Fundamentals of Sharpe Ratios in Storable Commodity Markets

by

Sami Attaoui and Pierre Six

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Fundamentals of Sharpe ratios in storable commodity markets

Sami Attaoui[1] and Pierre Six[2]

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Abstract:

We determine model free estimates of the Sharpe ratios of the spot price and the basis for storable commodities. We use these estimates to conduct a thorough analysis of these Sharpe ratios across four commodity classes - Energy, Base Metals, Grains and Precious Metals, that cover twelve individual commodities. We find similar results across the twelve commodities and the Sharpe ratios of the two risks in terms, of descriptive statistics, seasonality, mean-reversion and determinants. Our results have major implications for the understing of the risks of commodities under the physical probability measure.

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[1] Finance department, NEOMA Business School, sami.attaoui@neoma-bs.fr
[2] *Corresponding author*. Finance department, NEOMA Business School, pierre.six@neoma-bs.fr

Introduction

Recently, a strand of literature has highlighted and revived the interest in the dynamics of storable commodities under the true probability measure (Cortazar, Kovacevic, and Schwartz, 2015; Cortazar, Millard, Ortega, and Schwartz, 2019; Cortazar, Ortega, Rojas and Schwartz, 2021). These articles explain that the dynamics of commodity risks under the true probability lead to a poor forecast of prices, while on the contrary, the dynamics of the risks under the risk-neutral probability provides an accurate valuation of the cross-section of the prices of derivatives. However, these articles rely on an exogenous specification of the market price of risk, or the Sharpe ratio,[3] that are often chosen to obtain closed-form valuations of the price of derivatives. The Sharpe ratios constitute the difference between the risk-neutral and the true probability measures. The characterization and understanding of the dynamics of Sharpe ratios is then essential to understanding the prices' dynamics under the true probability.

This article aims at studying the Sharpe ratios of storable commodities empirically without any functional assumptions regarding their dynamics. To achieve our goal, we first provide model-free estimates of the Sharpe ratios of the fundamental risks of storable commodities, namely the basis and the spot price. We then compute our estimators and conduct statistical analyses of the two Sharpe ratios. We start with a statistical descriptive analysis. We then carry over a time series analysis. Finally, we conduct a microeconometrics analysis to find out about the determinants of the two Sharpe ratios. We conduct our empirical analyses on twelve individual commodities grouped into four different commodity classes: Energy, Base Metals, Grains and Precious Metals.

Sharpe ratios are important variables for commodities (Cortazar, Millard, Ortega, and Schwartz, 2019; Cortazar, Ortega, Rojas, and Schwartz, 2022). First, they are critical variables for commodity risk management, such as the computation of the Value at Risk or the Expected Shortfall of commodity risk exposure. Second, Sharpe ratios are fundamental for portfolio management and asset allocation. For the specific case of storable commodities, Sharpe ratios are also essential for price forecasting and inventory management by producers or transformers (Cortazar, Kovacevic, and Schwartz, 2015; Cortazar, Millard, Ortega, and Schwartz (2019). In

[3] The definition of the market price of risk or the Sharpe ratio is the same: the return of a tradable portfolio of assets perfectly correlated with this risk in excess of the risk-free rate and per units of risk, i.e. divided by the volatility of the portfolio. We use in this article the term Sharpe ratios. We assume in our framework that the market is complete and that all risks are tradable.

addition, Bec and De Gaye (2016) show that Central Banks' inflation forecasts are sensitive to oil price forecast errors.

Despite their importance, the systematic study of the Sharpe ratios in the commodity markets is scarce. On the one side, two strands of the literature theoretically analyze the dynamics of commodity prices. The first strand explains the dynamics of commodity prices using inventory as the primitive (e.g. Brennan, 1958; Deaton and Laroque, 1992, 1996; Routeldge, Seppi, and Spatt, 2000). To the best of our knowledge, in this strand of literature, investors are neutral to risk and then the Sharpe ratios are not studied. The second strand, models the Sharpe ratio as an exogenous variable that is usually statistically assessed via some state space filtering techniques (e.g. Gibson and Schwartz, 1990; Schwartz, 1997; Casassus and Collin-Dufresne, 2005; Cortazar, Kovacevic and Schwartz, 2015; Cortazar, Millard, Ortega and Schwartz, 2019; Cortazar, Ortega, Rojas and Schwartz, 2021).

However, this modeling of Sharpe ratios cannot be used for further analysis because it is not model free. In addition, the estimation of the Sharpe ratio usually provides parameters that are not statistically significant (Schwartz, 1997; Casassus and Collin-Dufresne, 2005; Cortazar, Millard, Ortega, and Schwartz, 2019). Different strategies have been adopted to better pin down the Sharpe ratios for commodities. For instance, Cortazar, Kovacevic, and Schwartz (2015) restrict the values taken by the Sharpe ratios. Cortazar, Millard, Ortega, and Schwartz (2019) and Cortazar, Ortega, Rojas, and Schwartz (2021) add analysts' forecasts to the usual information conveyed by futures prices.

On the other side, few empirical papers address the specific issue of the Sharpe ratios of storable commodities: Handika, Korn, and Trueck (2015), Hamilton and Wu (2014), and Cortazar, Ortega, Rojas, and Schwartz (2022). The analysis of Sharpe ratios in the three papers, however, are model-dependent. Furthermore, Hamilton and Wu (2014) analyze of the oil risk premia, but they do not study the determinants of the Sharpe ratios of oil. Cortazar, Ortega, Rojas, and Schwartz (2022) also study the difference between the risk-neutral probability and the true probability in the commodity markets. They nevertheless study this difference in terms of the commodity risk premium, defined as the difference between the expected spot price under the true probability minus the expected spot price under the risk-neutral probability, i.e. the futures price. They focus on the oil market.

Another strand of literature related to our work is the empirical asset pricing literature that adapts the seminal work on the equity markets of Fama and French (1992) to the commodity markets. The idea is to replace the equity universe with the commodity universe and to sort out the commodities to form portfolios based on signals that can be common to all classes of assets, such as the momentum factor, or specific to commodities such as inventory. The reader can refer to Miffre (2016) for a systematic review of the literature and to Sakas and Tessaromatis (2020) for a more recent account. Amongst this literature, Szymanowska, De Roon, Nijman, and Van Den Goorbergh (2014) who distinguish between a spot and term premia, are the closest to this article. However, the goal of this literature is to explain the returns of portfolio of commodities and not to study the Sharpe ratios of individual commodities.

As usual in the literature, e.g. Gorton, Hayashi, and Rouwenhorst, (2013), we identify the spot price to the nearest futures contract, i.e. the futures contracts with the shortest maturity. Because futures contracts are liquid assets without short sales constraints, the Sharpe ratio of the spot price is simply the expected return of the nearby futures[4] price divided by its volatility. These two moments can be replaced by their empirical counterparts for empirical purposes. The Sharpe ratio of the basis is more involved because the basis is not a tradable asset. We prove that a reverse calendar spread is perfectly correlated with the basis. Szymanowska, De Roon, Nijman, and Van Den Goorbergh (2014) also prove that a calendar spread can disentangle the term premia.

However, since the seminal paper of Gibson and Schwartz (1990), reduced-form asset pricing models rely on the (instantaneous) convenience yield to price commodity derivatives in lieu of the basis. And, while the basis is often used as a proxy for for the convenience yield in the literature, Fouquau and Six (2016) have shown that these two variables can differ. As a consequence, the Sharpe ratio of the convenience yield and that of the basis can only differ. We show in this article that the Sharpe ratio of the basis is a very good approximation of the Sharpe ratio of the convenience yield for Grains, Base Metals and Energy commodities. However, this result is not true for precious metals where the two Sharpe ratios differ.

As mentioned above, we conduct our empirical investigation in the Energy, Base Metals, Grains and Precious Metals markets. We choose commodities for which the futures price is available on Bloomberg, the open interest on the website of the Commodity Futures Trading Commission (CFTC), and the inventory on the website of the U.S. Administration of Energy information (AEI) or the U.S. Department of Agriculture. When inventory data are not available at the national level, we download inventory data from Bloomberg at the level of the

[4] The futures contract is a marked-to-market asset and does not include any cost of time.

Exchange Market where the futures contract is traded. Two Base Metals are traded on the London Metal Exchange: for these two metals, the open interest is not available. We choose three individual commodities per commodity classe: i) Energy: Oil, Heating Oil and Gasoline; ii) Base Metals: Copper, Aluminium and Zinc; iii) Grain: Soybean, Corn and Wheat; iv) Precious Metals: Gold, Silver and Platinum.

We show that both Sharpe ratios, that of the spot price and that of the basis, exhibit the same pattern for the twelve commodities. For, each Sharpe ratio and each commodity, we run a regression of the variation of the Sharpe ratio over the same Sharpe ratio at the beginning of the period. First, all the 24 Sharpe ratios exhibit a very strong, both economically and statistically, mean reverting pattern in line with the reduced form model of Casassus and Collin-Dufresne (2005). The assumption in many reduced-form models that the Sharpe ratios are constant is then a very strong one. Second, both Sharpe ratios for the twelve commodities change sign frequently: around fifty percent of the time, that is every two months for our monthly sample. The very frequent change of sign of the Sharpe ratios certainly explains the lack of statistical significance of the parametrization of Sharpe ratios in reduced form models.

We then move on to analyze the determinants of both Sharpe ratios for the twelve commodities. We select potential candidates in line with the empirical literature (Gorton, Hayashi, and Rouwenhorst, 2013; Kang, Rouwenhorst, and Tang, 2020). After checking for multi-collinearity and stationarity, we end up with a set of seven predictors for each commodity: the return of the spot price, the basis, the volatility of the spot price, the correlation between the spot price and the basis, the futures risk premium, the hedging pressure and the level of inventory. We run a linear regression for the Sharpe ratios of the spot price and the basis over the seven predictors for the four commodities. Our results are consistent among our twelve commodities. First, the seven determinants provide a better fit for the Sharpe ratio of the spot price than for the Sharpe ratio of the basis: the R^2 varies between 72.29% and 83.78% for the Sharpe ratio of the spot price while it varies between 8.19% and 28.07% for the Sharpe ratio of the basis. Second, the set of statistically significant predictors differ between the Sharpe ratio of the basis and that of the spot price. The Sharpe ratio of the basis is positively related to the basis, but the relation, while statistically very strong, is economically very imperfect. The Sharpe ratio of the spot price is positively related to the return of the spot price and this relation is economically and statistically very strong.

The reminder of the article is organized as follows. Section 1 provides the theoretical foundation for the risk-free estimate of the Sharpe ratios of the spot price and the basis. Section 2 provides the link between the Sharpe ratio of the convenience yield and that of the basis. Section 3 focuses on the empirical analyses of the Sharpe ratios. Section 4 offers general conclusions and extensions.

1. Model-free estimates of the Sharpe ratios

We consider a complete probability space (Ω, I, Ξ, P), with a continuous non-decreasing filtration $\Xi \equiv (\mathrm{I}_t: t \in [0; T_E])$. T_E is a positive constant that defines the end of the economy: $\mathrm{I}_{T_E} \equiv \mathrm{I}$. We assume that this filtration is generated by the augmented paths of Brownian motions, that do not need to be specified in this section – more mathematical details are given in the next section. $E_t[\ \ ] \equiv E[\ \ |_{I_t}]$ ($V_t[\ \ ] \equiv V[\ \ |_{I_t}]$) designates the conditional expectation (variance) of a random variable. For this specific section, we specify two Brownian motions, z_S and z_B, that drive the risk of the spot price and the risk of the basis, respectively. We assume that the augmented filtration generated by z_S and z_B is included in Ξ. All processes in this section are adapted to the filtration generated by z_S and z_B.

We denote by F_{Nt} and F_{Dt} the prices Nearby and the Distant futures contract, respectively. In the empirical part of the paper, we take the next nearby contract as the distant contract (Ederington, Fernando, Holland, Lee and Linn; 2021). We assume that the prices of the futures contracts are adapted to Ξ and that markets are perfect: no transaction costs and perfect divisibility of futures contracts. In particular, the market is complete in our framework and there exist a unique risk-neutral probability denoted by Q, Karatzas, Lehoczky and Shreve (1987). For future reference, we rely on the theorem of Girsanov and define by:

$$dz_{St}^{Q} \equiv dz_{St} + \lambda_{St} dt \tag{1a}$$

$$dz_{Bt}^{Q} \equiv dz_{Bt} + \lambda_{Bt} dt \tag{1b}$$

the Brownian motions that drive the risks of the spot price and the basis under the risk neutral probability, respectively. $\lambda_{St}(\lambda_{Bt})$ designates the market price of risk or the Sharpe ratio of the spot price (basis).

We define $\mathrm{B}_t \equiv \frac{\mathrm{F}_{Nt}}{\mathrm{F}_{Dt}}$ as the basis in our framework (Gorton, Hayashi and Rouwenhorst, 2013).[5] We also define a calendar spread, which price is denoted by Π_t. It consists in our framework of a 100% (-100%) investment, in proportion, in the Nearby (Distant) futures contract. Without loss of generality, we assume that this portfolio is not collateralized, i.e., it does not earn the risk-free rate. Using the well-known self-financing condition, the price of our calendar spread writes as follows:

$$\frac{d\Pi_t}{\Pi_t} = \frac{\mathrm{dF}_{Nt}}{\mathrm{F}_{Nt}} - \frac{\mathrm{dF}_{Dt}}{\mathrm{F}_{Dt}} \tag{2}$$

We are now equipped to state our theorem.[6]

Theorem 1.

The Sharpe ratio of the spot price is computed with the nearby futures contract:

$$\lambda_{\mathrm{St}} = \mathrm{E_t}\left[\frac{\mathrm{dF_{Nt}}}{\mathrm{F_{Nt}}}\right] \bigg/ \sqrt{\mathrm{V_t}\left[\frac{\mathrm{dF_{Nt}}}{\mathrm{F_{Nt}}}\right]} \tag{3a}$$

The Sharpe ratio of the basis is computed with the calendar spread:

$$\lambda_{\mathrm{Bt}} = \mathrm{E_t}\left[\frac{\mathrm{d\Pi_t}}{\Pi_t}\right] \bigg/ \sqrt{\mathrm{V_t}\left[\frac{\mathrm{d\Pi_t}}{\Pi_t}\right]} \tag{3b}$$

Poof just follows:

Consistent with the literature (e.g. Gorton, Hayashi and Rouwenhorst, 2013), we identify the nearby futures contract with the spot price. It is well-known that the futures price is a martingale under the risk-neutral probability, see e.g. (Duffie and Stanton, 1992). Then, by the martingale representation theorem, there exists an adapted process, σ_{Nt}, such that:

$$\frac{\mathrm{dF_{Nt}}}{\mathrm{F_{Nt}}} = \sigma_{Nt} dz_{St}^{Q} \tag{4a}$$

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[5] Gorton et la. uses $\left(\frac{\mathrm{F}_{Nt}}{\mathrm{F}_{Dt}} - 1\right)\frac{365}{\mathrm{D}_{Dt} - \mathrm{D}_{Nt}}$ as a precise definition of the basis where $\mathrm{D}_{Dt}, \mathrm{D}_{Nt}$ are the number of days remaining until maturity for the distant and nearby futures contracts, respectively. So, there is a one to one deterministic mapping between our definition of the basis and that of Gorton et al. (2013) definition.

[6] We do not need the definition of Π_t for our theorem to hold: the definition of Π_t is for illustrations purposes only – we just need the quantity $\frac{1}{\mathrm{F}_{Dt}}\mathrm{dF}_{Dt} - \frac{1}{\mathrm{F}_{Nt}}\mathrm{dF}_{Nt}$, which obviously holds in our case / reformuler.

Π_t is also a martingale under the risk-neutral probability as a linear combination of martingale under the risk-neutral probability. In addition, we apply Ito lemma to the basis $\mathrm{B}_t \equiv \frac{\mathrm{F}_{Nt}}{\mathrm{F}_{Dt}}$ and find that:

$$\frac{d\mathrm{B}_t}{\mathrm{B}_t} = \frac{dF_{Nt}}{F_{Nt}} - \frac{dF_{Dt}}{F_{Dt}} - \frac{dF_{Dt}}{F_{Dt}}\frac{dF_{Nt}}{F_{Nt}} + \frac{dF_{Nt}}{F_{Nt}}\frac{dF_{Nt}}{F_{Nt}} \tag{4b}$$

$$\frac{d\mathrm{B}_t}{\mathrm{B}_t} = \frac{\mathrm{d\Pi_t}}{\mathrm{\Pi_t}} - \frac{dF_{Dt}}{F_{Dt}}\frac{dF_{Nt}}{F_{Nt}} + \frac{dF_{Nt}}{F_{Nt}}\frac{dF_{Nt}}{F_{Nt}} \tag{4c}$$

By Ito calculus, $-\frac{dF_{Dt}}{F_{Dt}}\frac{dF_{Nt}}{F_{Nt}}$ and $\frac{dF_{Nt}}{F_{Nt}}\frac{dF_{Nt}}{F_{Nt}}$ are absolutely continuous with respect to the measure dt. As a consequence, the risk of the basis is the same as the risk of the calendar spread, and by the martingale representation theorem, there exists an adapted process, σ_{Bt}, such that:

$$\frac{\mathrm{d\Pi_t}}{\mathrm{\Pi_t}} = \sigma_{Bt} dz_{Bt}^{Q} \tag{4d}$$

Finally, we use the results of Girsanov's theorem as given by Eqs. (1a,b) to find the dynamics of the nearby futures contract and the calendar spread under the true probability, *P*:

$$\frac{dF_{Nt}}{F_{Nt}} = \lambda_{St}\sigma_{Nt}\mathrm{dt} + \sigma_{Nt} dz_{St} \tag{5a}$$

$$\frac{\mathrm{d\Pi_t}}{\mathrm{\Pi_t}} = \lambda_{Bt}\sigma_{Bt}\mathrm{dt} + \sigma_{Bt} dz_{Bt} \tag{5b}$$

By Ito calculs, $\mathrm{E_t}\left[\frac{\mathrm{dF_{Nt}}}{\mathrm{F_{Nt}}}\right] = \lambda_{St}\sigma_{Nt}\mathrm{dt}$, $\mathrm{E_t}\left[\frac{\mathrm{d\Pi_t}}{\mathrm{\Pi_t}}\right] = \lambda_{Bt}\sigma_{Bt}\mathrm{dt}$, $\mathrm{V_t}\left[\frac{\mathrm{dF_{Nt}}}{\mathrm{F_{Nt}}}\right] = \sigma_{Nt}^2 dt$ and $\mathrm{V_t}\left[\frac{\mathrm{d\Pi_t}}{\mathrm{\Pi_t}}\right] = \sigma_{Bt}^2 dt$. Theorem 1 immediately follows from these last four equations □

2. Relation with the Sharpe ratio of the convenience yield

Contrary to the basis that we defined with two observable futures prices, the instantaneous convenience yield is not observable and is then model dependent, Schwartz (1997). As already stated in the introduction, only reduced-form models compute the Sharpe ratio of the convenience yield / basis. We have demonstrated in the preceeding section that the Sharpe ratio of the basis can be computed model-free. We now consider several reduced form models to make the link between the Sharpe ratio of the convenience yield and that of the basis. Namely, we consider the models of Gibson and Schwartz (1990), Richter and Sorensen (2002), Nielsen and Schwartz (2004), and Casassus and Collin-Dufrense (2005). We refer to these models as GS, RS, NS and CCD, respectively.

These four models use alternatively as primitives the spot price, S, the convenience yield, δ,[7] the risk-free rate, r, and the volatility of the spot price, v. As a consequence, the filtration of section 1, $\Xi \equiv (\mathrm{I}_t: t \in [0; T_E])$, can be understood as the augmented filtration of the Brownian motions representing the risks of the spot price, z_S, the convenience yield, z_δ, the risk-free rate, z_r, and the volatility, z_v, respectively. The risk of the basis does not need to be explicated as in section 1 as the basis is a function of the state variables described above.

Because these four models infer the dynamics of the spot price, we define the basis in this section as $B(T) \equiv \frac{F(T)}{S}$. We also denote by, $X \equiv ln(S)$ With obvious notations the basis for the models $Y \in \{GS, RS, NS, CCD\}$ can be computed as follows – the proof is available from the authors upon request:

$$B_{GS}(T) = exp\big(\eta_{0,GS}(T) + \eta_{\delta,GS}(T)\delta\big), \quad \text{Eq.(6a)}$$
$$B_{RS}(T) = exp\big(\eta_{0,RS}(T) + \eta_{\delta,RS}(T)\delta + \eta_{v,RS}(T)v\big), \quad \text{Eq.(6b)}$$
$$B_{NS}(T) = exp\big(\eta_{0,NS}(T) + \eta_{\delta,NS}(T)\delta\big), \quad \text{Eq.(6c)}$$
$$B_{CCD}(T) = exp\big(\eta_{0,CCD}(T) + \eta_{X,CCD}(T)X + \eta_{\delta,CCD}(T)\delta + \eta_{r,CCD}(T)r\big) \quad \text{Eq.(6d)}$$

Where $\eta_{V,Y}(T)$ are the elasticities of the basis for a futures contract of maturity T to the state variable $V \in \{X, r, \delta, v\}$ and for model $Y \in \{GS, RS, NS, CCD\}$; with the convention that if, for a model a state variable does not affect the basis, the elasticity is set to zero: for example, $\eta_{X,GC}(T) \equiv 0$. The computation of the elasticities $\eta_V(T)$ is available from the authors upon request: these functions are deterministic function of time. The functions $\eta_{0,Y}(T), Y \in \{GS, RS, NS, CCD\}$ are also deterministic functions of time available from the authors upon request.

We apply Ito lemma to Eqs.(6a-d), to find the dynamics of the basis for the four models as a function of the elasticities $\eta_{V,Y}(T)$:

$$\frac{dB_Y(T)}{B_Y(T)} = [\quad]dt + \eta_{X,Y}(T)\sigma_{Xt}dz_{St} + \eta_{\delta,Y}(T)\sigma_{\delta t}dz_{\delta t} + \eta_{r,Y}(T)\sigma_{rt}dz_{rt} + \eta_{v,Y}(T)\sigma_{vt}dz_{vt} \quad \text{Eq.(7)}$$

Where the part $[\quad]dt$ absolutely continuous with respect to dt does not need to be specified for our Sharpe ratio target and where σ_{Vt}, is the volatility of state variable $V \in \{X, r, \delta, v\}$.

[7] Although the spot price and the convenience yield are not observable, these articles rely on state-space form techniques, using the observable term structure of futures prices to infer them.

For the GS and NS models, i.e. for $Y \in \{GS, NS\}$, the elasticities $\eta_{X,Y}(T), \eta_{r,Y}(T)$ and $\eta_{v,Y}(T)$ are equal to zero. As a consequence, for these two models the basis is perfectly corelated to the convenience yield and the Sharpe ratios of the basis and the convenience yield are identical. For the RS and NS models the elasticities $\eta_{v,CCD}(T), \eta_{X,RS}(T),$ and $\eta_{r,RS}(T)$ are equal to zero. We consign the values of the elasticities of the CCD model, $\eta_{X,CCD}(T), \eta_{\delta,CCD}(T)$ and $\eta_{r,CCD}(T)$ in Table 1 and the elasticities of the RS model, $\eta_{\delta,RS}(T)$ and $\eta_{v,RS}(T)$ in Table 2. The elasticities are computed for maturities T=1, 2, 3, 6 and 12 months.

[INSERT TABLE 1 AROUND HERE]

Typically, a maturity between two and three months is used to compute the Sharpe ratios in our empirical part in section four. We start by investing the oil market. We infer from Table 1 that, for maturities two or three months, the elasticity of the basis we respect to the convenience yield is between 25 and 39 higher than that of the (log) spot price. In addition, the volatility of the convenience yield for oil is slightly higher, 46.7%, than the volatility of the spot price, 39.7% (Casassus and Collin-Dufresne, 2005). As far as the risk-free rate is concerned, for the maturities two and three months, the elasticity of the convenience yield, in absolute value, is only between 1.08 and 1.13 higher than that . However, the volatility of the risk-free rate is only of 1%, that is around 40 times lower than the volatility of the convenience yield.

The results are similar for the copper market: Table 1 shows that the elasticity of the basis with respect of the convenience yield is between 47 and 75 times the elasticity of the basis with respect of the spot price. In addition, the volatility of the convenience yield for copper is of 20.1% while that of the spot price is 22.8%. As far as the risk-free rate is concerned, the elasticity of the convenience yield is only between 0.98 and 0.99 in absolute value that of the risk-free rate. However, the volatility of the convenience yield of the copper is more than 20 times that of the risk-free rate. As a consequence, looking at Eq. (7), we can say that for oil and copper, the Basis and the convenience yield are almost perfectly correlated and that the Sharpe ratio of the Basis is nearly the same as the Sharpe ratio of the convenience yield.

As far as the precious metals, gold, is concerned, the elasticity of the (log) of the spot price is zero whatever the maturity of the futures contract. However, the elasticity of the convenience yield, in absolute value and for maturity two and three months, is between that 0.98 and 0.97 that of the elasticity of the risk free rate. In addition, the volatility of gold is only of 1.5% that is only 1.5 the volatility of the risk-free rate. Similar results hold for silver. As a consequence, we can see that the risk-free rate impacts the basis in a similar magnitude as that of the convenience yield. As a consequence, we cannot say that the risk of the Basis and the risk of the convenience yield are similar. However, we strongly intuite that the elasticity of the risk-free rate with respect to the basis comes from the cost of time: we intuit that if we discount the futures price by the zero-coupon bond of the same maturity, then the corresponding (weak) basis will be almost perfectly correlated to the convenience yield. We leave this investigation for further research.

[INSERT TABLE 2 AROUND HERE]

Finally, in the case of the Soybean, the elasticity with respect to the convenience yield is between 32 and 53 times that of the eleasticity with respect to the volatility of the spot price. In addition, computation from the article of Richter and Sorensen (2002) available from the authors upon request, show that the volatility of the convenience yield is between 0.8 and 2.7 that of the volatility of the spot price. Consequently, for the

3. Empirical analysis

The preceeding section evidences the validity of the empirical identity between the Sharpe ratio of the basis and the Sharpe ratio of the convenience yield for Oil (Energy), Copper (Base metal), Soybean (Grains). However, the this empirical identity is questioned in the case of Gold and Silver (Precious metals) as we find that for these commodities that the impact of the interest rate is of the same magnitude as that of the convenience yield on the basis. However, we intuite that the difference between the Sharpe ratio of the basis and the Sharpe ratio of the convenience yield mainly comes from the cost of time. We leave this result for further research and nevertheless include them in our analysis for completeness with this stated caveat. Besides, we will study the impact of interest rates in our framework in the robusteness checks sections. With these caveat in mind, we know identify the Sharpe ratio of the basis with that of the convenience yield.

In addition, we include two additional commodities for each of the four categories, namely energy, base metals, precious metals and grains. The commodities are selected based on the availability of the data and the importance of their futures markets. i) For Energy, we add Heating Oil and Gasoline to Oil; ii) for Base Metals, we add Aluminium and Zinc to Copper; iii) for Precious Metals, we add Silver and Platinum to Gold. iv) For grains, we add

Corn and Wheat to Soybean. We are left with 12 storable commodities for our analysis of the Sharpe ratios.

3.1 Construction of the Sharpe ratios and their determinants

The periods of analysis for the commodities are based on the availability of the data needed to construct the Sharpe ratios and their determinants, namely: daily futures prices, monthly inventory and monthly open interest. The monthly open interest is available at the Commodity Futures Trading Commission since January 1986 except for Aluminium and Zinc that are traded on the London Metal Exchange where no data are available. Inventory is downloaded from the U.S. Energy Information Administration for the three Energy commodities and the U.S. Department of Agriculture for the three Grains commodities. Note that the record of grains inventory was discontinued in August 2014. For other commodities, the inventory is downloaded from Bloomberg and is the amount of inventory available at the warehouse of the exchange. Futures prices are downloaded from Bloomberg. In addition, one year of inventory is necessary to compute the first level of inventory (Gorton, Hayashi and Rouwenhorst).

[INSERT TABLE 3 AROUND HERE]

The reader can refer to Gorton, Hayashi and Rouwenhorst (2013), appendix A1 for the futures exchange of the commodities, appendix B for the choice of the inventory and appendix C for the positions of traders. As in Gorton, Hayashi and Rouwenhorst (2013) we construct monthly end of the month variables and rely on the nearest and next nearest for the nearby and distant futures contract, respectively. The choice of the variables and their constructions inspired by the empirical literature about commodities, Gorton, Hayashi and Rouwenhorst (2013) in particular. The construction of the variables is also summarized in Table 3. We are left with the following periods of investigation: Oil, 443 months from January 1986 to November 2022; Heating Oil, 436 months from August 1986 to November 2022; Gasoline, 433 months from October 1986 to October 1986; Copper, 351 months from September 1993 to November 2022; Aluminium, 303 months from August 1997 to November 2022; Zinc, 305 months from August 1997 to December 2022; Gold, 352 months from September 1993 to December 2022; Silver, 352 from September 1993 to December 2022; Platinum, 194 months from November 1996 to December 2012; Soybean, 344 months from January 1986 to August

2014; Corn, 344 months from January 1986 to August 2014; Wheat, 344 months from January 1986 to August 2014;

3.2 The dynamics of the Sharpe ratios

We first start by an eye-ball inspection of the time series of the Sharpe ratios for the 12 commodities. Results are similar across commodities and Sharpe ratios, except for the Sharpe ratio of the convenience yield for Gold and Silver. All the Sharpe ratios exhibit a strong mean-reverting pattern, frequently change sign and take values between -0.5 and 0.5. The Sharpe ratios of the convenience yield for Gold and Silver exhibit also some mean-reverting pattern and frequently change sign. However, these two Sharpe ratios can take higher absolute values: until 2 for Gold (Silver). The dynamics of the convenience yield are further studied in Tables 5 a,b,c below.

[INSERT Figure 1 here]

We then give a descriptive statistics study of the two Sharpe ratios, namely the Sharpe ratios of the spot price and the convenience yield. Specifically, we compute the the first four moments of the variation of the Sharpe ratios for the 12 commodities, Table 4a. The results are surprising stable inside commodity classes, between commodity classes and between the risk of the spot price and that of the convenience yield. All of the Sharpe ratio exhibit a mean that is very close to zero and a very high volatility. In addition, their distribution is symmetric with skewnesses that are very low and their kurtosis are very close to that of a normal distribution, 3. Finally, for all of the commodities and the Sharpe ratios, we can not reject the Null hypothesis of the Jarque-Bera test at 5% that the variation of the Sharpe ratios are Gaussian distributed.

[INSERT Table 4a around here]

Table 4a also displays the correlation between the variation of the Sharpe ratio of the two risks for the 12 commodities. All of the correlations are significant at the 5% level except for Gold and Silver. For these thwo commodities, the p-value of the correlation coefficients are more than 10%. All of the correlation coefficients are positive but the correlations are very imperfect: the maximum correlation reaches only 53.47%. This imperfect correlation explains the success of (reverse) calendar spread amongst commodity traders.[8] We believe we are the

[8] For an account on the popularity of spreads, including calendar spread, on futures markets, the reader can report to the webpage on futures spread of the Chicago Mercantile Exchange: https://www.cmegroup.com/education/courses/understanding-futures-spreads/futures-spread-overview.html

first to present this evidence that has important consequences for commodities portfolio management. Our analysis of the determinatns of the Sharpe ratios below explain this imperfect correlation.

[INSERT Table 4b around here]

For comparaison, we also plot the correlation between the monthly returns of the spot price and the monthly variation of the basis. As predicted by the theory of storage, the correlations are positive, except for two precious metals, Gold and Platinum, where the correlation is negative.[9] However, these two correlations coefficients are not significantly different from zero. This result is not surprising, as the demand in precious metals, also sometimes called financial commodities, tend to be more explained by investment than by any production process. Nevertheless, the correlations between the variation of the two Sharpe ratios and between the return and the variation of the basis tend to be higher for Energy commodities, than for Base Metals and Grains; and the latter tend to be higher than those for precious metals.

[INSERT Table 5a around here]

We now confirm the qualitative results of Fig. 1 with a quantitative analysis of the dynamics of the Sharpe ratio. We regress the variation of each Sharpe ratio on their lagged value and we count, in proportion, the number of times they change sign. Specifically, for each of the four commodities, we run the following regressions:

$$\Delta\lambda_{St} = \sum_{i=1}^{12} \alpha_i + \beta_S \lambda_{St} + \varepsilon \qquad \text{(R1a)}$$

$$\Delta\lambda_{\delta t} = \sum_{i=1}^{12} \alpha_i + \beta_\delta \lambda_{\delta t} + \varepsilon \qquad \text{(R1b)}$$

where α_i are monthly dummies where i indicates the month number during the year. We start by analysis of the seasonality of the Sharpe ratios in Table 5a. First, we see that the magnitude of the dummies is small for all commodities and for the Sharpe ratio of the spot price as well as the Sharpe ratio of the convenience yield. The economic impact of the seasonality on the Sharpe ratios is weak. Second, except for a few months that very between commodity classes as well as inside commodity classes, the dummies are not significant statistically. This is even true for Grains (Soybean, Corn and Wheat) which are highly seasonal commodities. The lack

[9] This correlation is lower than what is usually obtained in reduced form continuous time models: e.g. Schwartz (1997) obtains a correlation of more than 80% for oil between the convenience yield and the spot price. This difference arises because the correlation computed in Schwartz (1997) is instantaneous whereas we here compute correlations at a monthly frequency.

of impact of seasonality on the Sharpe ratio can easily be explained by the fact that, as its name indicates, the Sharpe ratio is a ratio of two quantities affected by seasonality.

[INSERT Table 5b around here]

Table 5b gives the result of the slope coefficients of the regressions given in Eq. (R1a) and Eq. (R1b). The results are identical across the 12 commodities and between the Sharpe ratios of the two risks, namely the convenience yield and the spot price. First, all the slope are extremely statiscally significant. Second, the economic power of the slopes is extremely high and negative: all of the Sharpe ratios exhibit a very strong and fast mean-reverting pattern. This strong mean-reverting pattern combined with the low mean of the variation of the Sharpe ratios outlined in Table 4a explains the result outlined in Table 5c. Whatever the risk considered, spot price or convenience yield, whatever the commodity, the Sharpe ratio changes sign between 40% and 60% of the time, that is roughly every two months!

[INSERT Table 5c around here]

3.3 The determinants of the Sharpe ratios

We run a regression of the Sharpe ratios of the spot price and the convenience yield with their determinants[10] presented in Table 3 as well as with monthly dummies for seasonality. We use the abbreviations given in Table 1 to describe the regression by Eq. (R2a) and Eq. (R2b):

$$\lambda_S = \sum_{i=1}^{12} \alpha_i + \beta_\rho \rho + \beta_\sigma \sigma + \beta_{r_S} r_S + \beta_B \mathrm{B} + \beta_{FutP} \mathrm{FutP} + \beta_{HP} \mathrm{HP} + \beta_{INV} \mathrm{INV} + \epsilon \qquad \text{(R2a)}$$

$$\lambda_\delta = \sum_{i=1}^{12} \alpha_i + \beta_\rho \rho + \beta_\sigma \sigma + \beta_{r_S} r_S + \beta_B \mathrm{B} + \beta_{FutP} \mathrm{FutP} + \beta_{HP} \mathrm{HP} + \beta_{INV} \mathrm{INV} + \epsilon \qquad \text{(R2b)}$$

First, we use Augmented Dickey-Fuller (ADF) and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) to verify the stationarity of our data. Second, we use the Variance Inflation Factor to check for multi-collinearity. Third we control for the autocorrelation of the residuals and some ARCH effect. We plot the autocorrelation function of the residuals with its bounds and we run a Ljung-Box test on the residuals and use an Engle test for heteroskedasticity on the residuals. Results are available from the authors upon request. Finally, we center and reduce all variables, i.e. we subtract the sample mean and divide by the sample standard deviation: the

[10] We have also considered other determinants such as: the Samuelson effect, defined as the volatility of the distant futures contract minus the volatility of the nearby contract, the logarithm of the nearby futures price and the return of the distant futures contract. However, either these variables were non stationary, either they added multicollinearity to the data after checking for their Variance Inflation Factor.

slope coefficients are then correlation coefficients and the economic strength of the relation can be assessed.

[INSERT Table 6 around here]

The results of our multivariate regressions R2a,b are consigned in Table 6, Panel A (B) for the Sharpe ratio of the spot price (convenience yield). For each estimated regression slope, we give its associated p-value. We do not report the monthly dummies. These monthly dummies are available from the authors upon request, but as mentioned in the analysis of section 3.2 their economic impact is not meaningful.

Several comments are in order. First, the Sharpe ratios of the convenience yield are much are harder to determine than those of the spot price. Indeed, for the same determinants, the adjusted R^2 varies between 73.35% and 83.81% for the Sharpe ratio fo the spot price but only between 8.19% and 23.33% for the Sharpe ratio of the convenience yield – as mentioned in section two, we do not consider the analysis of the Sharpe ratio of the convenience yield. In addition, only one determinant is economically and statistically significant per Sharpe ratio across all 12 commodities and these determinants differ. As far as the Sharpe ratio of the spot price is concerned, the return is an extremaly strong, both statiscally and economically, positive determinant. Regarding the Sharpe ratio of the convenience yield, the basis is a strong, both statiscally and economically, positive determinant – the return is also a strong positive determinant of the spot price, but its economic power is in general less important than the basis.

Other price driven determinants are economically weak, with correlation in absolute value less than ten percent. For example, the correlation between the basis and the spot price looks statistically significant for both Sharpe ratio in the case of oil, but the statistical significance disappears in a univariate regression for both Sharpe ratios. Similar behaviors happen for the volatility of the spot price: it is significant and positively related to the Sharpe ratio of the spot price for oil, but the sign is negative in a univariate analysis; as far as the oil Sharpe ratio of the convenience yield is concerned the volatility loses its significance in a univariate analysis. Similar results hold for the Futures Risk Premium.

Non price driven determinants such as the Hedging Pressure and the Inventory have an economically impact but it is nevertheless worth mentioning. For example, Hedging Pressure has a statistically negative impact for all commodities (except for Aluminium and Zinc because there is no data available) for the Sharpe ratio of the spot price and the statistics are still significants for univariate regressions. The impact of inventory on the Sharpe ratio of the spot price is commodity dependent: its impact is negative for energy commodities but varies for other types of commodities.

4. Conclusion

This article complements the work of Cortazar, Kovacevic and Schwartz (2015), Cortazar, Millard, Ortega and Schwartz (2019), Cortazar, Ortega, Rojas and Schwartz (2021) on the behavior of commodity prices under the physical measure. Specifically, we study the Sharpe ratios of the spot price and the convenience yield. Indeed, Sharpe ratios constitute the main difference between the risk neutral and physical measure and the spot price and the convenience yield are main risks attributed to commodity prices.

We first start by computing a model free estimate of the Sharpe ratios of the spot price and the convenience yield. The Sharpe ratio of the spot price is immediately computed by identifying the spot price with the nearest futures price. As far as the convenience yield is concerned, we find that a reverse calendar spread investment is perfectly correlated with the basis. As a consequence and because a (reverse) calendar spread is traded asset, the Sharpe ratio of the risk of the basis can be computed from the (reverse) calendar spread in a straight forward manner. In addition, we find that for commodities except precious metals, namely energy, base metals and grains, that the convenience yield is the main risk that constitutes the basis. We show that the impact of other state variables, namely spot price, interest rates and volatility, have a very small impact on the basis compare to the convenience yield. We can then identify the risk of the basis to the risk of the convenience yield; as well as their respective Sharpe ratios.

With our model-free estimates, we are able to conduct a thorough empirical studies of the Sharpe ratios of commodity risks. We conduct our empirical analysis on four different classes of commodities, namely, energy, base metals, grains and precious metals. For a thorough analysis we choose three commodities per commodity classes: Oil, Heating Oil and Gasoline (Energy); Copper, Aluminium and Zinc (Base Metals); Soybean, Corn and Wheat (Grains); Gold, Silver and Platinum (Precious Metals).

We start with a descriptive analysis of the variation of the two Sharpe ratios and find similar results across the two risks and the 12 commodities. The means of the Sharpe ratios are very close to zero and their standard deviations are very high and comparable. Their distribution does not exhibit any skewness nor excess kurtosis: for the 2*12 = 24 Sharpe ratios, we can not reject the nul hypothesis that the variation of the Sharpe ratios is distributed like a normal random variable. Finally, except for Gold and Silver where the correlation is not significant, we find a positive and imperfect correlation, at most 53%, between the variation of the Sharpe ratio of the spot price and that of the Sharpe ratio of the convenience yield.

We then conduct a time series analysis of the Sharpe ratios by conducting a regression analysis of the variations of the Sharpe ratios on their lagged value and monthly dummies. We find that the Sharpe ratios do not exhibit specific seasonal behavior. Of most interest, we find that the 2*12 = 24 Sharpe ratios exhibit a strong mean reversion pattern with a similar speed of mean reversion across the 24 Sharpe ratios of around one. Finally, we find that all of the 24 Sharpe ratios change sign extremely frequently, between 40% and 60%, that is on average once every two months for our monthly sample analysis.

Finally, we conduct a cross-section analysis of the Sharpe ratios. We regress the Sharpe ratios on various usual determinants used in storable commodities: return on the spot price, basis, correlation between the basis and the spot price, volatility of the spot price, futures risk premium, hedging pressure and inventory. We find that there is only one strong economically and statistically predictor of per Sharpe ratio of risk. For the Sharpe ratio of the spot price (convenience yield), the return of the spot price (basis) is a stong, both economically and statistically positive determinant for all the 12 commodities.

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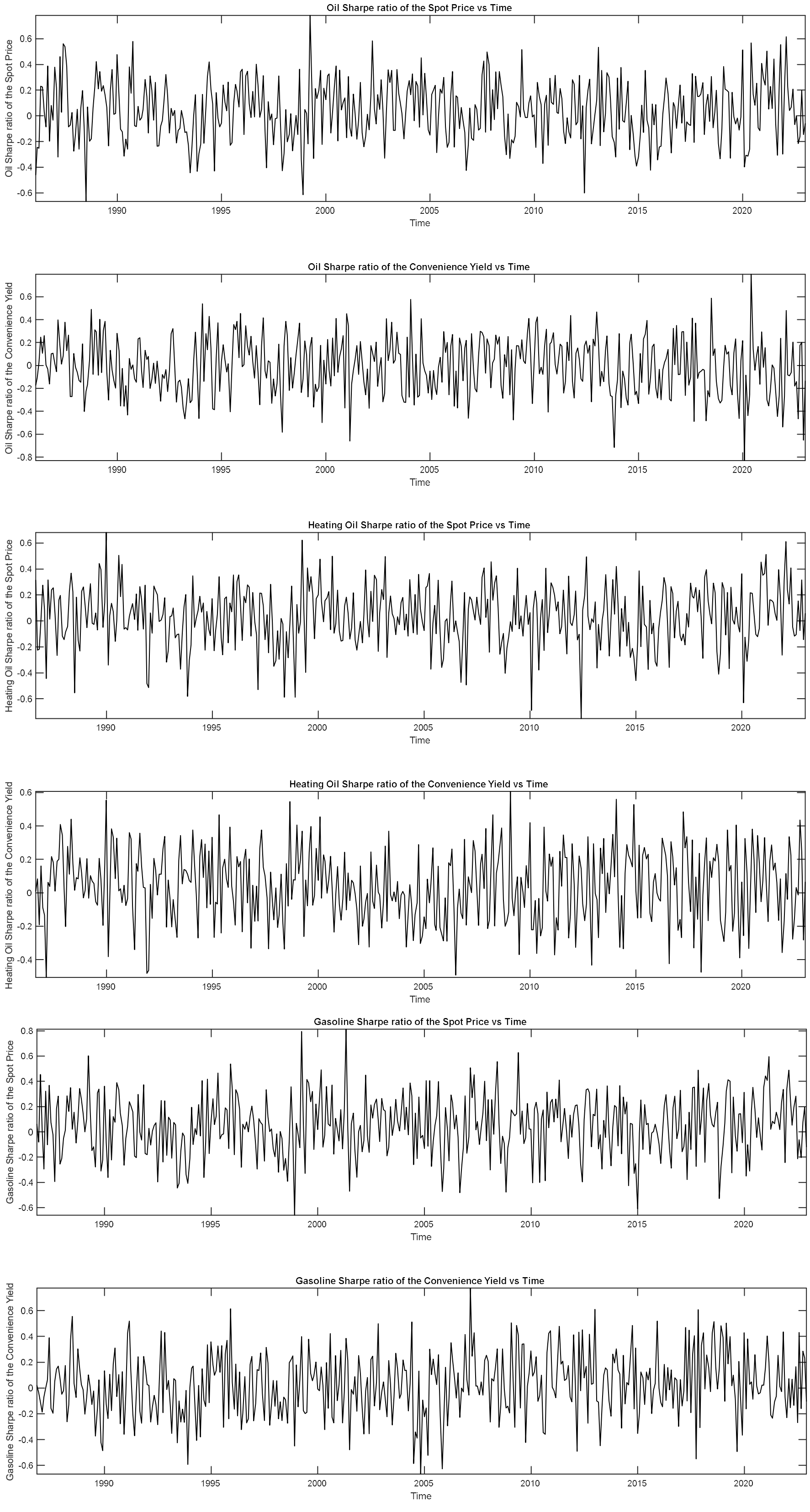
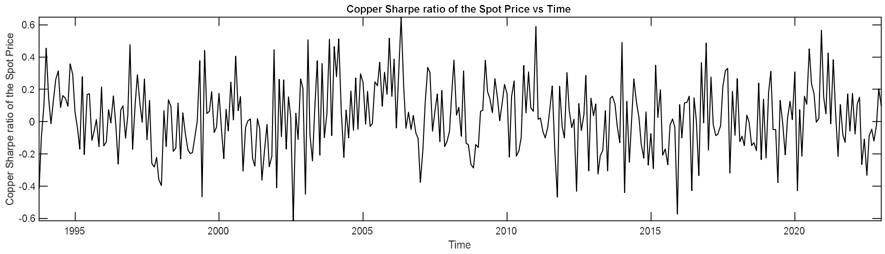
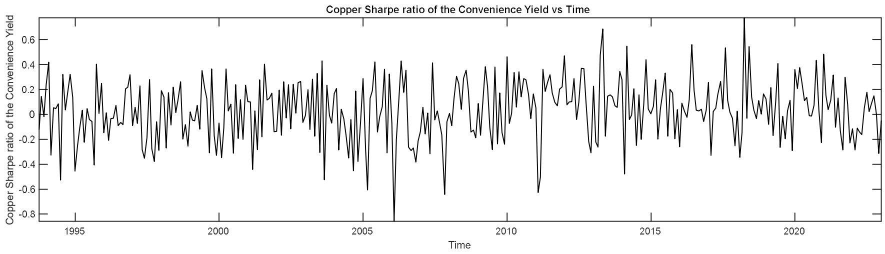
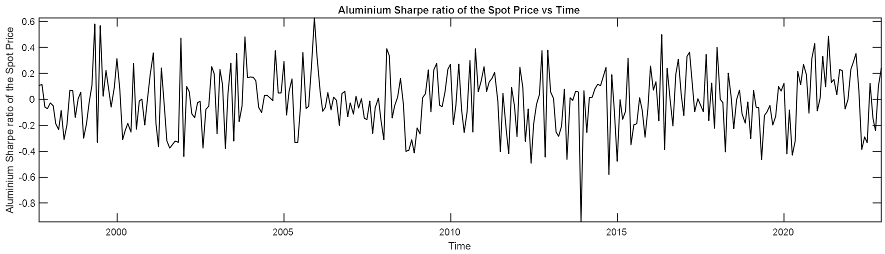
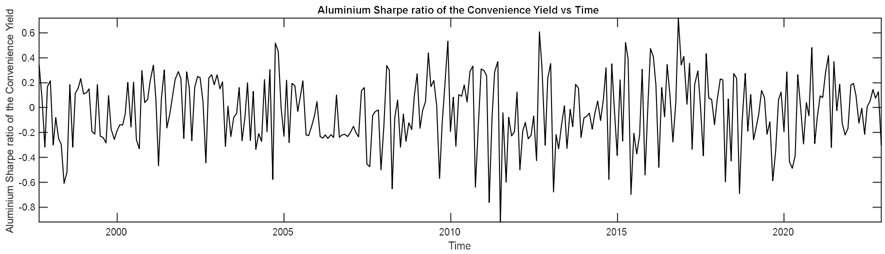
Figure 1 Dynamics of the Sharpe ratios




Copper Sharpe ratio of the Spot Price vs Time
Copper Sharpe ratio of the Spot Price
Time


Copper Sharpe ratio of the Convenience Yield vs Time
Copper Sharpe ratio of the Convenience Yield
Time


Aluminium Sharpe ratio of the Spot Price vs Time
Aluminium Sharpe ratio of the Spot Price
Time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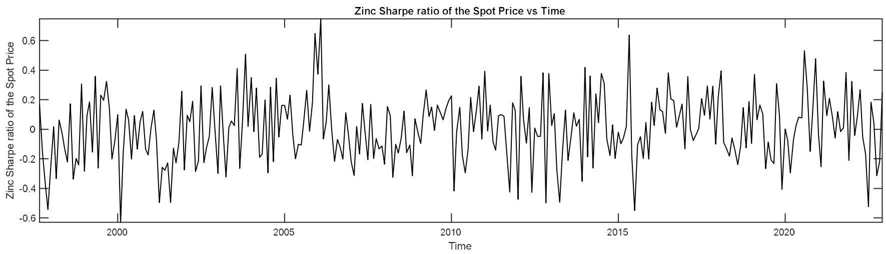
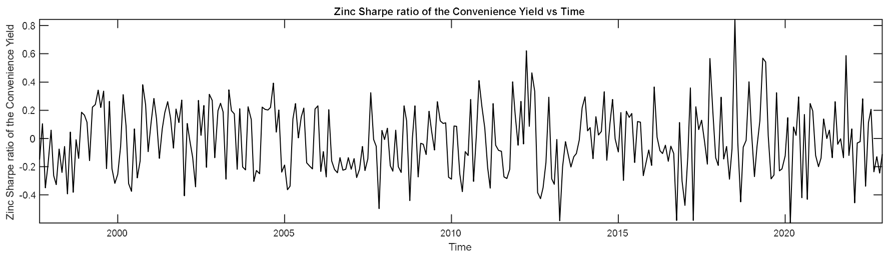
Aluminium Sharpe ratio of the Convenience Yield
Time


Zinc Sharpe ratio of the Spot Price vs Time
Zinc Sharpe ratio of the Spot Price
Time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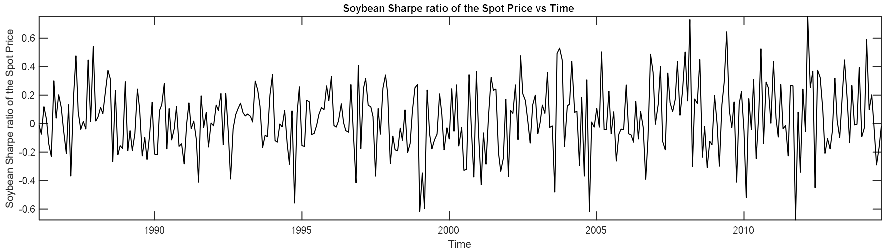
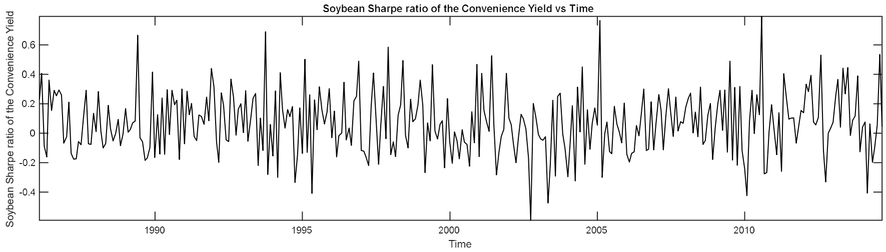
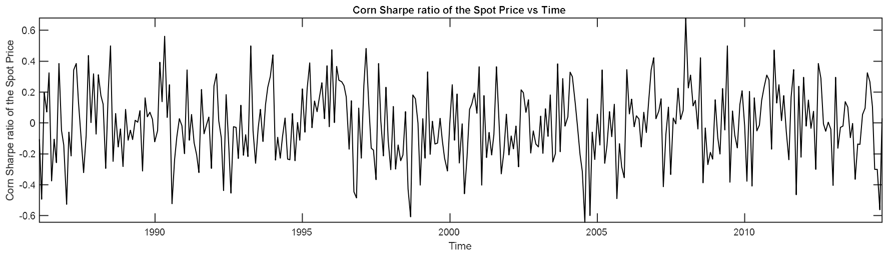
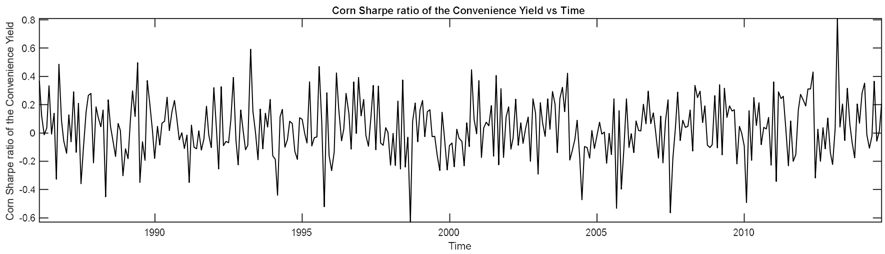
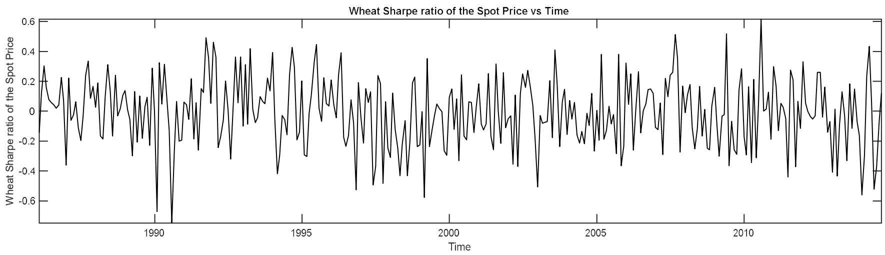
Zinc Sharpe ratio of the Convenience Yield
Time

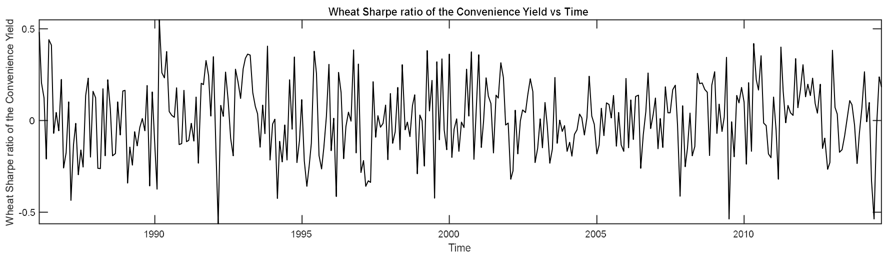

Soybean Sharpe ratio of the Spot Price vs Time
Soybean Sharpe ratio of the Spot Price
Time

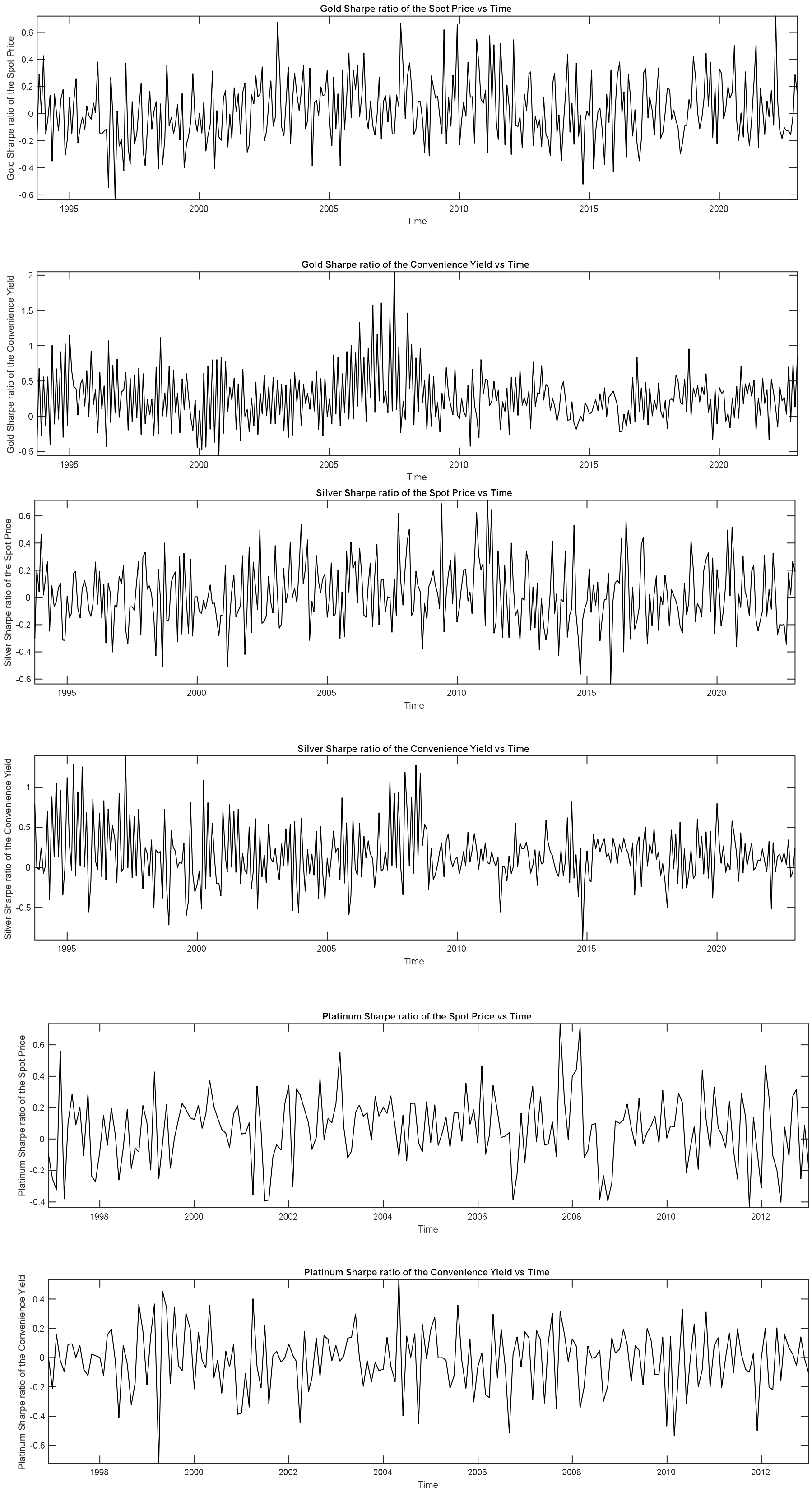

Soybean Sharpe ratio of the Convenience Yield vs Time
Soybean Sharpe ratio of the Convenience Yield
Time


Corn Sharpe ratio of the Spot Price vs Time
Corn Sharpe ratio of the Spot Price
Time


Corn Sharpe ratio of the Convenience Yield vs Time
Corn Sharpe ratio of the Convenience Yield
Time


Wheat Sharpe ratio of the Spot Price vs Time
Wheat Sharpe ratio of the Spot Price
Time


Wheat Sharpe ratio of the Convenience Yield vs Time
Wheat Sharpe ratio of the Convenience Yield
Time



Legend: This figure displays the evolution over time of the Sharpe ratio of the spot price and that of the convenience yield four the four commodities of the energy sector under investigation.

Tables

Table 1. Elasticities of basis vs the maturity of the distant contract – CCD's model

Panel (A): (log) spot price

TD (months)	1	2	3	6	12
Oil	-0,10%	-0,38%	-0,82%	-2,93%	-9,43%
Copper	-0,05%	-0,20%	-0,45%	-1,62%	-5,41%
Silver	0,00%	0,02%	0,04%	0,17%	0,68%
Gold	0,00%	0,00%	0,00%	0,00%	0,00%

Panel (B): convenience yield

Oil	-7,85%	-14,80%	-20,93%	-35,22%	-50,52%
Copper	-7,93%	-15,10%	-21,57%	-37,37%	-56,80%
Silver	-8,36%	-16,77%	-25,23%	-50,94%	-103,92%
Gold	-8,20%	-16,13%	-23,81%	-45,41%	-82,73%

Panel (C): interest rates

TD (months)	1	2	3	6	12
Oil	7,55%	13,66%	18,47%	26,44%	22,40%
Copper	7,99%	15,32%	22,06%	39,15%	62,69%
Silver	8,31%	16,59%	24,83%	49,33%	97,42%
Gold	8,28%	16,46%	24,54%	48,21%	93,21%

Legend: This table displays the elasticity of the basis with respect to the log spot price and the convenience yield for four commodities: oil, copper, silver and gold. These elasticities are inferred from the study of Casassus and Collin-Dufresne (2005).

Table 2. Elasticities of basis vs the maturity of the distant contract – RS's model

Panel (A): convenience yield

TD (months)	**1**	**2**	**3**	**6**	**12**
Soybean	-8,06%	-15,58%	-22,62%	-41,07%	-68,40%

Panel (B): volatility of the spot price

TD (months)	**1**	**2**	**3**	**6**	**12**
Soybean	-0,08%	-0,29%	-0,69%	-6,49%	-10,43%

Table 3. Variables and their construction

Variable	Name	Construction
Sharpe ratio of the spot price	λ_S	Sample moment analog of Eq. 3a using the last business month of daily data where a return is dropped if it uses futures contract with different maturities (Gorton, Hayashi and Rowenhorst, 2012 Appendix A.3)
Sharpe ratio of the convenient yield	λ_δ	Sample moment analog of Eq. 3b using the last business month of daily data where a return is dropped if it uses futures contract with different maturities (Gorton, Hayashi and Rowenhorst, 2012 Appendix A.3)
Basis	B	$B = \frac{365}{T_D - T_N} \times \left(\frac{F_N}{F_D} - 1\right)$, where T_D and T_N are the maturities in days of the distant and nearby futures contracts (Gorton, Hayashi and Rowenhorst, 2012 Appendix A.5)
Correlation spot price and convenience yield	ρ	Sample correlation between the return of the nearby futures contract and the variation of the basis. The sample uses the last business month of daily data where a return is dropped if it uses futures contract with different maturities (Gorton, Hayashi and Rowenhorst, 2012 Appendix A.3)
Volatility spot price	σ	Sample annualized standard deviation of the nearby futures return using the last business month of daily where a return is dropped if it uses futures contract with different maturities (Gorton, Hayashi and Rowenhorst, 2012 Appendix A.3)
Return on the spot price	r_S	Return on the nearby futures contract computed using F_{Nt} and $F_{N(t-1)}$ where the t is the end of the month under analysis. The same futures contract is used to compute the monthly return to avoid the roll-over effect.
Futures Risk Premium	FutP	Realized return on the nearby futures contract computed using $F_{N(t+1)}$ and F_{Nt} where the t is the end of the month under analysis. The same futures contract is used to compute the monthly return to avoid the roll-over effect. (Gorton, Hayashi and Rowenhorst, 2012 Appendix A.6)
Hedging Pressure	HP	The last published position of traders of the month under analysis is taken as the end of the month open interest. The Hedging pressure is equal to the net long position of commercial traders divided by the total open interest (Gorton, Hayashi and Rowenhorst, 2012 Appendix C)
Inventory	INV	We rely on the definition of the normalized inventory of (Gorton, Hayashi and Rowenhorst, 2012) defined as the ratio of the end of the month inventory divided by the one year moving average of the last monthly inventory data.

Legend: This table displays the names and the details of the construction of the dependent variables, the Sharpe ratios of the spot price and of the convenience yield, as well as their determinants.

Table 4a: Descriptive statistics of the Sharpe ratios

Panel A: Energy

	$\Delta\lambda_S$ (O)	$\Delta\lambda_\delta$ (O)	$\Delta\lambda_S$ (HO)	$\Delta\lambda_\delta$ (HO)	$\Delta\lambda_S$ (G)	$\Delta\lambda_\delta$ (G)
Mean	0,31%	-0,44%	-0,46%	-0,31%	0,13%	0,20%
standard deviation	1,32	1,35	1,36	1,39	1,38	1,43
skewness	2,82%	-6,08%	-5,65%	-2,62%	-3,27%	4,40%
kurtosis	2,90	3,08	3,10	2,67	2,54	2,88
Jarque-Bera	H0	H0	H0	H0	H0	H0
correlation	39,88%		33,42%		53,47%	

Panel B: Base Metals

	$\Delta\lambda_S$ (C)	$\Delta\lambda_\delta$ (C)	$\Delta\lambda_S$ (A)	$\Delta\lambda_\delta$ (A)	$\Delta\lambda_S$ (Z)	$\Delta\lambda_\delta$ (Z)
Mean	0,80%	-0,23%	0,19%	-0,77%	-0,39%	0,18%
standard deviation	1,43	1,43	1,38	1,42	1,41	1,37
skewness	-0,30%	10,67%	-1,03%	-25,69%	-2,32%	2,29%
kurtosis	2,95	2,96	3,65	3,31	2,70	2,82
Jarque-Bera	H0	H0	H0	H0	H0	H0
correlation	17,79%		27,88%		16,75%	

Panel C: Grains

	$\Delta\lambda_S$ (S)	$\Delta\lambda_\delta$ (S)	$\Delta\lambda_S$ (C)	$\Delta\lambda_\delta$ (C)	$\Delta\lambda_S$ (W)	$\Delta\lambda_\delta$ (W)
Mean	-0,01%	-0,25%	0,17%	-0,23%	0,17%	-0,23%
standard deviation	1,42	1,48	1,41	1,49	1,41	1,49
skewness	1,43%	-20,04%	5,35%	3,80%	5,35%	3,80%
kurtosis	3,11	3,34	2,64	2,91	2,64	2,91
Jarque-Bera	H0	H0	H0	H0	H0	H0
correlation	16,80%		31,68%		45,23%	

Panel D: Precious Metals

	$\Delta\lambda_S$ (G)	$\Delta\lambda_\delta$ (G)	$\Delta\lambda_S$ (Si)	$\Delta\lambda_\delta$ (Si)	$\Delta\lambda_S$ (P)	$\Delta\lambda_\delta$ (P)
Mean	0,37%	0,83%	0,60%	-0,42%	-0,20%	-0,27%
standard deviation	1,46	1,71	1,45	1,61	1,41	1,60
skewness	-1,61%	10,48%	-7,97%	11,04%	7,73%	3,48%
kurtosis	2,74	2,52	2,88	2,75	3,50	3,56
Jarque-Bera	H0	H0	H0	H0	H0	H0
correlation	0.28%		22,09%		11,51%	

Legend: This table displays the first four moments and the correlation of the Sharpe ratios of the spot price and the convenience yield for the four energy commodities under investigation. O, G, HO and NG stand for Oil, Gasoline, Heating Oil and Natural Gas, respectively.

Table 4b: Correlation between the convenience yield and the spot price

Panel A: Energy

Oil	Heating Oil	Gasoline
41,61%	40,41%	35,43%

Panel B: Base Metals

Copper	Aluminium	Zinc
22,29%	11,51%	19,31%

Panel C: Grains

Soybean	Corn	Wheat
7,45%	20,39%	22,82%

Panel D: Precious Metals

Gold	Silver	Platinum
-8,65%	14,54%	-1,54%

Legend: This table displays the monthly correlation between the spot price and the convenience yield for the four commodities under investigation.

Table 5a

	Energy						Base Metals						Grains						Precious Metals					
	Oil		Heating Oil		Gasoline		Copper		Aluminium		Zinc		Soybean		Corn		Wheat		Gold		Silver		Platinum	
	Slope	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*
αJan	0,04	0,28	0,06	0,11	-0,04	0,30	-4,E-04	0,99	0,21	0,28	-0,02	0,70	0,10	0,03	0,05	0,25	-0,06	0,13	-0,01	0,78	-0,20	0,26	0,05	0,36
αFeb	0,07	0,08	-0,04	0,29	0,08	0,03	0,07	0,07	0,14	0,46	0,02	0,69	0,04	0,31	0,05	0,24	-0,01	0,79	-0,01	0,89	-0,18	0,32	0,15	0,01
αMar	0,08	0,03	-0,05	0,21	0,04	0,30	0,03	0,44	0,05	0,81	0,03	0,50	0,09	0,05	-0,02	0,62	0,00	0,99	0,09	0,03	0,36	0,05	0,20	2,E-04
αApr	0,02	0,58	0,04	0,32	0,05	0,17	0,02	0,58	0,19	0,33	0,01	0,76	0,07	0,12	0,02	0,67	-0,01	0,78	0,08	0,07	0,32	0,09	-0,02	0,71
αMay	0,04	0,28	4,E-03	0,91	0,14	4,E-04	0,06	0,12	0,33	0,10	0,04	0,35	0,02	0,70	-0,07	0,11	-0,08	0,07	0,10	0,03	0,24	0,20	0,11	0,04
αJun	0,04	0,31	0,04	0,27	0,12	2,E-03	0,04	0,36	-0,37	0,06	0,04	0,43	-0,03	0,48	-0,10	0,03	0,02	0,62	-0,02	0,65	-0,03	0,87	0,04	0,45
αJul	0,03	0,42	0,09	0,02	0,04	0,37	0,06	0,17	-0,08	0,71	-0,06	0,22	0,05	0,27	0,00	0,96	0,06	0,19	0,03	0,53	-0,13	0,47	-0,03	0,60
αAug	0,07	0,08	0,10	0,01	-0,01	0,71	2,E-03	0,96	-0,14	0,48	0,03	0,52	-0,05	0,25	-0,07	0,10	0,05	0,22	-0,01	0,81	-0,09	0,62	0,04	0,43
αSep	-0,04	0,27	-0,01	0,74	0,05	0,17	5,E-03	0,91	-0,37	0,06	-0,08	0,09	0,02	0,62	0,02	0,59	-0,02	0,71	-0,03	0,51	-0,28	0,13	0,03	0,55
αOct	-0,06	0,09	0,02	0,51	0,04	0,25	0,07	0,07	0,33	0,09	-0,05	0,29	0,11	0,02	-0,04	0,36	-0,01	0,84	-0,02	0,72	0,08	0,67	0,07	0,20
αNov	0,09	0,02	0,04	0,35	0,05	0,16	-0,03	0,53	-0,25	0,21	0,07	0,13	0,01	0,90	0,04	0,38	0,00	0,92	0,08	0,07	-0,16	0,37	5,E-03	0,93
αDec	0,06	0,10	0,08	0,05	0,01	0,75	0,01	0,74	-0,09	0,66	-0,02	0,59	-0,02	0,67	0,04	0,37	-0,02	0,70	0,04	0,32	0,14	0,46	0,11	0,04

	Energy						Base Metals						Grains						Precious Metals					
	Oil		Heating Oil		Gasoline		Copper		Aluminium		Zinc		Soybean		Corn		Wheat		Gold		Silver		Platinum	
	Slope	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*
αJan	-0,02	0,56	0,08	0,03	0,01	0,76	-0,04	0,40	0,38	0,05	-2,E-03	0,97	0,00	0,92	0,03	0,42	-0,08	0,03	0,59	0,00	-0,71	1,E-06	-0,07	0,15
αFeb	-0,07	0,10	0,01	0,86	0,08	0,05	-0,04	0,32	0,08	0,69	0,02	0,72	0,03	0,41	0,09	0,01	0,03	0,35	0,01	0,83	-0,79	1,E-07	0,09	0,06
αMar	0,02	0,61	1,E-04	1,00	5,E-03	0,91	0,08	0,05	0,23	0,25	-8,E-04	0,99	0,04	0,31	0,07	0,08	-0,02	0,59	0,51	5,E-19	0,55	2,E-04	-0,08	0,07
αApr	0,01	0,78	0,03	0,48	0,01	0,75	-0,09	0,04	0,30	0,13	-0,07	0,12	0,17	2,E-05	0,10	0,01	0,08	0,02	0,16	0,01	-0,38	0,01	-0,12	0,01
αMay	-0,01	0,85	-0,02	0,60	0,04	0,28	-0,08	0,07	-0,14	0,48	-0,02	0,63	0,11	0,01	-0,05	0,20	-0,04	0,30	0,43	0,00	-0,49	8,E-04	0,16	6,E-04
αJun	#####	0,92	-0,02	0,59	0,05	0,27	0,15	4,E-04	-0,39	0,05	-0,07	0,12	0,10	0,01	0,06	0,09	0,07	0,06	0,01	0,91	0,80	8,E-08	-0,08	0,12
αJul	0,02	0,61	0,06	0,09	0,06	0,12	0,05	0,29	0,15	0,45	-0,06	0,18	0,06	0,13	-0,05	0,23	-0,01	0,80	0,51	0,00	-0,43	5,E-03	4,E-03	0,93
αAug	0,07	0,10	0,08	0,02	-0,04	0,39	0,16	2,E-04	0,04	0,83	0,05	0,30	0,03	0,48	0,07	0,08	0,15	6,E-05	0,04	0,56	0,80	8,E-08	0,06	0,16
αSep	-0,11	0,01	0,05	0,19	0,06	0,16	-0,02	0,63	-0,14	0,48	0,04	0,43	0,05	0,23	0,05	0,16	-0,05	0,19	0,55	2,E-21	-0,28	0,06	-0,09	0,04
αOct	-0,09	0,02	0,06	0,09	-0,01	0,87	0,14	0,00	-0,10	0,61	0,03	0,59	0,17	3,E-05	0,01	0,86	-0,01	0,76	0,06	0,34	0,76	3,E-07	-0,10	0,03
αNov	0,03	0,40	0,03	0,33	0,00	0,99	-0,05	0,20	-0,32	0,11	0,04	0,38	0,01	0,88	0,05	0,19	0,08	0,03	0,51	2,E-18	-0,42	0,01	0,11	0,02
αDec	0,07	0,10	0,11	0,00	0,07	0,11	0,11	0,01	-0,16	0,42	-0,14	0,00	0,13	0,00	-0,02	0,58	-0,10	0,01	-0,05	0,37	0,56	2,E-04	-0,04	0,41

Table 5b: Regression of the variation of the Sharpe ratios with their lagged value

Panel A: Energy

	β_S	**p-value**	**Adj R^2**	**β_δ**	**p-value**	**Adj R^2**
Oil	-0,88	3,E-56	44,59%	-0,92	3,E-59	47,01%
Heating Oil	-0,94	7,E-60	47,09%	-0,97	7,E-63	48,23%
Gasoline	-0,96	1,E-61	48,46%	-1,02	1,E-66	50,87%

Panel B: Base Metals

	β_S	**p-value**	**Adj R^2**	**β_δ**	**p-value**	**Adj R^2**
Copper	-1,01	4,E-54	50,48%	-0,98	8,E-51	56,30%
Aluminium	-0,94	8,E-42	48,57%	-1,02	1,E-46	51,13%
Zinc	-0,99	8,E-45	49,44%	-0,94	7,E-42	47,70%

Panel C: Grains

	β_S	**p-value**	**Adj R^2**	**β_δ**	**p-value**	**Adj R^2**
Soybean	-1,00	1,E-51	50,66%	-1,08	2,E-57	56,47%
Corn	-1,00	2,E-51	50,02%	-1,10	4,E-59	57,04%
Wheat	-0,97	1,E-49	47,84%	-0,92	1,E-46	48,98%

Panel D: Precious Metals

	β_S	**p-value**	**Adj R^2**	**β_δ**	**p-value**	**Adj R^2**
Gold	-1,07	5,E-58	53,23%	-1,05	2,E-56	80,43%
Silver	-1,06	2,E-57	52,92%	-1,09	1,E-59	76,81%
Platinum	-0,98	4,E-28	50,90%	-1,21	3,E-38	69,26%

Table 5c: Change of signs of Sharpe ratios

Panel A: Energy

	Sign change, λS	Sign change, λδ
Oil	47,96%	43,89%
Heating Oil	48,05%	46,67%
Gasoline	47,22%	50,93%

Panel B: Base Metals

	Sign change, λS	Sign change, λδ
Copper	51,71%	49,14%
Aluminium	44,37%	46,03%
Zinc	49,67%	42,76%

Panel C: Grains

	Sign change, λS	Sign change, λδ
Soybean	47,81%	47,23%
Corn	50,44%	53,05%
Wheat	48,69%	48,98%

Panel D: Precious Metals

	Sign change, λS	Sign change, λδ
Gold	52,14%	48,72%
Silver	50.43%	64,96%
Platinum	49,74%	60,62%

Legend: This table displays the mean reversion analysis of the Sharpe ratios of the spot price and of the convenience yield four the four energy commodities under investigation. Panel A displays the results of the regression of the variation of the Sharpe ratios on their lagged values. Panel B displays the number of times, in proportion, the Sharpe ratios change sign.

Table 6: Sharpe ratios as a function of their determinants

Panel A: Sharpe ratio of the Spot price

	Energy						**Base Metals**						**Grains**						**Precious Metals**					
	Oil		**Heating Oil**		**Gasoline**		**Copper**		**Aluminium**		**Zinc**		**Soybean**		**Corn**		**Wheat**		**Gold**		**Silver**		**Platinum**	
Adj R^2	73,35%		76,52%		73,61%		83,81%		78,59%		72,29%		79,15%		77,15%		78,85%		81,49%		83,78%		82,75%	
	Slope	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*
β_ρ	-0,05	0,06	-0,02	0,38	0,03	0,28	0,02	0,51	-0,06	0,04	-0,07	0,05	0,01	0,72	-0,08	0,01	0,02	0,51	0,05	0,07	0,02	0,41	0,02	0,55
β_σ	0,06	0,03	0,05	0,06	0,03	0,18	0,13	7,E-08	0,05	0,07	-0,02	0,54	0,01	0,62	0,02	0,64	0,05	0,08	-0,04	0,13	0,08	0,00	0,05	0,11
β_{rS}	0,83	2,E-103	0,88	1,E-104	0,84	1,E-95	0,89	9,E-122	0,87	3,E-93	0,85	7,E-82	0,87	8,E-105	0,80	1,E-88	0,85	8,E-101	0,85	7,E-109	0,89	2,E-126	0,89	4,E-58
β_B	0,02	0,58	-0,21	1,E-09	-0,12	1,E-03	0,00	0,88	0,03	0,29	0,11	2,E-03	0,02	0,48	0,04	0,19	0,05	0,09	0,02	0,36	0,03	0,30	0,03	0,46
β_{FutP}	-0,04	0,12	0,00	0,87	0,00	0,87	-0,04	0,05	-0,06	0,02	0,00	0,97	0,01	0,76	0,01	0,82	0,05	0,04	0,03	0,15	0,03	0,21	-0,05	0,12
β_{HP}	-0,09	4,E-04	-0,06	0,04	-0,12	1,E-05	-0,09	4,E-04	NA	NA	NA	NA	-0,06	0,03	-0,14	2,E-06	-0,06	0,02	-0,13	9,E-06	-0,08	0,00	-0,08	0,01
β_{INV}	-0,06	0,05	-0,18	2,E-07	-0,08	0,04	-0,10	3,E-05	0,01	0,75	0,10	3,E-03	-0,02	0,68	0,02	0,68	-0,02	0,57	-0,02	0,44	0,02	0,46	-0,05	0,17

Panel B: Sharpe ratio of the convenience yield

	Energy						**Base Metals**						**Grains**						**Precious Metals**					
	Oil		**Heating Oil**		**Gasoline**		**Copper**		**Aluminium**		**Zinc**		**Soybean**		**Corn**		**Wheat**		**Gold**		**Silver**		**Platinum**	
Adj R^2	14,43%		23,22%		28,07%		13,73%		14,27%		8,19%		8,29%		12,55%		19,72%		46,83%		40,19%		21,74%	
	Slope	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*	*Slope*	*p-value*
β_ρ	-0,05	0,29	-0,05	0,28	-0,05	0,22	-0,11	0,10	-0,10	0,09	-0,17	0,01	-0,01	0,91	-0,01	0,92	0,02	0,70	0,01	0,77	0,02	0,60	0,00	0,98
β_σ	0,05	0,34	-0,12	0,01	0,07	0,11	-0,07	0,18	-0,02	0,78	-0,05	0,43	-0,11	0,05	-0,01	0,87	0,02	0,73	-0,01	0,76	0,02	0,72	0,00	0,97
β_{rS}	0,24	5,E-06	0,12	0,02	0,20	6,E-05	0,09	0,09	0,13	0,02	0,05	0,37	0,08	0,15	0,23	4,E-05	0,30	3,E-08	-0,09	0,04	0,09	0,05	0,09	0,25
β_B	0,24	5,E-05	0,51	1,E-16	0,49	1,E-14	0,12	0,03	0,32	2,E-07	0,24	2,E-04	0,24	3,E-04	0,27	2,E-06	0,18	2,E-03	-0,20	1,E-05	-0,11	0,02	0,08	0,31
β_{FutP}	0,07	0,14	0,05	0,29	-0,07	0,11	0,08	0,11	0,02	0,73	0,09	0,10	-0,02	0,67	0,01	0,89	0,06	0,24	0,02	0,70	0,00	0,95	0,13	0,06
β_{HP}	0,05	0,31	0,09	0,05	-0,11	0,02	0,10	0,08	NA	NA	NA	NA	0,08	0,13	0,10	0,08	0,18	6,E-04	-0,12	0,02	0,04	0,38	0,07	0,33
β_{INV}	0,15	0,01	0,16	0,01	-0,06	0,35	0,05	0,42	0,14	0,02	0,12	0,05	0,07	0,49	0,12	0,11	0,10	0,13	-0,14	1,E-03	0,01	0,83	-0,11	0,12

Legend: This table displays the regression of the Sharpe ratios of the spot price and the convenience yield for the four energy commodities on their determinants. The determinants, are the correlation between the convenience yield and the spot price, the volatility of the spot price, the return of the spot price, the basis, the Futures Risk Premium, the Hedging Pressure and the Inventory. Panel A (B) is dedicated to the spot price (convenience yield).