NCCC-134

APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

Improved Value-at-Risk (VaR) Forward Curve Projection Using the Full Option Premium Profile

by

David W. Bullock and Edna M. Okoto

Suggested citation format:

Bullock, D. W., and E. M. Okoto. 2024. "Improved Value-at-Risk (VaR) Forward Curve Projection Using the Full Option Premium Profile." Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. St. Louis, MO. [http://www.farmdoc.illinois.edu/nccc134].

Improved Value-at-Risk (VaR) Forward Curve Projection Using the Full Option Premium Profile

David W. Bullock and Edna M. Okoto[1]

Paper presented at the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, 2024.

St. Louis, Missouri, April 22-23, 2024

[1] Research Associate Professor and Graduate Student in the Department of Agribusiness and Applied Economics, North Dakota State University. Please direct all correspondence to David W. Bullock, Corresponding Author, 624 Barry Hall, 811 2nd Avenue North, Fargo, ND 58102 or david.w.bullock@ndsu.edu.

Improved Value-at-Risk (VaR) Forward Curve Projection Using the Full Option Premium Profile

Abstract: *The predictive ability of two alternative forward price distribution forecasting methods based upon the full range of option premiums was developed and tested using 10 years of price and premium history for five traded commodities. The two models were a best-fit parametric distribution and a non-parametric linear interpolation fit. These were compared to two traditional approaches: historical time series and Black-76 option implied volatility. The forecast horizons ranged from 6 months to 1 week in duration. A modification of the theoretical results of King and Fackler (1985) nonparametric option pricing model was presented to justify the fitting of a price probability density function to the option premiums with the intrinsic value removed. Time series fits to the historical futures price indicted that the integrated ARCH (1) and GARCH (1,1) models were the most prevalent best fit to the data. For parametric fits to the option premiums, the Burr Type XII and Dagum distributions were the most prevalent best fits. Predictive ability was measured using 10-percent value-at-risk portfolio models for simple short and long futures positions where the number of actual exceptions was compared to the theoretical values. The predictive results indicated that the parametric and non-parametric distribution fits performed best on the short futures portfolios over the longer-term forecast horizons (6- and 3-months) while the Black-76 performed best over the same time horizon. For the shorter time horizons (1-month or less), the Black-76 and time series methods performed best. These results point to the possibility that a hybrid Black-76 and premium distribution fit approach (via a splice) might perform best for longer-term projections.*

Key Words: option pricing models, price distribution forecasting, statistical distribution fitting, value-at-risk

Introduction

Value-at-Risk (VaR) is defined as the maximum loss that can be expected on a portfolio of physical and/or financial assets given a specified forward time period and a probabilistic level of confidence (Holton 2003). A key component of VaR models is the forward curve projection, which is used to mark-to-market the actual or proposed portfolio and calculate the distribution of future profit/loss. The traditional approach to forward curve projection typically uses one or a combination of four basic projection methods: (1) time series modeling, (2) fundamental / econometric modeling, (3) expert opinion modeling, and/or (4) applying the Black-Scholes option pricing model to the current forward pricing curve with the forward price as the mean and the option implied volatility as the standard forecast error using a lognormal distribution for the forecast.

Developing a reasonably accurate model for forecasting the future distribution of prices is critical to developing a useful VaR model. Ideally, the frequency of occasions where the actual portfolio loss exceeds the VaR-projected loss (defined as an "exception") should match the frequency implied by the defined VaR probability parameter. For example, a 5% VaR model should ideally result in an exception frequency of 1 out of 20 periods. This accuracy should increase with the number of time periods such that as the number of periods approach infinity, the frequency rate should approach 5 percent of observations. In other words, the distribution of exceptions should follow a binomial (*n*, *p*) process with *n* equal to the number of observations and *p* equal to the specified VaR probabilistic level of confidence.

The primary objective of this study is to evaluate whether the employment of forward curves generated using information from the full option premium profile across all strike prices can offer significant improvement over the result from more traditional time series and Black-Scholes methodologies. King and Fackler (1985) demonstrated that the cumulative density function (cdf) of future commodity prices can be approximated from the slope of the option premium schedule over the range of offered strike prices. This study will evaluate two alternative models that are related to the King and Fackler methodology: (1) a parametric distribution fit (based upon minimum RMSE) to the option insurance premiums (i.e., market premium minus intrinsic value) across the complete range of strike prices, and (2) a non-parametric linear interpolation fit to the same insurance premium profile. Accuracy will be measured based upon placing the projections into a VaR model of a simple futures portfolio (either long one or short one futures contract) and comparing the number of actual VaR exceptions to those predicted by the model (10% VaR). The study will examine all of the major contract months for Chicago corn, soybeans, wheat, soybean meal, and lean hogs over the 2013 to 2022 calendar years and examine projections for 6 months, 3 months, 1 month, and 1 week prior to the option expiration date. Monte Carlo simulation will be used to generate the VaR projections. The proposed models will be compared to two traditional methods: (1) a best-fitting (based upon AIC) time series model and (2) the traditional Black-Scholes model mentioned earlier. Results will be segmented by projection duration and commodity.

In most current applications of VaR, the predominant methods of forward curve projection tend to be either time series modeling or the application of the Black-Scholes option pricing model using implied volatility. Developing forecasts using fundamental / econometric and/or expert opinion modeling tends to be time-consuming and expensive. Therefore, these methods tend to be utilized more frequently in risk analysis applications related to long-term project investments with durations of greater than 2 years. For most applications related to financial and commodity portfolios, the VaR models need to be run on a regular (daily, weekly, or monthly) basis which requires forecasting methods that are less time-intensive and have lower expense.

However, in the commodities industry, it is not unusual to see VaR models with projections up to 18 months into the future. This is particularly true for food processors who set their ingredient budgets on a monthly, quarterly, or annual basis. During the extreme price volatility of 2008-09, many food processors using time-series based VaR models found that their number of exceptions far exceeded the implied projections of their VaR models. This resulted in a shift to the use of either heteroscedastic time series models (ARCH, GARCH, and jump processes) or using Black-Scholes option-based projection methods. In particular, the latter method is used for commodities that have well-established forward markets in both futures (or swaps) and options, or have a suitable cross-hedge relationship with one of these commodities.

This study makes four substantive contributions to the existing literature. First, the analytical results of King and Fackler (1985) are extended to show that the implied cumulative density function of future prices is proportional to the option premiums at each strike price adjusted for the intrinsic value embedded in the option. This allows for the use of the option risk premium profile as a representation of the market's collective assessment of the likelihood (i.e., probability) of each price scenario occurring at option expiration. Second, using the preceding result, the study summarizes the implied best-fitting probability distributions (using maximum likelihood estimation and root mean squared error as the fitting criterion) across the wide range of futures contracts and forward periods analyzed (1,000 in all). Third, the study summarizes the best-fitting time series models (using a standard set of models and the Akaike Information Criterion as the fitting criterion) across the same contracts and time periods. Fourth, using a Monte Carlo simulation of a simple VaR portfolio model (i.e., long one or short one futures contract) the model compares the number of actual VaR exceptions to those implied by the model across the four sample forecasting methods (time series, Black-Scholes implied volatility, distribution best-fit to premium profile, and non-parametric best-fit to premium profile).

In the sections to follow, a brief review of the related literature is presented to set the groundwork for the methodological and empirical framework of this study. This is followed by a more detailed discussion of the King and Fackler (1985) framework for deriving a non-parametric representation of the implied forward pricing distribution using information contained in the option premium profile. The primary result of this study is then extended to illustrate how the cumulative density function (cdf) of forward prices is proportional to the option premiums at each strike price adjusted for the intrinsic value in the option. This is followed by a section discussing the data and empirical methods employed in this study to compare the forecasting accuracy of the four forward curve methods utilized in the study: (1) time series projection, (2) Black-Scholes option implied volatility, (3) a parametric best-fit to the option premium profile, and (4) a non-parametric linear spline fit to the option premium profile. Results are derived in the following section with a summary of the key findings in the concluding section.

Related Studies

Value-at-Risk

The conceptual framework of using derived percentiles as a measure of risk (value-at-risk or VaR) is not new and can trace its origins back to the origins of probability theory in the writings of mathematicians such as Blaise Pascal, Pierre de Fermat, Gottfried von Leibnitz, Jacob and Daniel Bernoulli, Abraham de Moivre, and Thomas Bayes (Bernstein 1998). The first modern applications can trace back to a capital test implemented by the New York Stock Exchange back in 1922 (Holton 2003). Over time, this approach was refined through the development of internal systems that utilized risk-adjusted return on capital (RAROC) measures. Following the Black Friday stock market crash of 1987, there was a wide recognition of the need for better platforms for measuring market risk which culminated with the introduction of the *RiskMetrics* software platform in 1994 (JP Morgan / Reuters 1996). In 1996, an amendment of the Basel Accords was passed that added the Internal Models (including VaR) approach as an approved method for setting international bank regulatory capital requirements (Jorion 2001).

A comprehensive introduction and primer to the use of VaR as a risk measurement methodology is contained in Linsmeier and Pearson (2000). They define VaR as "a single, summary statistical measure of possible portfolio losses … resulting from 'normal' market movements". The primary utility of using VaR in a business setting is that it "aggregates all of the risks in a portfolio into a single number suitable for use in the boardroom, reporting to regulators, or disclosure in an annual report". They go on to describe the three basic methods of calculating VaR: (1) the delta-normal approach, (2) historical simulation, and (3) Monte Carlo simulation. The delta-normal approach is based primarily upon the mean-variance portfolio model with non-linear derivative instruments, such as options, linearized by using their derivatives (such as option "delta"). The historical simulation approach randomly selects a past time period and draws all of the price history from that time period. This observation is placed into the portfolio model and a profit/loss is calculated. This is done over numerous random draws and the results are tabulated. The Monte Carlo simulation approach uses explicit pseudo-random number generation algorithms to simulate hypothetical draws from the price distributions. These are incorporated into the portfolio to calculate profit/loss and the results are statistically tabulated to derive the relevant percentile(s).

One of the common criticisms of using VaR is that it violates the subadditivity axiom for a coherent risk measure as defined in Artzner et al. (1999). Essentially, this axiom it met if the following relationship holds for a particular risk measurement ρ:

$$For\ X_1\ and\ X_2 \in \Theta,\ \rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2),$$

where X_1 and X_2 represent two separate investment portfolios, and θ represents the space of all possible portfolios. Essentially, if the preceding relationship does not hold, then an individual willing to take the risk of portfolios X_1 and X_2 may open two accounts, one for each, and incur a smaller margin requirement than under the combined portfolios.

Daníelsson et al. (2013) further evaluated whether VaR could meet the subadditivity axiom under certain conditions. They were able to product theoretical results where VaR met the subadditivity axiom when applied in the relevant tail region of a portfolio where asset returns are multivariate regularly varying which allows for dependent returns. Further, the noted that VaR estimated from historical simulations, in particular, may lead to violations of the subadditivity axiom. The proposed using a semi-parametric extreme value technique for VaR estimation instead of historical simulation.

A substitute measure for VaR that meets the conditions of subadditivity is conditional value-at-risk (CVaR) which is often also referred to as expected tail loss (ETL) or expected shortfall. Odening and Hinrichs (2002) demonstrated how ETL could be a useful compliment to VaR in cases of fat-tailed distributions such as the German hog market. Yamai and Yoshiba (2005) applied ETL to credit and exchange rate portfolios to examine the problem of tail risk. While they found that ETL provides several advantages over VaR, they also noted that estimation errors for ETL can be greater than those of VaR but this problem can generally be overcome by increasing the sample size of the simulation.

The potential for VaR in agricultural applications was discussed in the seminal works by Boehlje and Lins (1998) and Manfredo and Leuthold (1999). Boehlje and Lins identified some of the risks faced by the agriculture industry (such as financial, technological, political, and market) and how new technologies in risk analysis could be used to address them. These technologies included probability distribution techniques, option theory, risk scorecards, and value-at-risk. Manfredo and Leuthold noted that VaR has potential for improved risk analysis in making hedging decisions, managing cash flows, setting position limits, and portfolio selection and allocation. The also noted that VaR analysis was already being used in the AgRisk™ computer program developed by Ohio State University and the University of Illinois at Urbana-Champaign which advised producers in making alternative preharvest marketing decisions for corn, wheat, and soybeans. Also, they noted that the Commodity Futures Trading Commission (CFTC) was considering VaR for use in reporting risk under its agricultural trade options pilot program.

Manfredo and Leuthold (2001) examined the application of VaR for measurement of risks in large cattle feeding operation. The focus is on the margin between fed cattle sale prices and input prices for corn and feeder cattle. The study used cash prices from 1984 through 1997 that were converted into log-return format. A standard cattle feeding margin model was specified and a one-week ahead VaR was calculated using the delta-normal method. For the standard deviation projections, multiple methods were used including long-run historical averages, a 150-week moving average, a GARCH (1,1) time series, an exponentially weighted moving average (similar to that used in *RiskMetrics*), and option implied volatility using the Black-76 (Black 1976) option pricing model. Correlations were estimated using one of three historical estimation methods (historical average, 150-week moving average, and exponentially weighted moving average). A combination of standard deviation - correlation estimation combinations were evaluated in the study along with a pure historical simulation method. Forecast performance was evaluated using a comparison of the actual to predicted number of VaR exceptions over the study period. Statistical tests based upon the binomial distribution were used to evaluate the accuracy and biasedness of each projection method. Their results indicated that the *RiskMetrics*, historical moving average, and the historical simulation models provided coverage consistent with all three confidence levels (90%, 95%, and 99%) based upon the VaR exception tests. Of these models, the *RiskMetrics* based exponentially weighted moving average models performed the best.

Besides the Manfredo and Leuthold application to cattle feeding, VaR has been applied to numerous markets in agriculture and agribusiness including bakery procurement (Wilson et al. 2007), pork production (Hotz 2004), dairy (Bamba and Maynard 2004), freight rates (Angelidis and Skiadopoulos 2008), energy commodities (Hung et al. 2008), foodservice procurement (Sanders and Manfredo 2002), the Federal Crop Insurance program (Ramsey and Goodwin 2019), and water resource planning (Soltani et al. 2016) and many others. Additionally, there have been applications of using mean-VaR (E-VaR) analysis such as the application to the ethanol industry by Awudu et al. (2016).

Option Pricing Models

Option pricing models based upon the seminal works of Black and Scholes (1973) and Merton (1973) generally assume that underlying prices follow a Generalized Brownian Motion (GBM) random walk process. Black (1976) modified the general model to accommodate options that settled to underlying futures prices. The general implication of this family of models is that prices follow a lognormal distribution (or normal for log-returns). These distributions are two-parameter distributions that depend upon the first two centralized moments of the price distribution which are the mean and variance.

However, previous empirical studies have indicated that commodity prices tend to not follow a lognormal distribution. Gordon (1985) examined the distribution of futures price changes on a daily basis for several major commodity futures contracts. Over the short-run (2 months or less), the price changes tended to follow the constant volatility assumption of the Black-Scholes model. However, over longer time periods, the volatility varied based upon seasonality. This led to price changes following more leptokurtic distributions with fatter tails than those implied by the standard GBM process. Additional research by Hilliard and Reis (1999) also found that commodity futures prices were often characterized by a jump process rather than a standard GBM which can lead to fat tails in the distribution. Numerous studies (Baillie and Myers 1991; Bernard et al. 2008; Li and Chavas 2023) have found that commodity futures deviate significantly from the standard GBM model due to GARCH, jump processes, mean-reversion and other significant deviations from the assumptions behind the standard GBM model.

Given the previously stated issues with the use of the standard GBM model for commodity futures price projection calls into question the suitability of using the Black-Scholes / Merton family of option pricing models to derive forward curves for VaR application. Alternatives to Black-Scholes / Merton include stochastic volatility (Heston 1993), non-Gaussian distribution (Jondreau, Poon, and Rockinger 2007; De Domenico et al. 2023), and non-parametric distribution (King and Fackler 1985; Aït-Sahalia & Duarte 2003) option pricing models. By relaxing some of the restrictive assumptions of the Black-Scholes model, these approaches may provide improved forward curve projections for VaR applications.

Options and Market Information

The potential informational benefits of option trading in commodities was initially discussed by Gardner (1977) where the following was stated:

> *"A possibly more important result of options trading is the public information that can be inferred from the selling price of an option. Just as futures prices generate information about expectations of commodity prices, an option's price generates information about expectations of the variability of commodity prices."*

Gardner then goes on to analytically show, using a standard option valuation formula, how an estimate of the collective market expectation of the variance can be derived from the market premium. Under the additional assumption of log-normality, a closed-form model could be used to derive this estimate which is commonly known as *implied volatility*. Gardner also goes on to point out that this market estimate would have numerous applications in the agricultural sector including individual market risk management and government commodity programs.

Grossman (1977) noted the fact that futures markets exist in some commodities and not in others. The fact that these markets were incomplete in terms of spanning all commodities in all states in the world points to an explanation for the existence of these markets beyond just the Keynes-Hicks theory that they provide a mechanism for risk transfer. Grossman then goes on to develop, using noisy rational expectations theory, an explanation that these markets exist for the transfer of information from “informed” to “uninformed” market participants. In particular, these markets exist in cases where the spot price of a commodity fails to reveal all of the informed participants’ information because of the presence of additional “noise” in the spot market price in addition to the informed participants’ information.

Wolf (1987) examined the use of commodity options as a risk management tool using a simple two-period static model of an investor with risk preferences represented by a Hyperbolic Absolute Risk Aversion (HARA) utility function that can accommodate a wide range of risk preferences. First, a set of analytical results were derived using a negative exponential utility function that is linear in the mean and variance. Derivation of the optimal positions from the analytical model indicated that the optimal option positions contained both an informational part and a pure hedging part. The model was converted into expected net returns format for expected returns to the futures and option markets respectively.

From the analytical model, Wolf was able to derive three primary theoretical results: (1) a change in the expected net return to the futures market will result in a larger change in the option market position as compared to the futures market, (2) a change in the expected net return to the option market will result in a larger change in the option market position when compared to the futures, and (3) if the futures and option markets are both fairly priced and basis risk is not present, then the futures market will be used exclusively as the hedging instrument. Taken together, the results suggest that in the case of mean-variance preferences, the option market will be used primarily for speculation while the futures will be used primarily for hedging and to a lesser extent for speculation.

Wolf simulated the two-period portfolio model using the HARA utility function to derive more detailed results on the types of positions taken by the investor. Most of these positions tended to reflect variations of what is known as an “inverse straddle” in the option parlance. Wolf surmises that the straddle positions are reflective of the trader’s price expectations and preferences towards risk.

Wolf’s two-period mean-variance analytical results rely upon a generic representation of the variance-covariance matrix of returns between the futures and option markets. Using general results from conditional statistics theory, Bullock and Hayes (1992) were able to derive the explicit analytical representation of the variance-covariance matrix as a function of the mean and variance of the underlying prices. Using the model, they were able to replicate and prove the three theorems from Wolf’s paper regarding expected returns and hedging preference. However, when examining the impact at the statistical moment level, they found that the futures contract was the primary speculative instrument for information on the mean and the option market was the primary speculative instrument for information on the variance of the underlying commodity price. Also, they were able to show that the results were invariant to the presence of basis risk if it is assumed that the basis is uncorrelated with the spot price. Using Monte Carlo simulation of the two-period model, Bullock and Hayes also demonstrated that the optimal position would be either a short or long straddle based on their view that the investor was either “bearish” or “bullish” relative to market implied volatility. For information on the mean, the investor would choose a written a position representing a written option: a written call if the market was undervalued and a written put if the market was overvalued.

Bullock and Hayes (1993) followed up by examining the value of having access to an option market for an investor with mean-variance preferences based upon the content of the information set on the mean and variance of prices. They utilized a framework developed by Antonovitz and Roe (1986) to develop a Bayesian money metric for measuring the value of information. The results showed that the metric can never be negative and was only equal to zero in the case where the investor's expected marginal speculative return to the option is equal to the hedge-adjusted marginal expected speculative return to the futures contract. Simulation analysis of the model indicated that information on the mean of the underlying price was more significant than information on the variance when considering the access value of the option. However, the results also indicated that the impact of the variance upon the access value of the option was highly dependent upon the level of risk aversion and that there exists a level of risk aversion at which the access value would be maximized. Also, the results indicated that option access value became more significant as the level of market implied volatility increased. Therefore, there is an indication that there needs to be a sufficiently high level of volatility in the market to justify the introduction of option markets.

Bohmann (2020) examines the question of how important are commodity options relative to futures in the price formation process. He also examines who is more important in this price discovery process - speculators or hedgers. The commodities examined included corn, soybeans, crude oil, natural gas, gold, and silver traded on the Chicago Mercantile Exchange and used intraday data. To compare options to futures, the option premiums were converted to implied futures prices using the put-call parity relationship. A vector error correction model (VECM) was used to evaluate the price discovery leadership between the two markets. The primary results indicated that options tended to lead futures in reflecting new information in the corn, soybeans, crude oil, gold, and silver markets. Also, it was found that speculation was a more significant determinant of price discovery and increased speculation also lead to increases in the option contribution to price discovery.

Forward Projections of Commodity Price Distributions

Forecasting the forward distribution of potential commodity prices is important not only as a component of VaR modeling but also in option valuation. For both purposes, it is also important to have forecasting methods that easily implemented in a timely manner and are less costly due to the frequency of forecasting. Much of the previous literature has focused primarily upon the forecasting of the first two moments of the price distribution — the mean and standard deviation. When liquid futures or swaps markets are available, the common practice is to use these prices as a proxy for the mean. In terms of the standard deviation, the focus tends to be on two primary approaches: (1) time series methods and (2) option implied volatility. Therefore, much of the focus in the previous literature has been on comparing the two approaches in terms of forecasting future price volatility.

A complete summary of forecasting volatility in the financial markets is contained in the literature review articles by Poon and Granger (2003, 2005). They first identify the popular time series models used in forecasting volatility beginning with historical / moving average methods (such as used by *RiskMetrics*) followed by conditional heteroscedastic time series models (such as ARCH / GARCH and variants) and concluding with stochastic volatility models (such as maximum likelihood, generalized method of moments, and Monte Carlo integration). This is followed by a discussion of the option-based models with a primary focus on Black-Scholes and the assumptions behind the model. In particular, they discuss issues related to the distributional assumptions, effects of stochastic volatility, market microstructure, measurement errors, and investor risk preferences. Poon and Granger conducted a survey of 93 studies that made comparisons of volatility forecasting methods and concluded that the overall ranking suggested option implied volatility performed the best. Among the time series methods, there was no clear winner but a rough ranking had historical volatility as the most accurate followed by GARCH and stochastic volatility.

A more recent article by Rhee et al. (2012) compared the forecasting performance of four option-based methods in forecasting volatility in the KOSPI 200 stock price index. The four methods evaluated were Black-Scholes, the Heston stochastic volatility model, the Britten-Jones and Neuberger model-free implied volatility (MFIV), and the Korean version of the CBOE Market Volatility Index (VKOSPI). Their results indicated that the Heston model offered improvements over the Black-Scholes model while the other two methods offered no noticeable improvement over Black-Scholes.

In terms of commodity markets, Manfredo et al. (2001) compared time series (historical average, naive, GARCH, and RiskMetrics EWMA), option implied volatility, and composite forecasts that combined time series and implied volatility methods. The methods were evaluated using weekly cash market prices over a 14-year period (January 1984 through December 1997) for fed cattle, feeder cattle, and corn. They examined forecast accuracy across 1-week, 2-week, 4-week, and 20-week forecast horizons. Their results indicated that option implied volatility performed well for corn, particularly over the longer time horizons. For the fed and feeder cattle markets, the composite methods performed best particularly over the shorter time horizons.

Shao and Roe (2001) examined the performance of historical volatility, GARCH, and implied volatility in forecasting the feeding margin for live hogs by simulating the joint distribution of weekly hog, corn, and soybean meal cash prices over a ten-year period (1990 to 1999). A standard margin model was utilized and historical data was used for estimating the covariance structure between the three series. The prices were converted into log-return format and the forecasts were evaluated over forward horizons of 1, 2, 4, 8, 12, and 26 weeks. Their results indicated that the performance of the three forecast methods varied with commodity and forecast horizon. For live hogs, implied volatility performed best in the shorter time horizons while GARCH performed better over the longer time horizons. For corn and soybean meal, the historical volatility performed best in the short-term while implied volatility performed better in the longer term. They then incorporated the implied volatility into the margin model and simulated out-of-sample forecasts over the entire time horizon. Overall, they found the model to perform well with most realized margins falling within the 95 percent prediction interval.

Giot (2003) compared the informational content of lagged implied volatility to GARCH time series models of conditional volatility for a collection of commodities traded at the New York Board of Trade. The results of the study indicated that information contained in the lagged implied volatility improved the forecasts from the GARCH time series models. Giot concluded that the addition of option implied volatility could improve forward curve projections used in VaR models.

Manfredo and Sanders (2004) examined the forecasting performance of implied volatility in forecasting the 1-week ahead volatility in the nearby live cattle futures. Their results indicated that implied volatility is a biased and inefficient forecast despite encompassing all of the information from a time series alternative and showing improvement over time.

Chen et al. (2005) compared the performance of Black-76 to a stochastic volatility jump diffusion (SVJD) model in forecasting corn and soybean meal volatility. Performance was evaluated using three criteria: (1) in-sample pricing fitness, (2) out-of-sample prediction, and (3) volatility forecasting accuracy. The primary comparison metric was the root mean squared error (RMSE) from each model. Their results indicated that the SVJD model was superior to using Black-76 across both commodities and all forecasting horizons.

Egelkraut and Garcia (2006) examined the forecasting performance of Black-Scholes option implied forecasting models relative to three alternative time series approaches for five major U.S. commodities: corn, soybeans, soybean meal, wheat, and hogs. Their primary finding was that the implied volatility approach dominated the time series approaches based upon a set of forecast performance metrics. However, the power of this result varied across the commodities and was most prevalent in those markets that had well-established forward option markets that were unbiased and efficient. In less well-established markets (meal, wheat, and hogs) the volatility was less predictable as investors were able to extract a greater risk premium from the market which created a bias in option pricing.

Egelkraut et al. (2007) developed a flexible method to construct the term structure of volatility and applied this method to corn futures options with differing maturities. This method considers the potential for varying seasonality in corn futures volatility. Their results indicated that the term structure method using implied volatility provided forecasts that were superior to using historical volatilities.

Urcola and Irwin (2010, 2011) examined the pricing efficiency of options on live hogs, corn, soybeans, and wheat traded at the CME. They simulated expected returns from a variety of option strategies including buying puts and calls, and long straddle positions. Their results indicated that the pricing of options markets for all of the examined commodities were free of any bias and efficient. Brittain et al. (2011) evaluated the short-term (one-week ahead) forecasting performance of option implied volatility versus GARCH(1,1) time series for live and feeder cattle. Their results indicated that the option implied volatility forecasts were consistently upwardly biased and inefficient in forecasting realized volatility.

Trujillo-Barrera et al. (2018) examined ex-ante probability density forecasts in the lean hog futures markets using both option-based and time series forecasting models. Their results indicated that the forward-looking option-based techniques are better calibrated and have superior predictive accuracy when compared to GARCH time series models. Their calibration results also indicated the presence of short-term risk premiums, particularly in times of market turmoil.

Fernandez-Perez et al. (2019) developed an implied volatility index (DVIX) for New Zealand Exchange traded options on whole milk powder. Their in-sample estimation results indicated that the DVIX had a high information content with regards to the conditional variance and that the inclusion of historical information further improved the predicted power of the index. Their out-of-sample results indicated that DVIX provided substantial information regarding future realized volatility and the combination with historical volatility improved the forecast.

Much of the recent research has refocused upon how improvements to time series models could be used to improve forecasts of volatility. Approaches examined include the use of long-memory models (Alfeus and Nikitopoulos 2022), heterogeneous autoregressive (HAR) models (Degiannakis et al. 2022), and multi-factor seasonal models (Schneider and Tavin 2024).

Nonparametric Option Pricing Models

The development of a nonparametric approach to option pricing can be attributed to the pioneering work by King and Fackler (1985) who derived probabilistic price forecasts based upon option premium values. They define the general valuation of a European call option using the following equation:

$$V_C\ (K,t) = e^{-rt}\ \int_K^\infty f(P_t\ )(P_t - K)dP_t, \tag{1}$$

where r is the risk-free rate of return, P_t is the commodity price on the expiration date t, K is the option strike price, and *f(.)* is the probability density function (pdf) of price. Using the general properties of a pdf, equation (1) can be integrated to the following representation:

$$V_C\ (K,t) = e^{-rt}\ \int_K^\infty [1 - F(P_t)]dP_t, \tag{2}$$

where *F(.)* is the cumulative density function (cdf) of price. From equation (2) it follows that:

$$V_C\ (K_1,t) - V_C\ (K_2,t) = e^{-rt}\ \int_{K_1}^{K_2} [1 - F(P_t)]dP_t, \tag{3}$$

where $K_1 > K_2$. Application of the mean value theorem for integrals to the right-hand side of equation (3) and noting that $F(P_t)$ is monotonically increasing (by definition of a cdf), there exists a unique strike price, K^*, such that:

$$e^{-rt}\ \int_{K_1}^{K_2} [1 - F(P_t)]dP_t = e^{-rt}(K_2 - K_1)[1 - F(K^*)]. \tag{4}$$

For a sufficiently small distance between K_1 and K_2, a value for K^* can be approximated by $(K_1 + K_2)/2$. Making this substitution and combining equations (3) and (4) results in the following expression:

$$F[(K_1 + K_2)/2] = 1 - \frac{V_C(K_1, t) - V_C(K_2, t)}{e^{-rt}(K_2 - K_1)}, \tag{5}$$

which can be used to approximate one point on the CDF for P_t. King and Fackler go on to derive a similar relationship to equation (5) in the case of put options which can be represented as:

$$F[(K_1 + K_2)/2] = \frac{e^{-rt}[V_P(K_2, t) - V_P(K_1, t)]}{(K_2 - K_1)}. \tag{6}$$

Equations (5) and (6) essentially state that the option implied cumulative density function of the underlying commodity price is proportional to the relative slope of the option premiums to the change in strike prices. King and Fackler then go on to demonstrate how this model would work by applying it to quoted put and call premiums for soybeans, live cattle, and T-bonds. These equations later formed the basis for a computer software platform distributed by the University of Minnesota Extension Service called OPTIONS. Lawrence and Meyer (1993) evaluated the performance of this software using simple trading rules applied to the CME live cattle and live hog contracts over the 1989-1992 period. Their results indicated that the OPTIONS model performance was mixed with positive results noted in the live cattle contract and mostly negative results in the live hog contract. They noted that the hog contracts were generally more volatile over the time period examined and concluded that OPTIONS tended to work better in less volatile markets. The trading strategy examined was a simple buy/sell strategy based on the relationship of the current futures price to its percentile value.

Fackler and King (1987, 1990) followed up their 1985 paper by discussing how calibration methods could be used to evaluate and further refine option-based probability assessments. They discussed two approaches to calibration: (1) a parametric distribution-based approach where a general family of distributions such as the lognormal, beta, or Burr-12 could be used to calibrate the fit between the observed premiums and hypothesized distribution fit, and (2) a non-parametric approach where the hypothesized distribution could be derived via a linear interpolation fit to the observed premiums. Empirical goodness-of-fit tests (Kolmogorov, Cramer-von Mises, Kuiper, Watson, and Anderson-Darling) were applied to evaluate the reliability of the option-based assessments. These were applied to the futures markets for corn, soybeans, live cattle, and live hogs. The results were mixed with no evidence of problems for corn and live cattle. However, for soybeans, the methods tended to overstate volatility and, for hogs, understated the location. They noted that their results were preliminary given the relative newness of the option markets and suggested further study should be made as the markets mature in the future.

Silva and Kahl (1993) extended the Fackler and King analysis by focusing on the most liquid options contracts in corn and soybeans. They compared the reliability of probability assessments over two sub-periods (1985-1987, 1988-1990) with the latter representing a more liquid period. They compared the Black-76 implied volatility with the King and Fackler nonparametric approach using the calibration approach. Their empirical results indicated that for the earlier period, the Black-76 lognormal approach is generally favored over the non-parametric although the soybean results had to be adjusted for assessment problems. In the latter period, both the Black-76 and nonparametric approaches generally produced reliable estimates.

O'Brien et al. (1996) compared the ability of futures prices, fundamental modeling, and options-based assessments in projecting the harvest-time corn futures prices for four time periods during the 1992 through 1994 growing seasons. The fundamental model was a reduced-form version of the Shonkwiler and Maddala dynamic disequilibrium model. The options-based forecasts used the King and Fackler non-parametric approach as incorporated into the OPTIONS computer software. Their forecast results were mostly mixed with the fundamental model performing slightly better over the longer mid-June to Harvest time period while the options-based and futures price forecasts were slightly better over the shorter time periods.

Sherrick et al. (1996) used moment-based methods to assess various distribution candidates for parameterization of price distributions on soybean futures contracts from 1988-91. Their results suggested that the Burr III distribution as a good candidate as it generally produced lower pricing errors and better characterized ex ante price distributions.

Aït-Sahalia and Duarte (2003) developed a technique to use kernel estimators to derive the state price density implicit in the market prices of option. The technique involved placing constraints upon the values of the first and second derivatives of the estimators. Simulations using S&P 500 option prices demonstrated the efficacy of using their approach. More recent studies (Ivascu 2021; Almeida et al. 2023) have focused upon the use of machine learning (ML) tools to improve calibration of option probabilistic forecasts. The results indicate that these models can be used to help correct misspecification issues related to traditional model fitting.

Conceptual Model

The conceptual model used in this study relies heavily upon the results from King and Fackler (1985). Using conditional moments, we can rewrite equation (1) as follows:

$$V_C(K,t) = e^{-rt}[1-\Phi(K)] \cdot E[(P_t - K)|P_t > K], \tag{7}$$

where $\Phi(K)$ is the implied cdf of prices and E[.|.] is the conditional expectation operator. By substituting the market premium at strike price K [$\pi^*(K)$] for $V_C\,(K, t)$ and the conditional mean $\mu_P^+(K)$ for E [$P_t \mid P_t > K$], equation (7) can be rewritten as follows:

$$\pi^*(K) = e^{-rt}[1-\Phi(K)] \cdot [\mu_P^+(K) - K]. \tag{8}$$

Multiplying through and solving for $\Phi(K)$ gives the following relationship between the option implied cdf and the market quoted option premium:

$$\Phi(K) = 1 - \frac{\pi^*(K)}{e^{-rt} \cdot [\mu_P^+(K) - K]}. \tag{9}$$

Note, that by definition, the numerator in equation (9) is always positive since, by definition, $\mu_P^+(K) > K$ for all values of K. Note that equation (9) is similar to King and Fackler's equation (5) except in this case the cdf value is one minus the ratio of the quoted option premium at strike price K divided by the discounted difference between the conditional mean (given $P > K$) minus the strike price.

Note that if the futures price is regarded as the best forecast of the future price, the numerator term in equation (9) approaches the discounted intrinsic value on the option. Therefore, the derivative of equation (9) will be proportional to the relative change between the option premium in the numerator and the discounted intrinsic value term in the denominator. Therefore, the pdf of the option implied price distribution should be proportional to the option premium minus the intrinsic value at each strike price *K*. This remaining value is commonly referred to as the option *risk premium*.

If the market option premiums are considered risk neutral in the aggregate, it makes intuitive sense that the option premiums across strike prices should be proportional to the relative likelihood of the strike price scenario occurring. For example, if the premium at a strike price of $2.00 is twice the premium at $3.00, then the option writers are assessing that the likelihood of a $2.00 price at option expiry is twice as likely as a $3.00 expiration price.

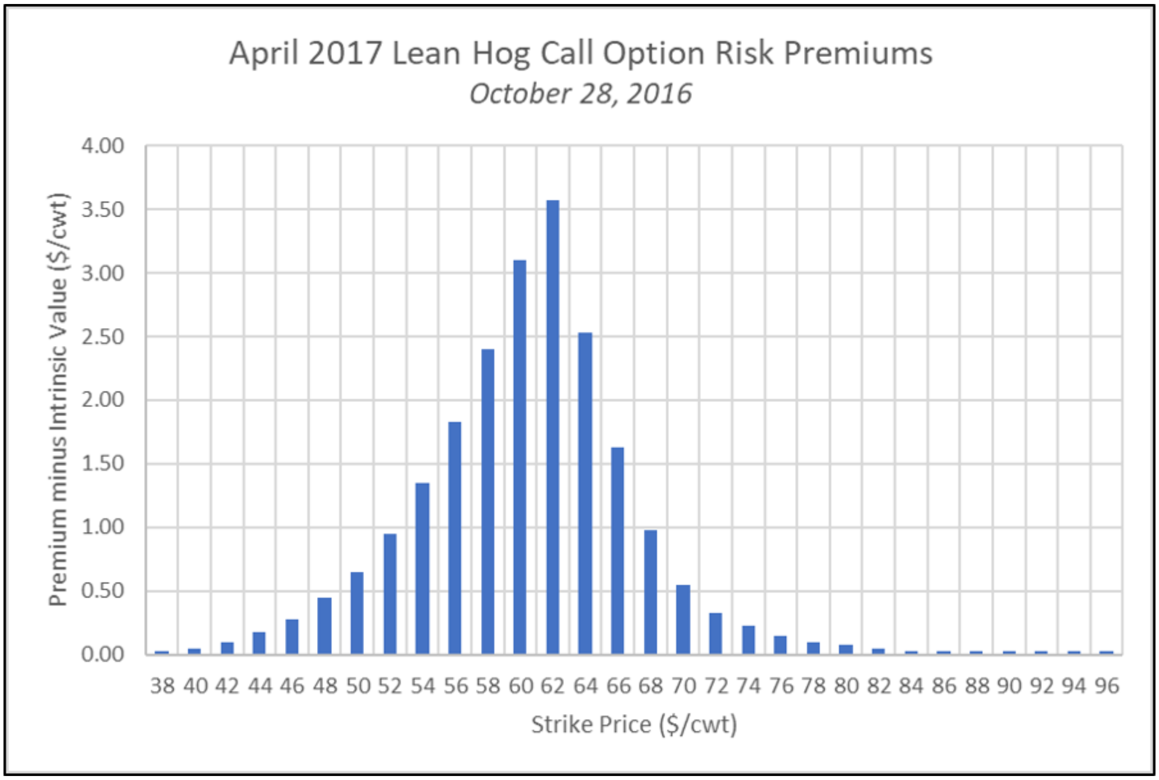

***Figure 1.* Call option risk premium profile for April 2017 lean hogs at 6 months until expiry.**

Figure 1 shows a plot of the option risk premium profile on the April 2017 CME lean hog call options at the close on October 28, 2016. The closing futures price was $61.65 per cwt. The risk premium profile indicates a modal value at $62 with a premium of $3.575 per cwt. At the $70 strike price, the premium is $0.55 per cwt indicating that the option traders assess the likelihood of an expiration futures price near $62 as 6.5 times more likely than the $70 scenario. The contract eventually settled at $62.12 on April 24, 2017 which indicates that the assessment was pretty accurate.

Data and Empirical Methodology

The data utilized in this study covers the CME corn, soybeans, wheat, soybean meal, and lean hog futures and options contracts from 2013 through 2022. Five months were chosen for each commodity: March, May, July, September, and December for corn and wheat; March, May, July, September, and November for soybeans; March, May, July, October, and December for soybean meal; and February, April, July, October, and December for lean hogs. This encompasses a total of 250 futures contracts over all of the commodities (50 each). Daily futures closing prices were extracted for each contract from the first to last trading day. Additionally, put and call option premiums were extracted across all traded strike prices for 4 separate dates per contract: 6 months, 3 months, 1 month, and 1 week prior to the expiration date of the option. All of the data was extracted from historical database available from the ProphetX (Data Transmission Network, 2023) online database. Daily spot interest rate data (3-month treasury bill) was extracted from the FRED online database (Federal Reserve Bank of St. Louis, 2023) for all days spanning the 10-year study period.

This study compares four different forward price probability projection methods. These methods are:

1. *Best Fitting Time Series Projection* - at each of the option pricing dates, a best fitting time series model was fitted to the past 60 days of futures price data using the best fitting time series feature from the *@Risk* (Palisade Software, 2023) simulation software. Software recommendations were used to determine the level of differencing (up to 2) and whether a log transformation was applied. The criterion for choosing the best fitting model was Akaike Information Criterion (AIC). All of the financial, Box-Jenkins, and stochastic volatility models available in the @Risk software were considered. The best fitting time series model was then simulated on a daily basis up to the option expiry date to derive the price forecast distribution.
2. *Black-Scholes Implied Volatility* - at each of the option pricing dates, the option implied volatility was extracted from the at-the-money call option premium through inversion of the Black-76 option pricing model. The implied standard deviation was then calculated from the implied volatility using the standard lognormal transformation based upon time until option expiration. The futures price and implied standard deviation are then input into a normalized lognormal distribution to provide for the simulation of the price forecast distribution at option expiry.
3. *Best-Fitting Parametric Distribution to the Option Risk Premiums* - this approach applies the @Risk distribution *Bestfit* procedure to fit candidate parametric distributions to the risk premium profile (option premium minus intrinsic value) from the options on each forecast date. The candidate distributions all had a minimum value of zero and the maximum value was set at undetermined. The distributions were fit based upon the "Density (X,Y) Points Unnormalized" option. The strike prices make up the X values while the option risk premiums were used for the Y values. The premium values are then normalized so that the area under the premium schedule integrates to a value of one. Distribution parameters are fit using maximum likelihood methods and the distribution that

has the smallest root mean squared error (RMSE) fit to the actual data was the one chosen. This distribution was then simulated to generate the price forecast distribution at expiry.

4. *Nonparametric Distribution Fit to Option Risk Premiums* - this approach applies the @Risk General distribution to the option risk premium profile. The strike prices make up the X values while the risk premiums make up the Y values. For the minimum and maximum, the strike prices were one strike increment below the lowest strike for which a premium is quoted and one strike above the highest strike price with a quoted premium. The General distribution constructs a linear interpolated fit to the risk premiums and normalizes so that the area underneath the curve integrates to one. This distribution was then used to simulate the price forecast distribution at option expiry.

To derive the VaR under each forecasting method, each of the four forecasting methods were incorporated into a Monte Carlo simulation model using @Risk. VaR was calculated on two separate portfolio models: (1) a simple long one futures contract position and (2) a simple short one futures contract position. Each portfolio was simulated 5,000 iterations and the same seed value was used to create comparable VaR estimates. The 10th percentile of the projected profit/loss (P/L) projection was used as the applicable VaR measure for evaluation.

The portfolio P/L using the actual futures prices at expiration was used for comparison to the simulated VaR value to determine if an exception had occurred. An exception occurs when the actual P/L value falls below the forecasted VaR P/L value. If the forecast model was accurate, it is expected that the actual loss frequency should be close to the VaR model frequency of 10 percent of the observations. Likelihood ratio and Z-score tests were applied to determine if the number of exceptions were consistent with the hypothesized values. These are the same tests that were applied in Manfredo and Leuthold (2001).

Results

Table 1 shows a summary of the best fitting time series models by commodity and in total. The once-differenced ARCH1 model was preferred in nearly 2/3 of the samples estimated. The second most prevalent model was the once-differenced GARCH11. The results were fairly consistent across commodities with the exception of lean hogs, where there was greater prevalence of other types of models. These results are fairly consistent with the past literature that has generally used ARCH/GARCH or their variants for generating time series projections. The results by forecast period are not shown; however, there was little variation across them.

Table 1. Best Fitting Time Series by Commodity

Model[a]	Count						Percent of Total
	Corn	Soybeans	Wheat	Soymeal	Lean Hogs	Total	
ARCH1(1)	127	147	135	129	108	646	64.6%
GARCH11(1)	27	31	39	37	38	172	17.2%
ARMA11(1)	4	0	1	7	17	29	2.9%
ARCH1(2)	5	6	6	5	3	25	2.5%
BMMRJD(1)	9	3	3	4	1	20	2.0%
MA2(2)	6	5	6	2	1	20	2.0%
Others	22	8	10	16	32	88	8.8%

[a]Number in parentheses is the number of differences applied to data before estimation.

Table 2 shows a summary of the distribution fitting results to the option risk premium profiles by commodity. The Burr-12 was the most frequently occurring distribution by a wide margin followed by the Dagum distribution. The results were consistent across commodities with the exception of live hogs where the Dagum occurred with a much higher frequency along with the Generalized Beta. The results across forecast horizons were fairly consistent; however, there was a greater frequency of the other distributions in the one-week forecast period.

Table 2. Best Fitting Parametric Distributions to Option Risk Premiums by Commodity

Distribution	Count						Percent of Total
	Corn	Soybeans	Wheat	Soymeal	Live Hogs	Total	
Burr12	196	178	196	185	105	860	86.0%
Dagum	3	19	3	8	79	112	11.2%
BetaGeneral	1	0	0	3	11	15	1.5%
Frechet	0	0	0	2	4	6	0.6%
LogLogistic	0	3	0	0	0	3	0.3%
Gamma	0	0	1	0	0	1	0.1%
Laplace	0	0	0	1	0	1	0.1%
Weibull	0	0	0	1	0	1	0.1%
LogNormal	0	0	0	0	1	1	0.1%

The Burr Type XII (Burr12) distribution is one of twelve distributions that make up the Burr system of distributions and is the most commonly used. It is also commonly referred to as the Singh-Maddala or the generalized log-logistic distribution (Rodriguez 1977). The @Risk version of the Burr12 is bounded at the lower end at zero, has no upper bound and is characterized by four parameters - a location parameter, a scale parameter, and two shape parameters. The Burr12 is a highly flexible distribution as it encompasses the gamma, lognormal, loglogistic, and the beta (bell and J-shaped) distributions. It also has the Weibull and Pareto Type I as limiting distributions. This flexibility allows it to fit a wide range of empirical data with different values of skewness and kurtosis. Due to its flexibility, it has found use in a wide variety of applications in the fields of finance, reliability, actuarial science, hydrology, and economics. Specific applications have included modeling household income, crop prices, insurance risk, travel times, flood levels, and failure data. This distribution is also mentioned in Fackler and King (1990), Sherrick et al. (1990), and Sherrick et al. (1996) as a good candidate for calibrating option-based future price distributions. Tejeda and Goodwin (2008) also determined that the Burr distribution was a good distribution for fitting crop prices.

Figure 2 shows the fit of the Burr12 distribution to the option premium profile for November 2019 soybeans with three months to option expiry. The General is representative of the overall shape of the actual option insurance premiums. The Burr12 does a better job of capturing the fatter upper tails in the option premiums as compared to the lognormal derived from the Black-76 model. Note that the data (General fit) is also more leptokurtic than what is implied by the lognormal, thus the more general Burr12 provides a better fit.

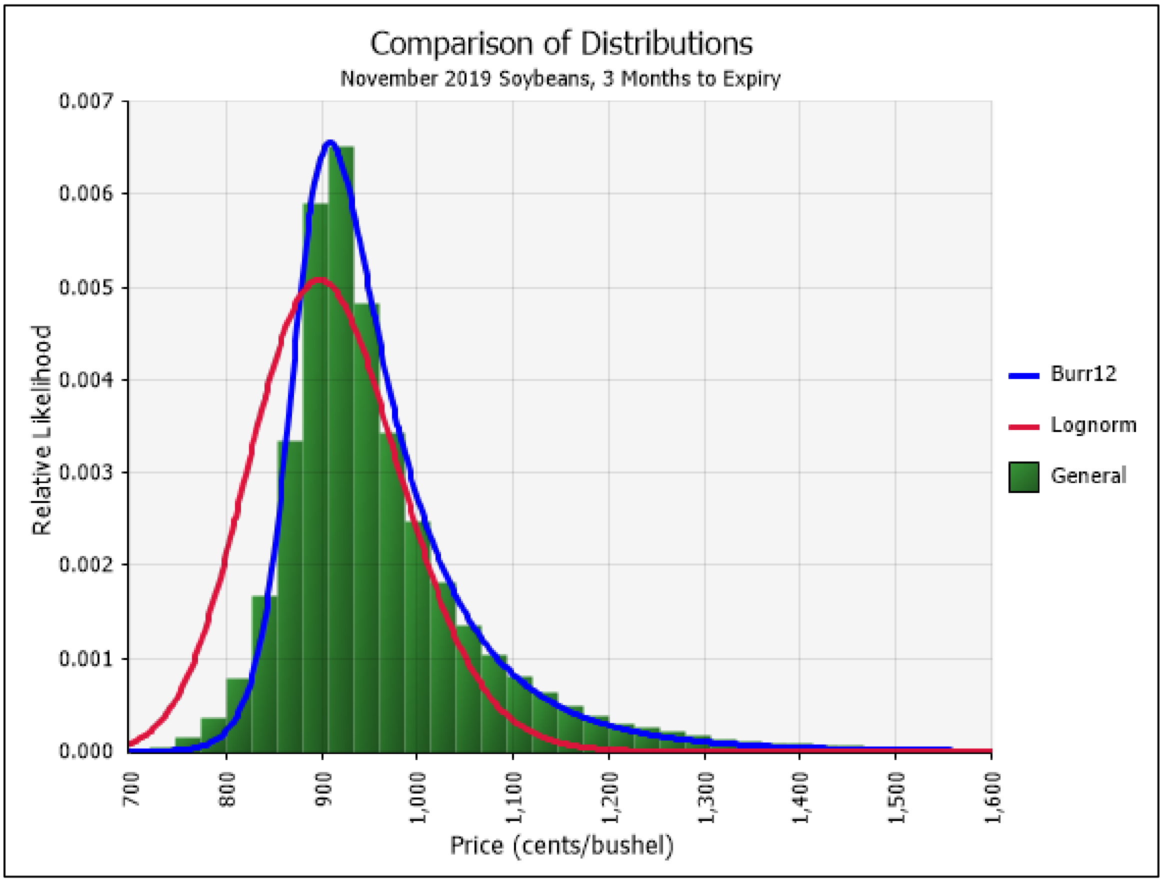

Figure 2. Distribution fits to November 2019 soybean risk premium profile for Burr12 (distribution fit model), Lognormal (Black 76 option model), and General (Nonparametric) distributions.

The Dagum distribution is closely related to the Burr12 distribution and is often referred to as the *inverse Burr distribution.* This distribution was originally proposed by economist Camilo Dagum to closely fit income and wealth distributions where other distributions, such as the Pareto and lognormal, were poor fits due to the presence of heavy tails (Kleiber 2007). Like the Burr12, the @Risk version of the Dagum distribution is bounded at the lower end at zero, unbounded at the upper end, and is characterized by four parameters - a location parameter, a scale parameter, and two shape parameters. Besides income and wealth distributions, the Dagum has also found use in actuarial and insurance applications.

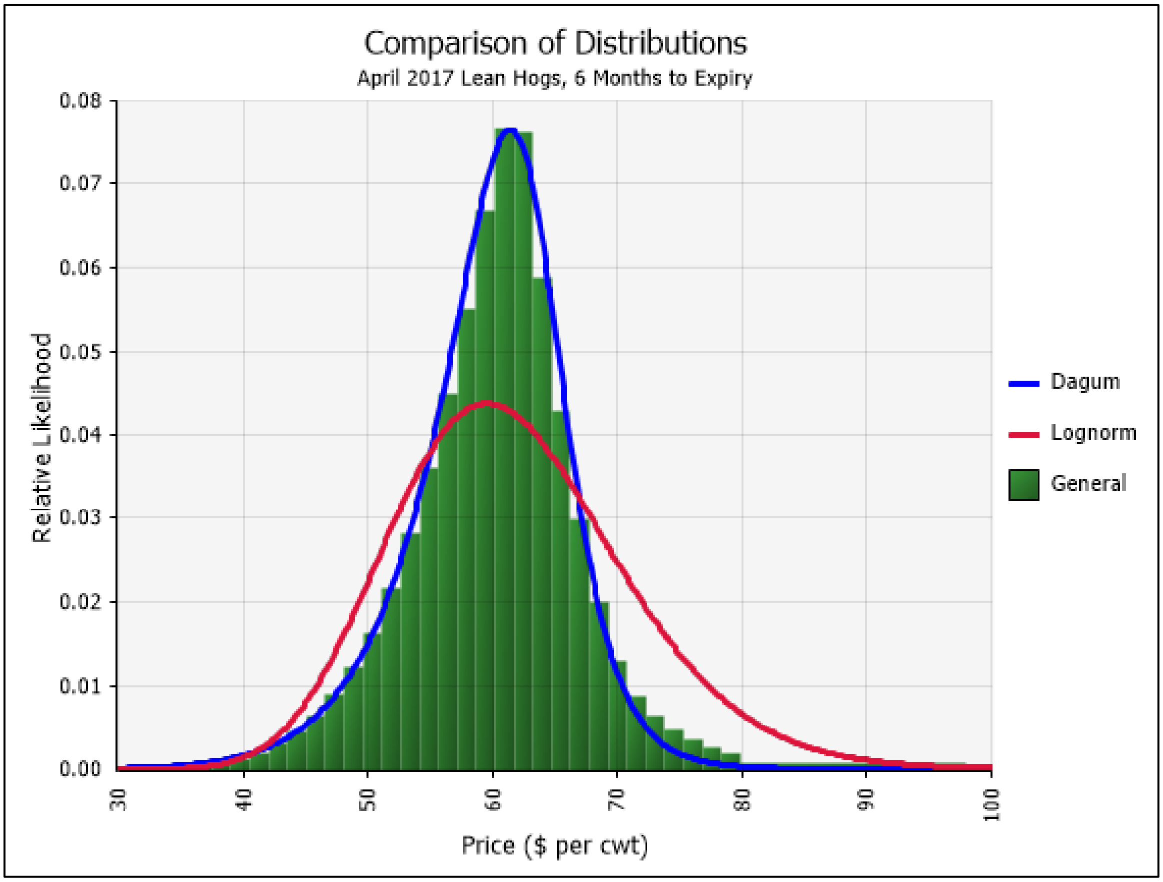

Figure 3. Distribution fits to November 2019 soybean risk premium profile for Dagum (distribution fit model), Lognormal (Black 76 option model), and General (Nonparametric) distributions.

Figure 3 shows the Dagum fit to the April 2017 lean hogs' option premium profile 6 months prior to expiration. In this case, the Black-76 tends to place more weight in the tails — particularly for the upper end. As with the Burr12, the Dagum does a better job of fitting the kurtosis of the actual data distribution (General) when compared to the Black-76 lognormal fit.

Table 3 shows the forecasting results across all commodities for the six-month forward forecast period. The number of VaR exceptions are distributed as a binomial(N,p) distribution with N equal to the total number of trials and p equal to the specified VaR percentile (0.10 in this case). The formula for the likelihood ratio statistic is as follows:

$$LR(p) = 2[\ln(\hat{p}^x(1-\hat{p})^{N-x} - \ln(p^x(1-p)^{N-x})], \tag{10}$$

where N is the total number of trials, x is the number of observed exceptions, $\hat{p}$ is equal to x/N, and p is the hypothesized VaR percentile. The statistic tests the null hypothesis that $\hat{p} = p$ and is distributed as a chi-squared with 1 degree of freedom. The Z-statistic relies upon the binomial approximation to the normal distribution and the formula is as follows:

$$Z_c = \frac{x - Np}{\sqrt{Np(1-p)}}, \quad (11)$$

where the variables as the same as equation (10). This statistic is distributed as a unit normal variate.

Table 3. VaR Exception Count for All Commodities - 6 Month Forward Forecast

	VaR Exceptions (N = 250)							
	Time Series Forecasts		Black76 IV Forecasts		Best Distribution Fit Forecasts		Nonparametric Distribution Fit	
Measure / Test	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1
VaR Exception Count	75	69	13	34	76	33	75	29
VaR Exception Frequency	30.0%	27.6%	5.2%	13.6%	30.4%	13.2%	30.0%	11.6%
Error (Count - Expected)	50	44	-12	9	51	8	50	4
Likelihood Ratio Statistic	76.83*	61.33*	7.63*	3.27	79.55*	2.61	76.83*	0.68
Z-Statistic	10.54*	9.28*	-2.53*	1.90	10.75*	1.69	10.54*	0.84

*Significant at 5% level. Likelihood Ratio distributed as chi-square with one degree of freedom (critical value = 3.84) and Z-statistic is unit normal (critical value = 1.96).

The results in Table 3 clearly show that none of the forecasting method VaR exception counts were in the acceptable range for the portfolio containing long 1 futures contract. Of the four alternatives, the Black-76 provides the lowest absolute Z-score for the long portfolio. For the short 1 contract portfolio, the Black-76, parametric distribution fit, and nonparametric distribution fit methods all provided exception counts that were in the acceptable bounds. The nonparametric provided the best accuracy with the lowest absolute Z-score among the alternatives.

Table 4. VaR Exception Count for All Commodities - 3 Month Forward Forecast

	VaR Exceptions (N = 250)							
	Time Series Forecasts		Black76 IV Forecasts		Best Distribution Fit Forecasts		Nonparametric Distribution Fit	
Measure / Test	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1
VaR Exception Count	52	67	15	31	69	31	65	28
VaR Exception Frequency	20.8%	26.8%	6.0%	12.4%	27.6%	12.4%	26.0%	11.2%
Error (Count - Expected)	27	42	-10	6	44	6	40	3
Likelihood Ratio Statistic	25.54*	56.48*	5.11*	1.50	61.33*	1.50	51.79*	0.39
Z-Statistic	5.69*	8.85*	-2.11*	1.26	9.28*	1.26	8.43*	0.63

*Significant at 5% level. Likelihood Ratio distributed as chi-square with one degree of freedom (critical value = 3.84) and Z-statistic is unit normal (critical value = 1.96).

Table 4 shows the three-month forward results across all commodities. The three-month results are essentially the same as the six-month results in terms of the relative ranking and hypothesis tests. The errors on the long 1 portfolio have come down slightly from the 6-month results while they are all slightly higher on the short 1 portfolio.

Table 5. VaR Exception Count for All Commodities - 1 Month Forward Forecast

	VaR Exceptions (N = 250)							
	Time Series Forecasts		Black76 IV Forecasts		Best Distribution Fit Forecasts		Nonparametric Distribution Fit	
Measure / Test	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1
VaR Exception Count	33	42	18	26	49	23	34	18
VaR Exception Frequency	13.2%	16.8%	7.2%	10.4%	19.6%	9.2%	13.6%	7.2%
Error (Count - Expected)	8	17	-7	1	24	-2	9	-7
Likelihood Ratio Statistic	2.61	10.90*	2.39	0.04	20.60*	0.18	3.27	2.39
Z-Statistic	1.69	3.58*	-1.48	0.21	5.06*	-0.42	1.90	-1.48

*Significant at 5% level. Likelihood Ratio distributed as chi-square with one degree of freedom (critical value = 3.84) and Z-statistic is unit normal (critical value = 1.96).

Table 5 shows the one-month forward results across all commodities. For the one-month results, all of the methods produce exception rates within the 95% confidence range for the long 1 portfolio with exception of the parametric distribution fit. The Black-76 has the lowest error rate followed closely by the time series and nonparametric distribution fit. For the short 1 portfolio, all are within the 95% confidence range with the exception of the time series. The best fit was the Black-76 followed closely by the parametric distribution fit.

Table 6. VaR Exception Count for All Commodities - 1 Week Forward Forecast

	VaR Exceptions (N = 250)							
	Time Series Forecasts		Black76 IV Forecasts		Best Distribution Fit Forecasts		Nonparametric Distribution Fit	
Measure / Test	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1	Long 1	Short 1
VaR Exception Count	24	25	6	15	26	11	15	1
VaR Exception Frequency	9.6%	10.0%	2.4%	6.0%	10.4%	4.4%	6.0%	0.4%
Error (Count - Expected)	-1	0	-19	-10	1	-14	-10	-24
Likelihood Ratio Statistic	0.04	0.00	22.44*	5.11*	0.04	10.79*	5.11*	44.04*
Z-Statistic	-0.21	0.00	-4.01*	-2.11*	0.21	-2.95*	-2.11*	-5.06*

*Significant at 5% level. Likelihood Ratio distributed as chi-square with one degree of freedom (critical value = 3.84) and Z-statistic is unit normal (critical value = 1.96).

Table 6 shows the one-week forward results across all commodities. For the one-week ahead results, the time series and parametric distribution fit have exception rates within the 95% confidence interval for the long 1 portfolio. Based upon the forecast errors, both were essentially equal in accuracy. For the short 1 portfolio, only the time series approach provided forecasts there were within the 95 percent confidence range.

Table 7. Summary of Best Forecasting Model(s) by Commodity and Futures Position

Position	Commodity	Time Period			
		6 Months	3 Months	1 Month	1 Week
Long 1	Corn	B76	B76	B76	TS
	Soybeans	B76	B76	TS	TS
	Wheat	B76	TS,B76	TS,NP	TS,NP
	Soybean Meal	B76	B76	TS, NP	TS
	Lean Hogs	B76	B76	TS, B76, PF, NP	NP
Short 1	Corn	B76, PF, NP	PF, NP	B76	TS
	Soybeans	PF, NP	B76, PF, NP	B76	TS
	Wheat	B76	B76, PF, NP	TS, B76, NP	TS
	Soybean Meal	PF, NP	NP	PF	TS
	Lean Hogs	B76	B76	TS	B76

Note: TS = time series model, B76 = Black-76 option implied volatility, PF = parametric distribution fit, NP = nonparametric distribution fit.

Table 7 shows the best forecasting model or models (ranked by Z-score) for each commodity by futures position (long or short one contract) and forecast period. Consistent with previous studies, the results are somewhat mixed across commodities. However, a couple of dominant themes emerge from the results. First, the Black-76 model dominates in performance on the long portfolio over the 3 and 6 month forecast periods. This indicates that the model does a very good job of modeling the downside price risk in the longer timer periods. Second, with the exception of lean hogs, the time series method dominates over the 1-week ahead time frame for both long and short positions. One explanation for the difference in the lean hog result may relate to the fact that it is the only cash settled contract among the five and therefore, the option expiry data coincides with the futures expiry.

For the short portfolio, the parametric and nonparametric fit models are the best performing for the corn and soy complex (beans and meal) in the longer periods while the Black-76 model performs best for wheat and lean hogs. In the 1-month time frame, the Black-76 seems to perform well with the time series also showing up for wheat and lean hogs.

Overall, the results indicate that the parametric and nonparametric distributional fits to the option risk premium profile tend to perform best in the longer time periods (one month or greater) for the short 1 futures portfolio position. This would indicate that these methods do a better job of capturing the upside price risks in the candidate markets over these longer time periods. This is likely due to their ability to better capture the upper fat tail of the price distribution (see example in Figure 2 above). None of the methods do an acceptable job of capturing the lower tail risk (i.e., long 1 portfolio) for the longer time periods (three and six months forward). Overall, the Black-76 method performs best for both sides of the market in the 1 month forecast period while the time series does best for the 1 week forecast. This suggests that, perhaps, for the longer (three- and six-month forward forecasts) time period, that a splice of the Black-76 (for lower half) and nonparametric or Burr12 fit (for the upper half) might perform best. For the 1-month forward, it appears that the Black-76 provides the best all-around method. For the shortest time period (1 week), the time series dominates over the other methods.

Summary and Conclusions

The projection of future market prices is a critical component in all applied value-at-risk (VaR) modeling. In particular, the challenge in VaR forecasting is that a method must be used that projects not just the mean (or other central moment) of the price but also the complete probability distribution of the price at a future point in time. This requires accurately assessing the volatility (second moment) and higher moments of the price distribution. The Black-Scholes option pricing model, introduced in 1973 (Black and Scholes 1973) and modified for application to futures contracts in 1976 (Black 1976), was revolutionary not only as a tool for incorporating individual traders' price risk expectations in the option premium but also provided a tool for extracting the collective market assessment of the future price volatility, a measure called *implied volatility*. Working within a two-parameter lognormal distribution of prices, this additional parameter along with the futures price as the mean, provides a complete distributional forecast of future prices.

However, past research testing the forecasting performance of the Black-76 implied volatility model has been mixed and the "volatility skew" present in commodity option premiums is strong evidence that lognormality may not be a reliable assumption in modeling commodity prices. This has led to greater research into other approaches to option pricing (stochastic volatility, jump processes, ARCH/GARCH, etc.) and deriving implied distributions from option prices (such as using other parametric and nonparametric distributional approaches).

One such innovative approach was by King and Fackler (1985) who mathematically derived a relationship between the cumulative density function of prices implied in the option premiums across strike prices and the slope of the option premium schedule. This approach was effectively incorporated into a software package distributed by the University of Minnesota Cooperative Extension Service. Later research by Fackler and King (1987, 1990) developed an approach for calibrating future price probability distributions to premiums in the options market.

In this study, the mathematical results of King and Fackler are extended to show that the cumulative density function of future prices is actually proportional to the option premiums across strike prices adjusted by their intrinsic value. Therefore, one can view the option risk premium (i.e., option premium minus current intrinsic value) profile across strike prices as representative of the shape of the market implied probability distribution function (pdf). This makes sense, since if one assumes risk-free valuation of the market, the relative height of the risk premiums should represent the relative likelihood of each strike price occurring at option expiration.

Using this result, two separate approaches were developed to derive the future price pdf using the option risk premium profile. The first approach fit alternative parametric distributions to the premium profile using normalization, maximum likelihood estimation, and minimum root mean squared error (RMSE) to choose the candidate distribution. The second approach utilized a nonparametric, linear interpolation fit (General distribution) to the option premium profile with the area under the fit normalized to one.

For forecast evaluation, the two distribution fit approaches, along with Black-76 and best fitting time series (using Akaike Information Criterion) were incorporated into a simple value-at-risk portfolio model where a one contract position (evaluated both long and short) is held until option expiration. The criterion for evaluation was the difference between the number of actual VaR exceptions (when actual loss exceeded VaR predicted loss) compared to the theoretical number implied by the percentile chosen for computing VaR. Forecasts were evaluated at 6 months, 3 months, 1 month, and 1 week prior to option expiration. Five commodity futures contracts (corn, soybean, wheat, soybean meal, and lean hogs) were evaluated over a ten year period (2013 to 2022) with five futures months per year for a total of 250 contracts evaluated (50 per commodity).

The time series fit results indicated that the first-differenced ARCH with lag 1 was the dominant model with the highest AIC for approximately 2/3rds (64.6%) of the observed results (1,000 total). The first-differenced GARCH11 came in second at 17.2 percent of the observations. This is consistent with most previous studies where ARCH/GARCH variants tend to occur most frequently.

For the best-fitting distributions to the option risk premiums, the results were dominated by the Burr Type XII distribution (86% of observations) followed by the Dagum (11%) which is often called the *inverse Burr distribution*. This is also consistent with previous literature that has found the Burr family of distributions having desirable properties for calibration due to their encompassing of many other distributions and their flexibility with regards to the range of skewness and kurtosis.

The forecasting results were mixed depending upon time frame and futures position (long or short), and also showed some variation across commodities. For the longer time periods (6 and 3 months), the parametric and nonparametric fitting procedures tended to perform best for the short positions while the Black-76 approach performed best for the long positions. This would suggest that the Burr12/Dagum and non-parametric distribution fits to the option risk premiums tend to do a better job of fitting the upper tails of the price distribution while the lognormal (Black-76) tends to do a better job fitting the lower tails. This would suggest a hybrid approach (spliced Black-76 with Burr12 or Dagum) might be the best approach for longer term price projections.

For the intermediate term (1 month forward) the Black-76 and nonparametric premium fit seem to have good all-around (long and short) performance with the Black-76 have a slight edge. In the short-term (1 week), the time series approach tends to dominate over the other approaches. These general results tend to hold up on a commodity by commodity basis, particularly the long portfolio dominance of Black-76 in the longer-term forecasts and the short-term dominance of the time series methods. Among the five commodities, lean hogs and wheat tend to deviate the most when compared to the other commodities.

There are several implications from these results for those involved in commodity analysis and public policy. First, for practitioners of VaR, these results indicate reinforce the use of time series methods (such as the *RiskMetrics* approach) only for calculating short-term VaR measures such as calculating daily margin requirements. For longer-term VaR modeling (1 month or longer), it is better to use an option-based forward projection method. The Black-76 method seems to be fairly robust for intermediate (1 month) to longer term modeling for commodity exposures that tend to be more long than short. For longer-term short exposures, the two methods developed in this study may be incorporated (or a splice for both long and short) to improve the accuracy of VaR projections. For those involved in very long-term forecasting (6 to 18 months), such as those involved in public policy and business planning, the incorporation of option premium based information may improve existing fundamental and technical forecasting methods.

There are also several limitations to this study that can lead to avenues for future research in this area. First, this study only compares forecasting performance using simple long one and short one futures speculative positions. It would be interesting to examine performance under more complex speculative and hedging positions. Second, the performance is evaluated based upon only one percentile of the price distribution (10th for long, 90th for short). Future studies could compare performance across a range of VaR percentiles and/or use the calibration approach advocated in Fackler and King (1987, 1990) to compare forecasting accuracy. Third, the nonparametric fit in this study uses a simple linear interpolation of the option risk premiums to determine the fit. Other methods, such as kernel density and wavelet functions, could be used to derive a candidate nonparametric model. Fourth, the option premium data utilized came from daily settlements reported in DTN ProphetX. It is suspected that many of the premiums quoted likely came from existing bid/offer prices rather than actual transactions (particularly for the deep in- and out-of-the-money strike prices). These issues may not be of major consequence, since these premiums are often small enough to not make a major difference in the distribution fit after normalization (i.e., become de minimis in stature) but it certainly warrants further investigation. Finally, one of the advantages of the Black-76 approach to implied volatility is the ability to time scale the forecast of the implied standard deviation using the relationship $\sigma = iv \cdot \sqrt{t} \cdot F$, where σ is the estimated standard deviation, *iv* is the Black-76 option implied volatility (annualized percent), *t* is the forecast forward period (in years), and *F* is the current futures price. The parametric and nonparametric premium fits used in this study are only used for option expiry. It would be very useful to develop a similar method to time scale these distributional forecasts to intermediate periods (perhaps by interpolating across multiple forward option expirations).

Literature Cited

Aït-Sahalia, Y., and J. Duarte. 2003. "Nonparametric Option Pricing Under Shape Restrictions." *Journal of Econometrics* 116(1–2): 9–47. doi: 10.1016/S0304-4076(03)00102-7.

Alfeus, M., and C.S. Nikitopoulos. 2022. "Forecasting Volatility in Commodity Markets with Long-Memory Models." *Journal of Commodity Markets* 28: 100248. doi: 10.1016/j.jcomm.2022.100248.

Almeida, C., J. Fan, G. Freire, and F. Tang. 2023. "Can a Machine Correct Option Pricing Models?" *Journal of Business & Economic Statistics* 41(3). Taylor & Francis: 995–1009. doi: 10.1080/07350015.2022.2099871.

Angelidis, T., and G. Skiadopoulos. 2008. "Measuring the Market Risk of Freight Rates: A Value-at-Risk Approach." *International Journal of Theoretical and Applied Finance* 11(05). World Scientific Publishing Co.: 447–469. doi: 10.1142/S0219024908004889.

Antonovitz, F., and T. Roe. 1986. "A Theoretical and Empirical Approach to the Value of Information in Risky Markets." *The Review of Economics and Statistics* 68(1): 105. doi: 10.2307/1924933.

Artzner, P., F. Delbaen, J.-M. Eber, and D. Heath. 1999. "Coherent Measures of Risk." *Mathematical Finance* 9(3). John Wiley & Sons, Ltd: 203–228. doi: 10.1111/1467-9965.00068.

Awudu, I., W. Wilson, and B. Dahl. 2016. "Hedging Strategy for Ethanol Processing with Copula Distributions." *Energy Economics* 57: 59–65. doi: 10.1016/j.eneco.2016.04.011.

Baillie, R.T., and R.J. Myers. 1991. "Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge." *Journal of Applied Econometrics* 6(2). John Wiley & Sons, Ltd: 109–124. doi: 10.1002/jae.3950060202.

Bamba, I., and L.J. Maynard. 2004. "Hedging-Effectiveness of Milk Futures Using Value-at-Risk Procedures." Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management of the St. Louis, MO, pp. 19. doi: 10.22004/ag.econ.19028.

Bernard, J.-T., L. Khalaf, M. Kichian, and S. Mcmahon. 2008. "Forecasting Commodity Prices: GARCH, Jumps, and Mean Reversion." *Journal of Forecasting* 27(4). John Wiley & Sons, Ltd: 279–291. doi: 10.1002/for.1061.

Bernstein, P.L. 1998. *Against the Gods: The Remarkable Story of Risk*. New York: John Wiley & Sons.

Black, F. 1976. "The Pricing of Commodity Contracts." *Journal of Financial Economics* 3(1): 167–179. doi: 10.1016/0304-405X(76)90024-6.

Black, F., and M. Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81(3). The University of Chicago Press: 637–654. doi: 10.1086/260062.

Boehlje, M.D., and D.A. Lins. 1998. “Risks and Risk Management in an Industrialized Agriculture.” *Agricultural Finance Review* 58: 1–16. CABI Databases.

Bohmann, M.J.M. 2020. “Price Discovery and Information Asymmetry in Equity and Commodity Futures Options Markets.” Ph.D. Dissertation. Sydney, Australia: University of Technology Sydney. Available online at https://opus.lib.uts.edu.au/bitstream/10453/140245/2/02whole.pdf. [Accessed Apr. 10, 2024].

Brittain, L., P. Garcia, and S.H. Irwin. 2011. “Live and Feeder Cattle Options Markets: Returns, Risk, and Volatility Forecasting.” *Journal of Agricultural and Resource Economics* 36(1). Western Agricultural Economics Association: 28–47. JSTOR.

Bullock, D.W., and D.J. Hayes. 1992. “Speculation and Hedging in Commodity Options: A Modification of Wolf’s Portfolio Model.” *Journal of Economics and Business* 44(3): 201–221. doi: 10.1016/S0148-6195(05)80013-X.

Bullock, D.W., and D.J. Hayes. 1993. “The Private Value of Having Access to Derivative Securities: An Example Using Commodity Options.” *International Review of Economics & Finance* 2(3): 233–249. doi: 10.1016/1059-0560(93)90002-8.

Chen, G., M.C. Roberts, and B. Roe. 2005. “Empirical Performance of Alternative Option Pricing Models for Commodity Futures Options.” Paper presented at the American Agricultural Economics Association Annual Meeting of the Providence, RI, pp. 17. doi: 10.22004/ag.econ.19183.

Daníelsson, J., B.N. Jorgensen, G. Samorodnitsky, M. Sarma, and C.G. de Vries. 2013. “Fat Tails, VaR and Subadditivity.” *Journal of Econometrics* 172(2): 283–291. doi: 10.1016/j.jeconom.2012.08.011.

Data Transmission Network. 2023. “ProphetX.” (version 4.11.1.6).

De Domenico, F., G. Livan, G. Montagna, and O. Nicrosini. 2023. “Modeling and Simulation of Financial Returns Under Non-Gaussian Distributions.” *Physica A: Statistical Mechanics and Its Applications* 622: 128886. doi: 10.1016/j.physa.2023.128886.

Degiannakis, S., G. Filis, T. Klein, and T. Walther. 2022. “Forecasting Realized Volatility of Agricultural Commodities.” *International Journal of Forecasting* 38(1): 74–96. doi: 10.1016/j.ijforecast.2019.08.011.

Egelkraut, T.M., and P. Garcia. 2006. “Intermediate Volatility Forecasts Using Implied Forward Volatility: The Performance of Selected Agricultural Commodity Options.” *Journal of Agricultural and Resource Economics* 31(3): 508–528.

Egelkraut, T.M., P. Garcia, and B.J. Sherrick. 2007. “The Term Structure of Implied Forward Volatility: Recovery and Informational Content in the Corn Options Market.” *American Journal of Agricultural Economics* 89(1): 1–11. doi: 10.1111/j.1467-8276.2007.00958.x.

Fackler, P.L., and R.P. King. 1987. "The Evaluation of Probability Distributions with Special Emphasis on Price Distributions Derived from Option Premiums." *Proceedings of Future Directions in Risk Analysis for Agricultural Firms* Paper presented at the University of Minnesota Department of Agricultural and Applied Economics, St. Paul, MN, pp. 108–129. Available online at https://ageconsearch.umn.edu/record/14030/files/p85-28.pdf. [Accessed Mar. 13, 2024].

Fackler, P.L., and R.P. King. 1990. "Calibration of Option-Based Probability Assessments in Agricultural Commodity Markets." *American Journal of Agricultural Economics* 72(1): 73–83. doi: 10.2307/1243146.

Federal Reserve Bank of St. Louis. 2023. "FRED Economic Data." Available online at https://fred.stlouisfed.org/.

Fernandez-Perez, A., B. Frijns, I. Gafiatullina, and A. Tourani-Rad. 2019. "Properties and the Predictive Power of Implied Volatility in the New Zealand Dairy Market." *Journal of Futures Markets* 39(5). John Wiley & Sons, Ltd: 612–631. doi: 10.1002/fut.21994.

Gardner, B.L. 1977. "Commodity Options for Agriculture." *American Journal of Agricultural Economics* 59(5): 986–992. doi: 10.2307/1239876.

Giot, P. 2003. "The Information Content of Implied Volatility in Agricultural Commodity Markets." *Journal of Futures Markets* 23(5). John Wiley & Sons, Ltd: 441–454. doi: 10.1002/fut.10069.

Gordon, J.D. 1985. *The Distribution of Daily Changes in Commodity Futures Prices*. Technical Bulletin 1702. Washington, DC: USDA Economic Research Service. Available online at https://ageconsearch.umn.edu/record/156817/files/tb1702.pdf. [Accessed Sep. 22, 2023].

Grossman, S.J. 1977. "The Existence of Futures Markets, Noisy Rational Expectations and Informational Externalities." *The Review of Economic Studies* 44(3): 431. doi: 10.2307/2296900.

Heston, S.L. 1993. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies* 6(2): 327–343. doi: 10.1093/rfs/6.2.327.

Hilliard, J.E., and J.A. Reis. 1999. "Jump Processes in Commodity Futures Prices and Options Pricing." *American Journal of Agricultural Economics* 81(2). John Wiley & Sons, Ltd: 273–286. doi: 10.2307/1244581.

Holton, G.A. 2003. *Value-at-Risk: Theory and Practice*. San Diego, CA: Academic Press.

Hotz, J.C. 2004. "An Evaluation of Value at Risk in the Alberta Pork Production Industry." Master's Thesis. Edmonton, Alberta: University of Alberta. Available online at https://era.library.ualberta.ca/items/05a727ae-3623-45dd-930b-f5db7f5bb647/download/6cadd491-20a0-4079-8e57-3c033e8833a5. [Accessed Apr. 9, 2024].

Hung, J.-C., M.-C. Lee, and H.-C. Liu. 2008. "Estimation of Value-at-Risk for Energy Commodities Via Fat-Tailed GARCH Models." *Energy Economics* 30(3): 1173–1191. doi: 10.1016/j.eneco.2007.11.004.

Ivașcu, C.-F. 2021. "Option Pricing Using Machine Learning." *Expert Systems with Applications* 163: 113799. doi: 10.1016/j.eswa.2020.113799.

Jondreau, E., S.-H. Poon, and M. Rockinger. 2007. *Financial Modeling Under Non-Gaussian Distributions*. London: Springer.

Jorion, P. 2001. *Value at Risk: The New Benchmark for Managing Financial Risk*, 2nd ed. McGraw-Hill Finance Series. New York: McGraw-Hill.

J.P. Morgan / Reuters. 1996. "RiskMetrics(Tm) -- Technical Document." Morgan Guaranty Trust Company and Reuters, Ltd. Available online at https://my.liuc.it/MatSup/2006/F85555/rmtd.pdf. [Accessed Apr. 18, 2022].

King, R.P., and P.L. Fackler. 1985. *Probabilistic Price Forecasts Based on Commodity Option Values*. Department of Agricultural and Applied Economics Staff Paper Series P85-28. St. Paul, MN: University of Minnesota. Available online at https://ageconsearch.umn.edu/record/14030/files/p85-28.pdf. [Accessed Aug. 25, 2021].

Kleiber, C. 2007. *A Guide to Dagum Distributions*. WWZ Working Paper 23/07. Basel, Switzerland: University of Basel. Available online at https://edoc.unibas.ch/61230/1/20180305141821_5a9d439d76be5.pdf. [Accessed Apr. 4, 2024].

Lawrence, J.D., and S. Meyer. 1993. "Distilling Option Premium Information into Simple Decision Rules." *Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management*. Available online at http://www.farmdoc.uiuc.edu/nccc134. [Accessed Mar. 13, 2024].

Li, J., and J.-P. Chavas. 2023. "A Dynamic Analysis of the Distribution of Commodity Futures and Spot Prices." *American Journal of Agricultural Economics* 105(1). John Wiley & Sons, Ltd: 122–143. doi: 10.1111/ajae.12309.

Linsmeier, T.J., and N.D. Pearson. 2000. "Value at Risk." *Financial Analysts Journal* 56(2). Routledge: 47–67. doi: 10.2469/faj.v56.n2.2343.

Manfredo, M.R., and R.M. Leuthold. 1999. "Value-at-Risk Analysis: A Review and the Potential for Agricultural Applications." *Applied Economic Perspectives and Policy* 21(1). John Wiley & Sons, Ltd: 99–111. doi: 10.2307/1349974.

Manfredo, M.R., and R.M. Leuthold. 2001. "Market Risk and the Cattle Feeding Margin: An Application of Value-at-Risk." *Agribusiness* 17(3). John Wiley & Sons, Ltd: 333–353. doi: 10.1002/agr.1020.

Manfredo, M.R., and D.R. Sanders. 2004. "The Forecasting Performance of Implied Volatility from Live Cattle Options Contracts: Implications for Agribusiness Risk Management." *Agribusiness*

20(2). John Wiley & Sons, Ltd: 217–230. doi: 10.1002/agr.20003.

Manfredo, M.R., R.M. Leuthold, and S.H. Irwin. 2001. "Forecasting Fed Cattle, Feeder Cattle, and Corn Cash Price Volatility: The Accuracy of Time Series, Implied Volatility, and Composite Approaches." *Journal of Agricultural and Applied Economics* 33(3). Cambridge University Press: 523–538. Cambridge Core. doi: 10.1017/S1074070800020988.

O'Brien, D., M. Hayenga, and B. Babcock. 1996. "Deriving Forecast Probability Distributions of Harvest-Time Corn Futures Prices." *Applied Economic Perspectives and Policy* 18(2): 167–180. doi: 10.2307/1349430.

Odening, M., and J. Hinrichs. 2002. "Using Extreme Value Theory to Estimate Value-at-Risk." *Agricultural Finance Review* 63(1). MCB UP Ltd: 55–73. doi: 10.1108/00215000380001141.

Palisade Software. 2023. "@Risk: Advanced Risk Analysis for Microsoft Excel." (version 8.3.2 [Build 42] Industrial Edition). Ithaca, NY: Palisade Software, Inc.

Poon, S.-H., and C. Granger. 2005. "Practical Issues in Forecasting Volatility." *Financial Analysts Journal* 61(1). Routledge: 45–56. doi: 10.2469/faj.v61.n1.2683.

Poon, S.-H., and C.W.J. Granger. 2003. "Forecasting Volatility in Financial Markets: A Review." *Journal of Economic Literature* 41(2): 478–539. doi: 10.1257/002205103765762743.

Ramsey, A.F., and B.K. Goodwin. 2019. "Value-at-Risk and Models of Dependence in the U.S. Federal Crop Insurance Program." *Journal of Risk and Financial Management* 12(2): 65. doi: 10.3390/jrfm12020065.

Rhee, D.W., S.J. Byun, and S. Kim. 2012. "Empirical Comparison of Alternative Implied Volatility Measures of the Forecasting Performance of Future Volatility*: Forecasting Performance of Future Volatility." *Asia-Pacific Journal of Financial Studies* 41(1): 103–124. doi: 10.1111/j.2041-6156.2011.01066.x.

Rodriguez, R.N. 1977. "A Guide to the Burr Type XII Distributions." *Biometrika* 64(1): 129–134. doi: 10.1093/biomet/64.1.129.

Sanders, D.R., and M.R. Manfredo. 2002. "The Role of Value-at-Risk in Purchasing: An Application to the Foodservice Industry." *Journal of Supply Chain Management* 38(1): 38–45. doi: j.1745-493X.2002.tb00128.x.

Schneider, L., and B. Tavin. 2024. "Seasonal Volatility in Agricultural Markets: Modelling and Empirical Investigations." *Annals of Operations Research* 334(1): 7–58. doi: 10.1007/s10479-021-04241-7.

Shao, R., and B. Roe. 2001. "Underpinnings for Prospective, Net Revenue Forecasting in Hog Finishing:" Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management of the St. Louis, MO, pp. 15. doi:

10.22004/ag.econ.18954.

Sherrick, B.J., D.L. Forster, and S.H. Irwin. 1990. "The Calibration of Expected Soybean Price Distributions: An Option Based Approach." Paper presented at the Agricultural and Applied Economics Annual Meeting of the Vancouver, British Columbia, pp. 16. doi: 10.22004/ag.econ.270919.

Sherrick, B.J., P. Garcia, and V. Tirupattur. 1996. "Recovering Probabilistic Information from Option Markets: Tests of Distributional Assumptions." *Journal of Futures Markets* 16(5). John Wiley & Sons, Ltd: 545–560. doi: 10.1002/(SICI)1096-9934(199608)16:5<545::AID-FUT3>3.0.CO;2-G.

Silva, E., and K.H. Kahl. 1993. "Reliability of Soybean and Corn Option-Based Probability Assessments." *The Journal of Futures Markets* 13(7): 765–779.

Tejeda, H., and B.K. Goodwin. 2008. "Modeling Crop Prices Through a Burr Distribution and Analysis of Correlation Between Crop Prices and Yields Using a Copula Method." Paper presented at the Agricultural and Applied Economics Annual Meeting of the Orlando, Florida, pp. 39. doi: 10.22004/ag.econ.6061.

Trujillo-Barrera, A., P. Garcia, and M.L. Mallory. 2018. "Short-Term Price Density Forecasts in the Lean Hog Futures Market." *European Review of Agricultural Economics* 45(1): 121–142. doi: 10.1093/erae/jbx026.

Urcola, H.A., and S.H. Irwin. 2010. "Hog Options: Contract Redesign and Market Efficiency." *Journal of Agricultural and Applied Economics* 42(4): 773–790. doi: 10.1017/S1074070800003953.

Urcola, H.A., and S.H. Irwin. 2011. "Are Agricultural Options Overpriced?" *Journal of Agricultural and Resource Economics* 36(1): 63–77. doi: 10.22004/ag.econ.105525.

Wilson, W.W., W.E. Nganje, and C.R. Hawes. 2007. "Value-at-Risk in Bakery Procurement." *Review of Agricultural Economics* 29(3): 581–595. doi: 10.1111/j.1467-9353.2007.00373.x.

Wolf, A. 1987. "Optimal Hedging with Futures Options." *Journal of Economics and Business* 39(2): 141–158. doi: 10.1016/0148-6195(87)90013-0.

Yamai, Y., and T. Yoshiba. 2005. "Value-at-Risk Versus Expected Shortfall: A Practical Perspective." *Risk Measurement* 29(4): 997–1015. doi: 10.1016/j.jbankfin.2004.08.010.