}(O, f^I)}{\text{var}(f^I)}$ is the optimal hedge ratio when

cross-hedging output price risk through input futures. The minimum-variance hedge strategy

adjusted is again the weighted average of β_1 and β_2 .

For demand-driving cases (DO) and (DI) in which major buyers play the price leadership roles, price transmission suggests that

$$I_1 = \chi_0 + \sum_{i=1}^s \chi_i I_{1-i} + \sum_{j=1}^k d_j O_{1-j} , \quad (12)$$

where χ_i is the input price effect of its lags and d_j measures the impact of lagged output price on input prices.

Due to the absence of available input futures contracts, a DO firm's problem using output futures contract is

$$\begin{aligned} & \min_{h_o^d} \{Var(\Pi)\} \\ s.t. \quad & I_1 = \chi_0 + \sum_{i=1}^s \chi_i I_{1-i} + \sum_{j=1}^k d_j O_{1-j} \\ & \Pi = (O_2 - (f_2^O - f_0^O)h_o^d) - I_1 q - m|h_o^d| \end{aligned} \quad (13)$$

The optimal ratio is then

$$h_o^d = \beta_1 \quad (14)$$

which is positive when selling output futures. h_o^d is the same as the one-side hedge solution.

The reason is that under demand-driving price links, input price at time 1 is associated with the time-0 price of output and its own. It follows that the only exposure a DO-firm facing is output price at time 2.

For a DI company with input futures available, its problem is

$$\begin{aligned} & \min_{h_l^d} \{Var(\Pi)\} \\ s.t. \quad & I_1 = \chi_0 + \sum_{i=1}^s \chi_i I_{1-i} + \sum_{j=1}^k d_j O_{1-j} , \\ & \Pi = O_2 - (I_1 q - (f_1^I - f_0^I)h_l^d) - m|h_l^d| \end{aligned} \quad (15)$$

and the solution is

$$h_t^d = -\beta_2, \quad (16)$$

where h_t^d is the optimal input futures positions to buy and a positive h_t^d suggests to long input futures. With the presence of price transmissions, the adjusted minimizing-variance strategy for the DI firm is to cross hedge output price risks through shorting β_2 input futures. This is because dynamics of input prices at time 1 are driven by output prices at time 0, making output exposure to be the problem to handle with.

2.3 Economic Setup and dynamic hedging policy

In this economy, the two-factor model of Gibson and Schwartz (1990), Schwartz (1997) and others is employed to describe price dynamics. The spot price S_t and convenience yield δ_t are assumed to follow the joint diffusion processes

$$\begin{aligned} dS_t &= (\mu - \delta_t)S_t dt + \sigma_s S_t dZ_s \\ d\delta_t &= \kappa_\delta (\alpha_\delta - \delta_t) dt + \sigma_\delta dZ_\delta \end{aligned} \quad (17)$$

where the stochastic mean, μ , convenience yield, δ_t , and volatility σ_s , are deterministic parameters of S_t . κ_δ is the speed adjustment parameter, α_δ is the average long run convenience yield, σ_δ is the instantaneous volatility of the convenience yield processes. dZ_s and dZ_δ are standard Wiener processes, and $dZ_\delta = \rho_{s,\delta} dZ_s$, where $\rho_{s,\delta}$ is the correlation coefficient between the two processes.

For hedging horizon $[0, T]$, the dynamics for the futures price, F , is modeled as¹

¹ See, e.g., Bjerk Sund (1991), Brennan and Crew (1995), Jamshidian and Fein (1990) and Schwartz (1997).

$$F_t = F(S_t, \delta_t, t) = S_t A(T-t) e^{r(T-t) - H_\delta(T-t)\delta_t}, \quad (18)$$

where r is the short-term risk-free rate,

$$A(T) = \exp \left[\frac{(H_\delta(T) - T) \left(\kappa_\delta^2 \alpha_\delta - \kappa_\delta \lambda_\delta \sigma_\delta - \frac{1}{2} \sigma_\delta^2 + \rho_{s,\delta} \sigma_s \sigma_\delta \kappa_\delta \right)}{\kappa_\delta^2} - \frac{\sigma_\delta^2 H_\delta^2(T)}{4\kappa_\delta} \right], \quad (19)$$

λ_δ is the market price of risk for the convenience yield², and $H_\delta(T) = \frac{(1 - e^{-\kappa_\delta T})}{\kappa_\delta}$.

The hedger chooses a direct minimum-variance hedging policy, β_1 , as

$$\begin{aligned} \beta_1 &= \frac{\text{Cov}(S_t, F_t)}{\text{Var}(F_t)} \\ &= \frac{\text{Cov}\left(S_t, S_t A(T-t) e^{r(T-t) - H_\delta(T-t)\delta_t}\right)}{\left(A(T-t) e^{r(T-t)}\right)^2 \text{Var}\left(S_t e^{-H_\delta(T-t)\delta_t}\right)}. \end{aligned} \quad (20)$$

Substituting (18) into (20) yields

$$\begin{aligned} \beta_1 &= \frac{e^{-r(T-t) + \delta_0 e^{-\kappa_\delta t} + \alpha_\delta (1 - e^{-\kappa_\delta t})} \text{Cov}(e^{\ln w_t}, e^{\ln u_t})}{A(T-t) \text{Var}(e^{\ln u_t})} \\ &= \frac{e^{-r(T-t) + \delta_0 e^{-\kappa_\delta t} + \alpha_\delta (1 - e^{-\kappa_\delta t})} e^{\mu_w - \mu_u + \frac{1}{2}(\sigma_w^2 - \sigma_u^2)} (e^{\sigma_{wu}} - 1)}{A(T-t) (e^{\sigma_u^2} - 1)}, \end{aligned} \quad (21)$$

where $u_t = S_t e^{-H_\delta(T-t)\delta_t}$ and $w_t = S_t e^{\sigma_\delta e^{-\kappa_\delta t} \int_0^t \kappa_\delta v dZ_\delta^*(v)}$. μ_u is the expectation of $\ln u_t$, σ_u^2 is the volatility of $\ln u_t$. μ_w and σ_w^2 are the expectation and volatility of $\ln w_t$, respectively. σ_{wu} represents covariance between $\ln w_t$ and $\ln u_t$. The direct hedge, β_1 , is comprised of the

² See Gibson and Schwartz (1990) and Schwartz (1997).

diffusion parameters of spot price S_t , convenience yield δ_t , and futures price F_t (see Appendix A).

Following Bertus, Godbey and Hilliard (2009), we postulate the relationship between spot prices of asset with and without futures contracts (denoted by S_t and P_t , respectively) to be

$$P_t = S_t e^{c_t} . \quad (22)$$

where c_t is the log spread. The spread is assumed to follow the dynamics

$$dc_t = \kappa_c (\alpha_c - c_t) dt + \sigma_c dZ_c , \quad (23)$$

where κ_c is the speed adjustment parameter, α_c is the average long run spread, σ_c is the instantaneous volatility, $dZ_c = \rho_{s,c} dZ_s$ is the increment of a standard Winer process, and $dZ_\delta = \rho_{c,\delta} dZ_c$.

The cross-hedging ratio according to variance-minimizing criterion, β_2 , is then

$$\begin{aligned} \beta_2 &= \frac{\text{Cov}\left(P_t, F_t\right)}{\text{Var}\left(F_t\right)} \\ &= \frac{\text{Cov}\left(S_t e^{c_t}, S_t A(T-t) e^{r(T-t) - H_\delta(T-t)\delta_t}\right)}{\left(A(T-t) e^{r(T-t)}\right)^2 \text{Var}\left(S_t e^{-H_\delta(T-t)\delta_t}\right)} . \end{aligned} \quad (24)$$

Equivalently,

$$\begin{aligned} \beta_2 &= \frac{e^{-r(T-t) + c_0 e^{-\kappa_c t} + \alpha_c (1 - e^{-\kappa_c t})} \text{Cov}(y_t, x_t)}{A(T-t) \text{Var}(x_t)} \\ &= \frac{e^{-r(T-t) + c_0 e^{-\kappa_c t} + \alpha_c (1 - e^{-\kappa_c t})} e^{\mu_y - \mu_x + \frac{1}{2}(\sigma_y^2 - \sigma_x^2)} (e^{\sigma_{xy}} - 1)}{A(T-t) (e^{\sigma_x^2} - 1)} . \end{aligned} \quad (25)$$

where $x_t = S_t e^{-H_\delta(T-t)\delta_t}$ and $y_t = S_t e^{\sigma_c e^{-\kappa_c t} \int_0^t e^{-\kappa_c v} dZ_c}$. μ_x is the expectation of $\ln x_t$, σ_x^2 is the volatility of $\ln x_t$. μ_y and σ_y^2 are the expectation and volatility of $\ln y_t$, respectively. σ_{xy} stands for covariance between $\ln y_t$ and $\ln x_t$. The cross hedging ratio, β_2 , is comprised of parameters in the dynamics of spot price S_t , convenience yield δ_t , futures price F_t and of the spread c_t (see Appendix B).

3. Comparisons of Hedging Models

This section aims to compare performance between the one-sided and two-sided hedges in incomplete markets. More specific, we consider a hypothetical firm that uses light sweet crude oil to produce jet fuel. The firm intends to reduce price exposures with a futures contract on light sweet crude oil (the input).

3.1 Data

The data used to test the models consist of weekly observations in the period from April 4, 1990 to August 16, 2015. Two futures contracts for light sweet crude oil for delivery to Cushing, OK, spot prices for New York Harbor jet fuel, and spot price for light sweet crude oil are obtained from the Energy Information Administration. The price data used are described in Table 1. F1 is the contract closest to maturity and F3 is the third contract closest to maturity. Since the contracts have a fixed maturity date, the time to maturity changes as time grows.

[Table 1 is about here]

3.2 Parameter Estimation for the two-factor model

Following the general procedure of Schwartz (1997), we use the Kalman filtering methodology to obtain the parameter estimation for the two-factor diffusion model. The estimation uses contracts F1 and F3. The time to maturity of nearby futures contract is denoted by T_1 . Subsequent contracts are represented in a similar manner. P_t is the price for jet fuel. By writing the joint diffusion of the two-factor model in state space form, we have the measurement equation to be (Bertus, Godbey and Hilliard 2009):

$$y_t = Z_t \alpha_t + d_t + \eta_t, t = 1, \dots, T \quad (26)$$

where

$$\begin{aligned}
y_t &= \ln[\ln(F1_t), \ln(F3_t), P_t]' \\
Z_t &= \begin{bmatrix} 1 & -H(T_1) & 0 \\ 1 & -H(T_3) & 0 \\ 1 & 0 & 1 \end{bmatrix} \\
d_t &= \begin{bmatrix} \ln(A(T_1)) + rT_1 & \ln(A(T_3)) + rT_3 & 0 \end{bmatrix}' \\
\alpha_t &= \begin{bmatrix} \ln(S_t) & \delta_t & c_t \end{bmatrix}'
\end{aligned}$$

η_t is a 3×1 vector of serially uncorrelated disturbances with $E(\eta_t) = 0$ and $\text{var}(\eta_t) = H_t$.

Parameter estimations are given in Table 2.

[Table 2 is about here]

3.3 Estimation of PT Parameters

Estimation based on a vector autoregression (VAR) model is used to describe the PT mechanism for this jet fuel producer since VAR model is designed for analyzing multivariate time series and has the structure that each variable is a linear function of past lags of itself and past lags of the other variables. Specifically, we use the following VAR model to estimate the direction and magnitude of PT:

$$I_t = \chi_0 + \chi_1 I_{t-1} + d_1 O_{t-1} + \varepsilon_{1t} \quad (27)$$

$$O_t = \theta_0 + b_1 I_{t-1} + \theta_1 O_{t-1} + \varepsilon_{2t} \quad (28)$$

where I_t is the log spot price of input, or the log spot price of the light sweet crude oil; and O_t stands for the log spot price of output, i.e., the price of jet fuel. ε_{1t} and ε_{2t} are error terms. All price time series pass the integration test.

[Table 3 is about here]

Estimation results for the VAR model is reported in Table 3. As is shown, PT parameter estimates in both equations are statistically significant, suggesting that the evolution of output (input) price statistically depends on its own lags and the lags of the input (output) price. These results capture the linear interdependencies among input and output price series and therefore

indicate the existence of price transmission.

The identification of the direction of PT mechanism is based on the sign of $b_1(d_1)$, which estimates the impact of current output (input) price on the future input (output) price. The positive effect of lagged input price on output price (b_1) suggests that the change in output price is driven by the fluctuation of lagged input price, since increases in current input price could result in increase in the next-period output price. The estimate of d_1 , however, is negative, indicating that when current output price increases, future input price tends to decrease. Thus, the direction of PT for this jet fuel producer is *cost driving* in the sense that current output price is *positively* affected by lagged input price. Since the available futures contracts are only written on the input (futures for light sweet crude oil), we categorize the hedging problem of the jet fuel produce as CI case, i.e., cost-driving PT with input futures contracts.

3.4 Comparisons of hedging policy and hedging effectiveness

Based on the parameter values reported in Table 2 and Table 3, we compute the direct and cross hedging ratio β_1 and β_2 . The optimal two-sided hedge policy for the CI firm, h_I^c , is then calculated via equation (11). Since the CI firm only has available futures contracts for inputs, its one-sided hedge policy is

$$h_{I,one-sided}^c = q\beta_1, \quad (29)$$

and positive $h_{I,one-sided}^c$ corresponds to long positions. The effectiveness of the two-sided model,

Eff_{two} , is calculated as $Eff_{two} = -\left(\text{var}(\Pi_{two}) - \text{var}(\Pi_{unhedged})\right)$ and the effectiveness of the

one-sided model is $Eff_{one} = -\left(\text{var}(\Pi_{one}) - \text{var}(\Pi_{unhedged})\right)$, where Π_{one} , Π_{two} , and $\Pi_{unhedged}$ stand for profits under one-sided model, two-sided model and unhedged positions, respectively.

[Table 4 is about here]

Table 4 gives the results for the comparison. We report hedging policy and hedging effectiveness for each model under four horizons from 4 weeks up to two years. The hedges are not adjusted during the horizon. Panel A depicts results from matching horizons. Since the last trading day for crude oil futures is the third business day prior to the 25th calendar day of the month preceding the delivery month, it almost impossible for the maturity of the futures contract for crude oil to match the jet fuel hedging horizon $t \neq T$. We follow Bertus, Godbey and Hilliard (2009) and assume that the hedge expires two weeks before the maturity of futures

contracts. We report the results for the unmatched case in Panel B.

When horizons are matched as in Panel A, the two-sided hedging policy ranges from 0.1599 (two years) to 86.4184 (4 weeks), and is greater than policy of the one-sided model in each case. The effectiveness of the two-sided model ranges from 0.0013 (two years) to 15.5523 (4 weeks); while the one-sided effectiveness ranges from 0.0011 (two years) to 12.3592 (4 weeks). For every horizon, the two-sided model outperforms the traditional one-sided model.

Comparison results changes little when futures expiration is two weeks longer than the hedging horizon. The effectiveness of two-sided model in Panel B ranges from 0.0012 to 9.3698, compared to 0.0001 to 7.4074 matching horizons in Panel A. Similar to Panel A, the two-sided model has greater effectiveness than the one-sided hedging technique for all horizons in Panel B.

4. Conclusions

Firms are looking for improved methods to more efficiently hedge risks that are uncorrelated with fundamental cash flows. By directly accounting for price transmission between the input and the output, this paper develops an incomplete-market hedging strategy through which a firm may minimize both input and output price fluctuations through usable input/output futures. This strategy is conditional on the direction and magnitude of price transactions between raw materials and products, as well as on the availability of futures market. The optimal hedge ratio is the weighted average of the classic minimizing strategy of direct hedging ratio, and the cross hedging policy.

Using data for crude oil futures and jet fuel spot prices, we compare hedging ratios and performance between a traditional one-sided hedge and a two-sided strategy. These dynamic hedging strategies include stochastic convenience yields and a mean-reverting spread. We find that the two-sided model results in a more effective hedge. These findings thus suggest that jet fuel producers may reduce more profit fluctuations by using a hedging model that directly accounts for movements of both the input and output prices. Meanwhile, since price transmission is evidenced over many supply chains, the two-sided hedging policy we proposed could be employed by many firms in multiple industries in their incomplete-market risk management applications.

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Appendix A -- Solution of hedging policy h_o^c

The problem a CI firm with cost-driving PT and input futures market is

$$\begin{aligned} \min_{h_o^c} \{ & Var(\Pi) \} \\ s.t. \quad & O_2 = \theta_0 + \sum_{i=1}^p \theta_i O_{2-i} + \sum_{j=1}^q b_j I_{2-j} \\ & \Pi = (O_2 - (f_2^o - f_0^o)h_o^c) - I_1 q - m |h_o^c| \end{aligned} \quad . \quad (A1)$$

The variance of profits Π thus is

$$\begin{aligned} Var(\Pi) = & \theta_1^2 Var(O) + h_o^{c^2} Var(f^o) + (b_1 - q)^2 Var(I_1) \\ & + \theta_1 h_o^c Cov(O, f^o) + \theta_1 (b_1 - q) Cov(O, I) + h_o^c (b_1 - q) Cov(f^o, I) \end{aligned} \quad . \quad (A2)$$

Solving (A1) yield

$$h_o^c = \theta_1 \frac{\text{cov}(O, f^o)}{\text{var}(f^o)} - (q - b_1) \frac{\text{cov}(I, f^o)}{\text{var}(f^o)} \quad (A3)$$

Defining $\beta_1 = \frac{\text{cov}(O, f^o)}{\text{var}(f^o)}$ and $\beta_2 = \frac{\text{cov}(I, f^o)}{\text{var}(f^o)}$, we have (9). Solutions to h_l^c , h_o^d , and h_l^d can be derived via similar procedures.

Appendix B -- Solution to β_1

The hedger chooses a direct minimum-variance hedging policy, β_1 , as

$$\begin{aligned} \beta_1 = & \frac{\text{Cov}(S_t, F_t)}{\text{Var}(F_t)} \\ = & \frac{\text{Cov}\left(S_t, S_t A(T-t) e^{r(T-t) - H_\delta(T-t)\delta_t}\right)}{\left(A(T-t) e^{r(T-t)}\right)^2 \text{Var}\left(S_t e^{-H_\delta(T-t)\delta_t}\right)} \end{aligned} \quad , \quad (A4)$$

or

$$\begin{aligned}\beta_1 &= \frac{e^{-r(T-t)+\delta_0 e^{-\kappa_\delta t} + \alpha_\delta (1-e^{-\kappa_\delta t})} \text{Cov}(e^{\ln w_t}, e^{\ln u_t})}{A(T-t) \text{Var}(e^{\ln u_t})} \\ &= \frac{e^{-r(T-t)+\delta_0 e^{-\kappa_\delta t} + \alpha_\delta (1-e^{-\kappa_\delta t})} e^{\mu_w - \mu_u + \frac{1}{2}(\sigma_w^2 - \sigma_u^2)} (e^{\sigma_{wu}} - 1)}{A(T-t)(e^{\sigma_u^2} - 1)},\end{aligned}\quad (\text{A5})$$

where $u_t = S_t e^{-H_\delta(T-t)\delta_t}$ and $w_t = S_t e^{\sigma_\delta e^{-\kappa_\delta t} \int_0^t \delta_s^\nu dZ_\delta^*(s)}$. Gibson and Schwartz (1990) and Schwartz (1997), among others, have shown that the expectation of $\ln u_t$, μ_u is

$$\begin{aligned}\mu_u &\equiv E[\ln u_t] = E[\ln S_t] - \int_0^t (\delta_0 e^{-\kappa_\delta s} + \alpha_\delta (1-e^{-\kappa_\delta s})) ds + H_\delta(T-t)E(\delta_t) \\ &= \ln S_0 + \left(\mu - \frac{1}{2}\sigma_s^2\right)t - H_\delta(t)(\alpha_\delta - \delta_0)\delta - \alpha_\delta t \\ &\quad - H_\delta(T-t)(\delta_0 e^{-\kappa_\delta t} + H_\delta(t)\alpha_\delta \kappa_\delta)\end{aligned}\quad , \quad (\text{A6})$$

and the volatility of $\ln u_t$, σ_u^2 is

$$\begin{aligned}\sigma_u^2 &\equiv \text{Var}[\ln u_t] \\ &= \text{Var}[\ln S_t] + H_\delta^2(T-t) \left(\frac{\sigma_\delta^2}{2\kappa_\delta} (1-e^{-2\kappa_\delta t}) \right) \\ &\quad - 2H_\delta(T-t) \left(\rho_{s,\delta} \sigma_\delta \sigma_s H_\delta(t) - \frac{1}{2} \sigma_\delta^2 H_\delta^2(t) \right)\end{aligned}\quad , \quad (\text{A7})$$

where

$$\text{Var}[\ln S_t] = -\left(H_\delta(t) - t\right) \frac{\sigma_\delta^2}{\kappa_\delta^2} - \left(\frac{\sigma_\delta^2}{2\kappa_\delta} H_\delta^2(t)\right) + \sigma_s^2 t + 2 \frac{\rho_{s,\delta} \sigma_s \sigma_\delta}{\kappa_\delta} (H_\delta(t) - t) \quad (\text{A8})$$

The expectation of $\ln w_t$, μ_w , is

$$\begin{aligned}\mu_w &\equiv E[\ln w_t] \\ &= \ln S_0 + \left(\mu - \frac{1}{2} \sigma_s^2 \right) t + H_\delta(t) (\alpha_\delta - \delta_0) - \alpha_\delta t\end{aligned}\quad , \quad (\text{A9})$$

and its variance σ_w^2 , is

$$\begin{aligned}\sigma_w^2 &\equiv \text{Var}[\ln w_t] \\ &= \text{Var}[\ln S_t] + \sigma_\delta^2 \left(\frac{1 - e^{-2\kappa_\delta t}}{2\kappa_\delta} \right) \\ &\quad - \sigma_\delta^2 H_\delta^2(t) + 2\rho_{s,\delta} \sigma_s \sigma_\delta H_\delta(t)\end{aligned}\quad . \quad (\text{A10})$$

Covariance between u_t and w_t , σ_{uw} , is

$$\begin{aligned}\sigma_{uw} &\equiv \text{Cov}(\ln u_t, \ln w_t) \\ &= \text{Var}[\ln S_t] - \frac{1}{2} \sigma_\delta^2 H_\delta^2(t) \\ &\quad - H_\delta(T-t) \left(\rho_{s,\delta} \sigma_s \sigma_\delta H_\delta(t) - \frac{1}{2} \sigma_\delta^2 H_\delta^2(t) \right) \\ &\quad + \rho_{s,\delta} \sigma_s \sigma_\delta H_\delta(t) \\ &\quad - H_\delta(T-t) \sigma_\delta^2 \left(\frac{1 - e^{-2\kappa_\delta t}}{2\kappa_\delta} \right)\end{aligned}\quad . \quad (\text{A11})$$

Plugging (A6)-(A11) into (21) yields β_1 .

Appendix B -- Solution to β_2

The cross-hedging ratio according to variance-minimizing criterion, β_2 , is

$$\begin{aligned}\beta_2 &= \frac{\text{Cov}\left(P_t, F_t \right)}{\text{Var}\left(F_t \right)} \\ &= \frac{\text{Cov}\left(S_t e^{c_t}, S_t A(T-t) e^{r(T-t) - H_\delta(T-t)\delta_t} \right)}{\left(A(T-t) e^{r(T-t)} \right)^2 \text{Var}\left(S_t e^{-H_\delta(T-t)\delta_t} \right)}\end{aligned}\quad , \quad (\text{A12})$$

or,

$$\begin{aligned}\beta_2 &= \frac{e^{-r(T-t)+c_0e^{-\kappa_c t}+\alpha_c(1-e^{-\kappa_c t})} \text{Cov}(y_t, x_t)}{A(T-t) \text{Var}(x_t)} \\ &= \frac{e^{-r(T-t)+c_0e^{-\kappa_c t}+\alpha_c(1-e^{-\kappa_c t})} e^{\mu_y - \mu_x + \frac{1}{2}(\sigma_y^2 - \sigma_x^2)} (e^{\sigma_{xy}} - 1)}{A(T-t)(e^{\sigma_x^2} - 1)}.\end{aligned}\quad (\text{A13})$$

where $x_t = S_t e^{-H_\delta(T-t)\delta_t}$ and $y_t = S_t e^{\sigma_c e^{-\kappa_c t} \int_0^t e^{-\kappa_c v} dZ_c}$. Bertus, Godbey and Hilliard (2009) show that the expectation and volatility of $\ln x_t$ and $\ln y_t$ are

$$\begin{aligned}\mu_x &\equiv E[\ln x_t] = \ln S_0 + \left(\mu - \frac{1}{2} \sigma_s^2 \right) t - H_\delta(t) (\alpha_\delta - \delta_0) \delta - \alpha_\delta t \\ &\quad - H_\delta(T-t) (\delta_0 e^{-\kappa_\delta t} + H_\delta(t) \alpha_\delta \kappa_\delta) \\ \sigma_x^2 &\equiv \text{Var}[\ln x_t] \\ &= \text{Var}[\ln S_t] + H_c^2(T-t) \left(\frac{\sigma_\delta^2}{2\kappa_\delta} (1 - e^{-2\kappa_\delta t}) \right) \\ &\quad - 2H_c(T-t) \left(\rho_{s,\delta} \sigma_\delta \sigma_s H_\delta(t) - \frac{1}{2} \sigma_\delta^2 H_\delta^2(t) \right), \quad (\text{A14}) \\ \mu_y &\equiv E[\ln y_t] = \ln S_0 + \left(\mu - \frac{1}{2} \sigma_s^2 \right) t + H_\delta(t) (\alpha_\delta - \delta_0) - \alpha_\delta t \\ \sigma_y^2 &\equiv \text{Var}[\ln y_t] = \text{Var}[\ln S_t] + \sigma_c^2 \left(\frac{1 - e^{-2\kappa_c t}}{2\kappa_c} \right) \\ &\quad + 2 \left(\frac{\sigma_s \sigma_c \rho_{\delta c}}{\kappa_\delta + \kappa_c} (H_\delta(t) e^{-\kappa_c t} - H_c(t)) + \rho_{sc} \sigma_s \sigma_c H_c(t) \right)\end{aligned}$$

where $H_c(t) = (1 - e^{-\kappa_c t}) / \kappa_c$. Covariance between $\ln x_t$ and $\ln y_t$ is

$$\begin{aligned}
\sigma_{x,y} \equiv \text{Cov}[\ln x_t, \ln y_t] &= \text{Var}[\ln S_t] \\
&+ \frac{\rho_{c\delta} \sigma_c \sigma_\delta}{\kappa_c + \kappa_\delta} \left(H_\delta(t) e^{-\kappa_c t} - H_c(t) \right) \\
&- H_c(T-t) \left[\rho_{s\delta} \sigma_\delta \sigma_s H_\delta(t) - \frac{1}{2} \sigma_\delta^2 H_\delta^2(t) \right]. \quad (\text{A15}) \\
&+ \rho_{s,c} \sigma_s \sigma_c \frac{1 - e^{-\kappa_c t}}{\kappa_c} \\
&- H_c(T-t) \left(\rho_{c\delta} \sigma_\delta \sigma_c \left(\frac{1 - e^{-t(\kappa_\delta + \kappa_c)}}{\kappa_\delta + \kappa_c} \right) \right)
\end{aligned}$$

Plugging (A14)-(A15) into (25) yields β_2 .

Table 1 Futures and Spot Price Data

	Mean Price (Standard Error)	Mean Maturity (Standard Error)
Futures Contract: F1	47.15 (0.868)	0.042 (0.245) years
Futures Contract: F3	47.33 (0.876)	0.209 (0.248)
Jet fuel	1.37 (0.027)	
Light Sweet Crude Oil	47.15 (0.868)	

Note. This table presents data description. The data consist of weekly observations in the period from April 4, 1990 to August 16, 2015, including two futures contracts for light sweet crude oil for delivery to Cushing, OK (F1 and F3, where F1 is the contract closest to maturity and F3 is the third contract closest to maturity.), spot prices for New York Harbor jet fuel, and spot price for light sweet crude oil. All data are obtained from the Energy Information Administration.

Table 2 Parameter Estimates For The Two-Factor Diffusion Model

Parameter	Estimates
α_δ	0.5031
κ_δ	1.0427
σ_δ	0.2681
σ_s	0.5040
$\rho_{s\delta}$	0.1500
λ_δ	0.4893
α_c	0.4997
κ_c	4.0238
σ_c	0.4825
ρ_{sc}	0.5322
$\rho_{c\delta}$	0.3987

Note. This table presents the Kalman filter parameter estimates for the two-factor diffusion model. Parameters are estimated using weekly data from April 4, 1990 to August 16, 2015. Futures prices for light sweet crude oil for delivery to Cushing, OK, and spot prices for New York Harbor jet fuel are obtained from the Energy Information Administration.

Table 3 Estimation Results for the PT Mechanism

Parameter	Estimate (Std. Error)
Equation: $I_t = \chi_0 + \chi_1 I_{t-1} + d_1 O_{t-1} + \varepsilon_{1t}$	
χ_1	1.2778*** (0.2468)
d_1	-1.3185*** (0.2346)
χ_0	-0.9171 (0.8797)
Adjusted R-squared	0.03195
Equation: $O_t = \theta_0 + b_1 I_{t-1} + \theta_1 O_{t-1} + \varepsilon_{2t}$	
b_1	1.2443*** (0.2609)
θ_1	-1.2565*** (0.2480)
θ_0	-4.3598*** (0.9300)
Adjusted R-squared	0.02201
Num. of Observations	1306

Note. This table presents the parameter estimation for the PT mechanism. We use the vector autoregression (VAR) model to describe interdependencies between input and output price time series. The VAR model has the following structure:

$$\begin{aligned} I_t &= \chi_0 + \chi_1 I_{t-1} + d_1 O_{t-1} + \varepsilon_{1t} \\ O_t &= \theta_0 + b_1 I_{t-1} + \theta_1 O_{t-1} + \varepsilon_{2t} \end{aligned} .$$

where I_t is the log spot price of input, or the log spot price of the light sweet crude oil; and O_t stands for the log spot price of output, i.e., the price of jet fuel. ε_{1t} and ε_{2t} are error terms. All price time series pass the integration test. Significance codes: <0.0001 '***', 0.01 '**', 0.1 '*'.

Table 4 Comparisons of Hedging Models

Horizon	Hedging Policy		Effectiveness	
Panel A: Futures expiration matches hedging horizon				
	Two-sided	One-sided	Two-sided	One-sided
4 weeks	86.4184	47.2607	15.5523	12.3592
13 weeks	20.9923	11.3716	2.8846	2.2787
26 weeks	7.1514	3.8739	0.6653	0.5256
One year	1.0408	1.6443	0.8907	0.0701
Two years	0.1599	0.0866	0.0013	0.0011
Panel B: Futures expiration is two weeks longer than hedging horizon				
	Two-sided	One-sided	Two-sided	One-sided
4 weeks	55.2736	29.9775	9.3698	7.4074
13 weeks	17.1626	9.2970	2.2210	1.7545
26 weeks	6.2567	3.3893	0.5482	0.4331
One year	1.4914	0.8079	0.0599	0.0473
Two years	0.1478	0.0800	0.0012	0.0001

Note. This table presents the hedging performance for the one-sided and two-sided model. We report hedging policy and hedging effectiveness for each model under four horizons from 4 weeks up to two years. The hedges are not adjusted during the horizon. The optimal two-sided hedge policy for the CI firm is $h_t^c = (q - b_1)\beta_1 - \theta_1\beta_2$ and

the one-sided hedge policy is $h_{t,one-sided}^c = q\beta_1$. Effectiveness of the two-sided and one-sided model are

$$Eff_{two}^c = -\left(\text{var}(\Pi_{two}) - \text{var}(\Pi_{unhedged})\right) \text{ and } Eff_{one}^c = -\left(\text{var}(\Pi_{one}) - \text{var}(\Pi_{unhedged})\right), \text{ respectively. } \Pi_{one}, \Pi_{two}, \text{ and}$$

$\Pi_{unhedged}$ stand for profits under one-sided model, two-sided model and unhedged positions, respectively.