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The Economic Value of Intraday Data in Hedging Commodity Spot Prices

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The Economic Value of Intraday Data in Hedging Commodity Spot Prices

This article shows how high-frequency market data relates to low frequency events by examining the economic value of using intraday data to hedge commodity spot prices in the futures market. We use the realized minimum-variance hedging ratio (RMVHR) framework, which depends on the realized futures-cash covariance matrix forecast. We focus on the crude oil crack and soybean crush industries and consider both multiple and single-commodity portfolios, as well as different forecast strategies based on intraday data. We use the Naïve hedging ratio as the benchmark to investigate the performance of intraday data-based hedging models. Our results suggest that for each portfolio considered, there is usually one intraday data-based hedging strategy that outperforms the Naïve. Superior performance, however, is not always statistically significant, for the crack industry. Our estimates place the advantage of using intraday data between $7,155.00 and $287.50 per contract and year on average, with these values representing the decline in the portfolio's standard deviation achieved through hedging. This points at a promising path to improving the performance of hedging in the commodity space based on intraday data.

Key Words: intraday data, hedging effectiveness, economic value, crude oil crack, soybean crush.

1. Introduction

Since 2020, energy and food prices have experienced one of the largest increases since the beginning of the 21st century. Resulting from a combination of supply and demand factors, crude oil prices increased by 440% from April 2020 to June 2022, and soybean prices doubled from April 2020 to June 2022 (see figure in Appendix A). The energy and food price shocks have caused rising global inflation, slowing global economic growth, and heightened food insecurity and social unrest (Neufeld, 2022).

Increased price volatility has also induced greater marketing and operation risks for producers, users and processors in the energy and food supply chains. This has placed the need for hedging at the center of commodity-related businesses and increased the interest in derivative markets. Futures are a popular price risk management instrument among commodity traders. In a simple scenario where the hedger holds a single physical asset, hedging consists of building a portfolio that combines the hedger's position in the physical asset and an offsetting position in the futures market. An important hedging decision concerns the optimal position in the futures market that meets the objectives of the hedger. The popular minimum variance hedging ratio (MVHR) establishes the position in the futures market that minimizes the portfolio risk (Ederington, 1979). MVHR is expressed as the ratio of the futures position relative to the spot position and depends on the joint dynamics of futures and cash prices. Specifically, the MVHR is calculated as the ratio of the covariance between the spot and futures returns and the variance of futures returns over the investment's holding period. As a result, the MVHR's success in minimizing the portfolio risk depends on the hedger's ability to forecast the price (co)variance over the holding period.

An extensive literature has proposed different approaches to model the futures and spot price joint dynamics to accurately forecast the covariance matrix. Wang et al. (2015) review this literature and compare 18 of these models against the *Naïve* hedging strategy using weekly spot and futures price data across 24 futures markets. The *Naïve* hedging strategy fully offsets spot positions with futures contracts, it does not require to forecast risk, is equivalent to a MVHR=1 and assumes that futures and spot prices are perfectly correlated over the holding period. Wang et al. (2015) conclude that no strategy dominates the *Naïve* approach consistently and significantly. Thus, hedgers should gain little value from investing in sophisticated hedging strategies, which speaks volumes about our ability to forecast risk.

Over the past decade, the finance literature has achieved superior risk forecasting measures using intraday price data which has become widely available as electronic trading in financial markets has expanded to dominate floor trading. The Heterogeneous Autoregressive (HAR) model proposed by Corsi (2009) has emerged as the workhorse in volatility forecasting given its consistent performance in different empirical settings. The HAR model forecasts daily realized variance (*RV*) based on a linear function of past *RV*s averaged at daily, weekly and monthly horizons, with *RV* being the sum of finely sampled squared returns within the day. While different studies have attempted to improve the simple HAR-RV model, they have achieved mixed results, with HAR usually performing as well, if not better than more sophisticated alternatives (Sévi, 2014; Ma et al., 2014; Qiu et al., 2019; Audrino et al., 2018; Ding et al., 2021).

Recent articles have applied the HAR structure to forecasting price return covariance matrices. This poses challenges related to the curse of dimensionality that may result in lower forecasting performance, and the need to ensure positive semi-definiteness (psd) of the covariance matrix. An array of solutions imposes different restrictions on the covariance matrix. Some approaches ensure psd by relying either on log transformations (Bauer & Vorkink, 2011), covariance Cholesky decompositions (Symitsi et al., 2018), or the Wishart Autoregressive model (Gouriéroux et al., 2009). Zhang et al. (2019) minimize the curse of dimensionality by building on Audrino & Corsi (2010) and Asai & McAleer (2017). To prevent overparameterization, they extend the HAR-RV models to directly forecast the realized covariance between two price returns, rather than using the full realized covariance matrix for this purpose.

The growing literature based on high-frequency market data has often failed to show how these data relate to low-frequency economic events. A few exceptions have built on improvements in the ability to forecast risk and revisited optimal hedging by using intraday data (Harris et al., 2010). Markopoulou et al. (2016) propose the realized minimum variance hedging ratio (RMVHR) based on the realized variance and covariance calculated using 5-minute intraday data. Then, they assess the RMVHR's predictability using an array of econometric models including the HAR model structure. They compare hedging performance of RMVHR relative to MVHR based on lower frequency data. They focus on a sample of two asset classes, stocks and exchange rates, and conclude that the use of intraday data outperforms daily data in risk management. Like Markopoulou et al. (2016), Qu et al. (2019) and Kuang (2022) show that high-frequency data help generating more accurate covariance forecasts and hedging ratios for the CSI 300 Index and US equity-oil cross-hedges than daily data.

However, the utilization of high-frequency data in commodity hedging remains unexplored due to the lack of intraday spot prices. We propose an approach to overcome this issue and investigate for the first time whether intraday data can improve commodity hedging. In our analysis, we generate intraday cash prices using the cost of carry model for storable commodities (Pindyck, 2001; Main et al., 2018) which establishes the relationship between spot and futures prices through the carrying costs. The latter represent the expenses associated with carrying a commodity into the future and include forgone interest, physical storage costs and the convenience yield. Specifically, we assume that the cost of carry is constant within each day but changes across days, which allows us to derive intraday cash prices by subtracting the cost of carry from intraday futures prices. We then study whether the RMVHR strategy can outperform the *Naïve* hedging, thus shedding light on whether intraday data have any value for hedgers in the commodity space..

We focus on the crude oil cracking and the soybean crushing industries whose margins are measured through the crack and crush spread, respectively. Consequently, our analysis offers valuable insights for commercial traders along these supply chains who hedge in the futures markets. Improving hedging outcomes can also enhance welfare by increasing individual firms' utility, improve allocation of commodities over time, as well as investment decisions (Willems and Morbee, 2010). Crude oil can be cracked into gasoline and diesel/heating oil, for transportation and heating. Soybean can be crushed into soybean meal and oil, for livestock feeding and cooking/food manufacturing. Their margin spreads are among the most highly traded in the Chicago Mercantile Exchange (CME) where futures contracts available allow simulating the financial aspects of their businesses. In terms of all energy futures traded, crude oil, gasoline and heating oil represent the 1^{st,} 3rd and 4th most traded contracts in the CME, reflecting the economic relevance of this industry. Soybeans, soybean oil and soybean meal rank 2nd, 4th and 5th, with the complex as a whole being first in terms of agricultural commodities' trading volume (CME, 2022). Consideration of the crack and crush spreads requires a multivariate portfolio approach that is substantially more complex than the single-asset model used by the research pioneering hedging based on intraday data (Harris et al., 2010; Markopoulou et al., 2016; Qu et al., 2019; Kuang, 2022). This allows us to investigate whether complexity pays off.

The multivariate portfolio model was initially proposed by Anderson & Danthine (1980, 1981) and allows for the covariation of cash and futures input and output prices. Since spot commodity prices are rarely available at intraday frequency, empirical studies on the crack and crush spreads have generally used daily (Collins, 2000; Liu et al., 2017), weekly (Tzang & Leuthold, 1990; Garcia et al., 1995; Haigh & Holt, 2002; Alexander et al., 2013), or even monthly (Fackler &

McNew, 1993) prices. By relying on generated intraday spot prices, we calculate the optimal hedging positions that allow minimizing the portfolio return variance based on the assumption of portfolio rebalancing on a daily basis. Drawing on HAR-type models, we consider forecasting the necessary (co)variances following different approaches, from independently forecasting the components of the (co)variance matrix (Zhang et al., 2019), to jointly forecasting spot and futures (co)variance for each market while imposing psd (Bauer & Vorkink, 2011, Qu et al., 2019, Symitsi et al., 2018), and even directly forecasting the RMVHR as in Markopoulou (2016), Qu et al. (2019) or Kuang (2022).

To provide information on the value of using intraday data for hedging purposes, we compare our results with the *Naïve* hedging ratio that Wang et al. (2015) have shown to outperform other more sophisticated hedging strategies. We choose the *Naïve* hedging ratio as a benchmark because it does not require forecasting risk and is thus independent from the data frequency and modeling approach.[2]. We provide a monetary estimation of this value by relying on hedging effectiveness measures.

We use both futures and cash prices for the commodities involved in the crack and crush spreads. Futures prices are observed intraday, while cash prices are observed daily and transformed into intraday prices. Our period of analysis starts on 01/02/2009 and ends on 06/17/2022. In general, results suggest the combination of advanced hedging strategies and high-frequency data produces results that outperform the *Naïve* strategy. Superior performance is statistically significant for agricultural commodities, but less so for energy commodities. Our estimates place the advantage of using intraday data between \$7,155.00 and \$287.50 per contract and year on average, with these values representing the decline in the portfolio's standard deviation achieved through hedging.

2. Methodology

In this section we provide an overview of the methods used in the paper. We start with the description of the hedging framework, which allows identifying the optimal hedging ratios. We then move into the modeling and forecasting of risk during the hedgers' holding period. Lastly, we present the hedging efficiency measures.

2.1. The hedging framework

Our hedging strategy aims at minimizing the return variance of a portfolio that includes cash and

[2] *Naïve* is the best hedging strategy in Wang et al. (2015). We do not compare hedging results with strategies based on daily data as these cannot rely on HAR models. Instead, they require other approaches such as the ones described in Wang et al. (2015). As a result, differences across hedging performance can either be due to data or to the modeling approach.

futures positions of outputs and inputs in the crude oil crack and soybean crush industries. We normalize output prices on a per unit of input (i.e., barrel (bushel) of crude oil (soybeans)) using the technical specifications of the production process as follows:

$P_{z,t} = \left[log\left(p_{z,t}^{a}\right) \quad log\left(p_{z,t}^{b}(b/a)\right) \quad log\left(p_{z,t}^{c}(c/a)\right)\right]'$, where $z = s, f$ denotes either spot or futures prices and $log(.)$ is the natural logarithm function that we apply on prices to reduce heteroskedasticity and promote normality in price distribution. Coefficients a, b, c are technical coefficients representing the crude oil crack (soybean crush) production functions; a represents the units of crude oil (soybeans) needed to produce b units of gasoline (soybean meal) and c units of heating oil (soybean oil). Typically, the cracking industry generates 2 barrels of gasoline and 1 barrel of heating oil from 3 barrels of crude oil. Using the futures contracts available from CME (2017), the spread can be hedged using the 3:2:1 rule (long $a = 3$ crude contracts and short $b = 2$ and $c = 1$ gasoline and heating oil contracts, respectively), with each crude oil contract representing 1,000 barrels. Similarly, the soybean crushing industry produces 11 pounds of soybean oil and 44 pounds of soybean meal from 1 bushel of soybeans. Considering the soybean complex futures contracts' technical specifications, the soybean crush margin can be hedged using the 10:11:9 rule (long $a = 10$ soybean contracts and short $b = 11$ and $c = 9$ soybean meal and oil contracts, respectively) (CME, 2020), where each soybean futures contract represents 5,000 bushels. While a, b and c coefficients vary by industry, for ease of notation we do not use different symbols. P and p represent transformed and untransformed prices and superscripts a, b and c are indicators for the different output and inputs.

Assume the hedger rebalances the portfolio daily; on day t-1 the crack (crush) hedger longs crude oil (soybeans) and shorts gasoline and heating oil (soybean meal and oil) in the futures market. On day t the hedger buys crude oil (soybeans) and sells gasoline and heating oil (soybean meal and oil) in the spot market and simultaneously cancels the futures positions. This implies that the crude oil crack and soybean crush industries keep relatively dynamic portfolios that adjust frequently to changing market conditions. Hedged portfolio returns $\left(r_{\pi,t}\right)$ on day t can be expressed as:

$$r_{\pi,t} = -P_{s,t}^{a} + P_{s,t}^{b} + P_{s,t}^{c} + \beta^{a}\left(P_{f,t}^{a} - P_{f,t-1}^{a}\right) - \beta^{b}\left(P_{f,t}^{b} - P_{f,t-1}^{b}\right) - \beta^{c}\left(P_{f,t}^{c} - P_{f,t-1}^{c}\right), \qquad (1)$$

where β^{i} $i = a, b, c$ are the futures' positions in each commodity i expressed as the proportion of cash positions that are hedged using futures contracts. We express the returns in matrix form as

$$r_{\pi,t} = 1_{m}' P_{s,t} + \beta' r_{f,t}, \qquad (2)$$

where 1_m is a 3×1 vector of ones, $P_{s,t} = \left[-P_{s,t}^{a}\ P_{s,t}^{b}\ P_{s,t}^{c}\right]'$, $\beta = [\beta^{a}\ \beta^{b}\ \beta^{c}]'$, and $r_{f,t} = \left[(P_{f,t}^{a} -\right.$

$P^{a}_{f,t-1}$) - ($P^{b}_{f,t} - P^{b}_{f,t-1}$) - ($P^{c}_{f,t} - P^{c}_{f,t-1}$)$\big]'$. While prices at t-1 are known at the time the decision to hedge is taken, prices at t are not. Hence the expectation of expression (2) is:

$$E\big[r_{\pi,t}\big] = 1'_m E\big[P_{s,t}\big] + \beta' E\big[P_{f,t}\big] - \beta' P_{f,t-1} \tag{3}$$

where $P_{f,t} = \big[P^{a}_{f,t} \; -P^{b}_{f,t} \; -P^{c}_{f,t}\big]'$. The variance of the portfolio returns can be derived as:

$$V\big[r_{\pi,t}\big] = E\left[\left(r_{\pi,t} - E\big[r_{\pi,t}\big]\right)^2\right] = E\left[\left(1'_m\left(P_{s,t} - E\big[P_{s,t}\big]\right) + \beta'\left(P_{f,t} - E\big[P_{f,t}\big]\right)\right)^2\right]. \tag{4}$$

Following the standard practice in the literature, we assume naïve daily spot price expectations, conditional on the information available at day t-1 and an unbiased futures market, which results in: $E\big[P_{s,t}\big] = P_{s,t-1}$ and $E\big[P_{f,t}\big] = P_{f,t-1}$. Hence, the portfolio return variance can be alternatively expressed as:

$$V\big[r_{\pi,t}\big] = 1'_m V\big[r_{s,t}\big] 1_m + \beta' V\big[r_{f,t}\big]\beta + 2\beta' Cov\big[r_{f,t}, r_{s,t}\big] 1_m \tag{5}$$

where $r_{s,t} = P_{s,t} - P_{s,t-1}$ and $r_{f,t} = P_{f,t} - P_{f,t-1}$, $V\big[r_{s,t}\big]$ and $V\big[r_{f,t}\big]$ are the 3×3 symmetric spot and futures return covariance matrices, respectively while $Cov\big[r_{f,t}, r_{s,t}\big]$ is a non-symmetric 3×3 covariance matrix between spot and futures prices. Notice that since prices are expressed in logs, returns are in percent, which makes results robust to any price data transformation. To utilize intraday data in this framework, we replace the spot and futures return (co)variance matrices with realized (co)variance matrices, and thus $V\big[r_{\pi,t}\big]$ becomes the realized variance of the portfolio. Realized (co)variances are calculated based on finely sampled intraday returns and details are offered in section 2.2.

The realized minimum variance hedging ratio (RMVHR) is obtained by minimizing expression (5), which results in:

$$\beta = -V\big[r_{f,t}\big]^{-1} Cov\big[r_{f,t}, r_{s,t}\big] 1_m. \tag{6}$$

In matrix form, expression (6) is:

$$\begin{bmatrix}\beta^a\\ \beta^b\\ \beta^c\end{bmatrix} = -\begin{bmatrix}RV_{r_{f,t}^a r_{f,t}^a} & RV_{r_{f,t}^a r_{f,t}^b} & RV_{r_{f,t}^a r_{f,t}^c}\\ RV_{r_{f,t}^a r_{f,t}^b} & RV_{r_{f,t}^b r_{f,t}^b} & RV_{r_{f,t}^b r_{f,t}^c}\\ RV_{r_{f,t}^a r_{f,t}^c} & RV_{r_{f,t}^b r_{f,t}^c} & RV_{r_{f,t}^c r_{f,t}^c}\end{bmatrix}^{-1} \begin{bmatrix}RC_{r_{f,t}^a r_{s,t}^a} & RC_{r_{f,t}^a r_{s,t}^b} & RC_{r_{f,t}^a r_{s,t}^c}\\ RC_{r_{f,t}^b r_{s,t}^a} & RC_{r_{f,t}^b r_{s,t}^b} & RC_{r_{f,t}^b r_{s,t}^c}\\ RC_{r_{f,t}^c r_{s,t}^a} & RC_{r_{f,t}^c r_{s,t}^b} & RC_{r_{f,t}^c r_{s,t}^c}\end{bmatrix} \begin{bmatrix}1\\1\\1\end{bmatrix}, \tag{7}$$

where $RV_{r_{f,t}^i r_{f,t}^j}$ and $RC_{r_{f,t}^i r_{s,t}^j}$ denote the forecast realized (co)variances over the holding period, where i, j = a, b, c. Expression (7) is demanding in terms of the number of forecasts that need to be generated. This increases the risk of forecasting errors and low hedging performance. We also consider an alternative to (7), which assumes that matrices $V[r_{f,t}]$ and $Cov[r_{f,t}, r_{s,t}]$ are diagonal matrices. While this alternative may reduce forecast errors, it ignores the correlation between prices across different markets in RMVHR calculation (Markopoulou, 2016). In this simplified alternative, we derive the RMVHR for each input and output as follows:

$$\beta^i = -\frac{RC_{r_{f,t}^i r_{s,t}^i}}{RV_{r_{f,t}^i r_{f,t}^i}}. \tag{8}$$

We evaluate portfolio performance under the complex and simplified alternatives, which allows us assessing whether complexity helps in forecasting. In the following subsection we describe how we model realized (co)variances.

2.2. Realized (co)variances

Realized (co)variance provides a measure of price return(s) (co)variance within a day. For example, it informs on the daily joint variability of cash and futures price returns. Let spot and futures prices be observed on trading day t = 1,…,T in evenly-spaced intervals of k-minutes, with K being the number of k-minute intervals within the day. We define futures price intraday returns as the natural log difference of intraday prices in percentage form:

$$r^i_{f,t+\frac{k}{K}} = \left[\log(p^i_{f,t+\frac{k}{K}}) - \log\,(p^i_{f,t+\frac{k-1}{K}})\right] \times 100. \tag{9}$$

We define intraday spot price returns $r^i_{s,t+\frac{k}{K}}$ analogously.[3] Realized futures price (co)variances ($RV_{f,t}$) and spot price-futures price realized (co)variances ($RC_{sf,t}$) on day t are defined as:

[3] Since returns are expressed in percent, the coefficients characterizing the production process (a, b and c) in $P_{zt} = [log(p^a_{z,t}) \quad log(p^b_{z,t}(b/a)) \quad log(p^c_{z,t}(c/a))]'$ become irrelevant.

$$RV_{f,t}^{ij} = \sum_{k=1}^{K}\left(r_{f,t+\frac{k}{K}}^{i} r_{f,t+\frac{k}{K}}^{j}\right), \text{ and} \tag{10}$$

$$RC_{sf,t}^{ij} = \sum_{k=1}^{K}\left(r_{f,t+\frac{k}{K}}^{i} r_{s,t+\frac{k}{K}}^{j}\right) \tag{11}$$

For ease of notation, we suppress superindices *ij* in the rest of the article.

2.3. Forecasting futures risks

We follow Zhang et al. (2019) and forecast the individual variances and covariances in (7) and build the RMVHR afterwards by replacing each individually-generated forecast into (7). We also forecast the RMVHR, i.e., $\beta = [\beta^a \quad \beta^b \quad \beta^c]'$ directly through a HAR structure, as opposed to forecasting its individual components, a strategy proposed by Andersen et al. (2005) and adopted by Harris et al. (2010) and others. To produce the necessary forecasts, we consider an array of HAR-based models which we describe below.

2.3.1. *HAR model*

The HAR-RV models the interrelations of volatility aggregated over different frequencies to capture persistence and non-normality, both known to be pervasive in volatility modeling (Corsi, 2009). It mirrors the asymmetric behavior of traders such that short-term (long-term) traders will (not) consider the level of long-term (short-term) volatility:

$$RV_{f,t+h} = \gamma_0 + \gamma_d RV_{f,t} + \gamma_w RV_{f,t}^{w} + \gamma_m RV_{f,t}^{m} + \varepsilon_{f,t+h}\,, \tag{12}$$

where *RV* is defined in expression (10) and superindices *w* and *m* represent the average *RV* over the past 5 and 22 trading days, respectively, with h=1 day being the forecast horizon, which corresponds to 1-day holding period. Following Asai & McAleer (2017) and Zhang et al. (2019), we use the same structure to model realized covariance (equation 11), which results in the HAR-RC model:

$$RC_{sf,t+h} = \alpha_0 + \alpha_d RC_{sf,t} + \alpha_w RC_{sf,t}^{w} + \alpha_m RC_{sf,t}^{m} + \varepsilon_{sf,t+h}, \tag{13}$$

We apply models (12) and (13) to forecast each of the elements in the realized variance and covariance matrices in expression (7). Forecast values are reassembled into (7) to calculate the optimal hedging ratios. As an alternative to using (7), we also generate hedging ratios as in (8) by ignoring price covariances across markets. These hedging ratios are then used to evaluate the overall portfolio performance.

Finally, following Markopoulou et al. (2016) and others, we apply the HAR model structure to forecasting each hedging ratio directly, calculated as in (8). This results in the HAR-Beta model:

$$\beta_{t+h} = \delta_0 + \delta_d\beta_t + \delta_w\beta_{t+h}^w + \delta_m\beta_{t+h}^m + \varepsilon_{\beta,t+h}. \tag{14}$$

Expression (14) allows for long memory in the hedging ratio, though if the long memory in underlying (co)variances was common, the betas may be only weakly persistent (Andersen et al., 2005). As is common practice, we estimate equations (12)-(14) by OLS.

2.3.2. *Vector HAR*

The Vector HAR (VHAR) model allows forecasting the (2 x 2) covariance matrix between futures and spot price returns for each commodity and is especially relevant to single-commodity portfolios. We use the matrix logarithmic transformation to ensure psd of the covariance matrix (Bauer & Vorkink, 2011 and Qu et al., 2019):

$$A_t = logm\begin{bmatrix} RV_{s,t} & RC_{sf,t} \\ RC_{sf,t} & RV_{f,t} \end{bmatrix} = \begin{bmatrix} X_{s,t} & X_{sf,t} \\ X_{sf,t} & X_{f,t} \end{bmatrix}, \tag{15}$$

where A_t is a real and symmetric matrix. We take the half vectorization of A_t:

$$X_t = vech(A_t) = (X_{s,t}, X_{sf,t}, X_{f,t}), \tag{16}$$

The VHAR-Log model is then expressed as:

$$\begin{bmatrix} X_{s,t+h} \\ X_{sf,t+h} \\ X_{f,t+h} \end{bmatrix} = \begin{bmatrix} \varphi_0 \\ \eta_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} \varphi_{d,s} & \varphi_{d,sf} & \varphi_{d,f} \\ \eta_{d,s} & \eta_{d,sf} & \eta_{d,f} \\ \theta_{d,s} & \theta_{d,sf} & \theta_{d,f} \end{bmatrix}\begin{bmatrix} X_{s,t} \\ X_{sf,t} \\ X_{f,t} \end{bmatrix}$$

$$+ \begin{bmatrix} \varphi_{w,s} & \varphi_{w,sf} & \varphi_{w,f} \\ \eta_{w,s} & \eta_{w,sf} & \eta_{w,f} \\ \theta_{w,s} & \theta_{w,sf} & \theta_{w,f} \end{bmatrix}\begin{bmatrix} X_{s,t}^w \\ X_{sf,t}^w \\ X_{f,t}^w \end{bmatrix} + \begin{bmatrix} \varphi_{m,s} & \varphi_{m,sf} & \varphi_{m,f} \\ \eta_{m,s} & \eta_{m,sf} & \eta_{m,f} \\ \theta_{m,s} & \theta_{m,sf} & \theta_{m,f} \end{bmatrix}\begin{bmatrix} X_{s,t}^m \\ X_{sf,t}^m \\ X_{f,t}^m \end{bmatrix} + \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{sf,t} \\ \varepsilon_{f,t} \end{bmatrix}, \tag{17}$$

where super indices *w* and *m* represent the average X_t over the past 5 and 22 days, respectively. Each of the equations in (17) is estimated by OLS and forecasts for $X_{s,t+h}$, $X_{sf,t+h}$ and $X_{f,t+h}$ are generated for *h*=1. As opposed to the HAR models (12) and (13), the Vector HAR allows for spillovers across spot and futures markets. The forecast realized (co)variances are retrieved by applying the inverse of half vectorization on the forecast for $X_{t+h} = (X_{s,t+h}, X_{sf,t+h}, X_{f,t+h})$ and taking the matrix exponential:

$$\begin{bmatrix} RV_{s,t+h} & RC_{sf,t+h} \\ RC_{sf,t+h} & RV_{f,t+h} \end{bmatrix} = expm[invvech(X_{t+h})] \tag{18}$$

As an alternative to the matrix logarithmic transformation, to ensure positive definiteness we also consider Symitsi et al.'s (2018) Cholesky decomposition. See Appendix B for details.

2.4. Out-of-sample forecasting

We generate all out-of-sample forecasts ($t+h$, h=1) based on a rolling window containing a fixed number of days equal to ϖ. We estimate the forecasting models over the initial subsample (first ϖ observations). These allow forecasting the (co)variance for ϖ +1 and generate the hedging ratios according to (7) and (8) using the forecast strategies described above. Based on equations (2) and (5), we produce the portfolio returns and their realized variance, respectively for day ϖ +1. Then, the subsample is rolled one observation ahead by adding one new observation and omitting the oldest one, and the models are re-estimated to produce the next forecast. Hence, we generate T- ϖ- h portfolio returns and return realized variances over our sample of T days. This allows conducting statistical tests on hedging performance (see section 2.4.1.). The process is repeated until the sample observations are exhausted. We use ϖ = 800, which roughly corresponds to 3 years and allows generating 1267 (2566) out-of-sample hedging performance measures for the crack (crush) complex.[4]

2.4.1. *Hedging effectiveness measure*

The hedging effectiveness (*HE*) measure compares return variances of a hedged with an unhedged portfolio and quantifies the variance reduction achieved through hedging. Let $r^{\varpi}_{\pi,t+h}$ denote the returns obtained within a rolling window ϖ and a hedging horizon h=1 day. An unhedged portfolio defines β in (2) as $\beta = \mathbf{0}'$, where $\mathbf{0}$ is a 3×1 vector of zeros and corresponding intraday portfolio returns are denoted as $r^{\varpi,u}_{\pi,t+h+\frac{k}{K}}$. Under *Naïve* hedging, β is set to $\beta = \mathbf{1}'$, with $\mathbf{1}$ being a 3×1 vector of ones, and corresponding intraday portfolio returns are represented as $r^{\varpi,n}_{\pi,t+h+\frac{k}{K}}$. The *HE* quantifies the variance reduction of a hedged over an unhedged position. The following expression calculates the *HE* for both the RMVHR and *Naïve* strategies:

$$\begin{bmatrix} HE^{h}_{naive} \\ \\ HE^{h}_{RMVHR,\gamma} \end{bmatrix} = \begin{bmatrix} \left(1 - \dfrac{V\left[r^{\varpi,n}_{\pi,t+h+\frac{k}{K}} \right]}{V\left[r^{\varpi,u}_{\pi,t+h+\frac{k}{K}} \right]} \right) \\ \\ \left(1 - \dfrac{V\left[r^{\varpi}_{\pi,t+h+\frac{k}{K}} \right]}{V\left[r^{\varpi,u}_{\pi,t+h+\frac{k}{K}} \right]} \right) \end{bmatrix}. \tag{19}$$

[4] In section 4.2. we conduct a robustness check by setting ϖ = 400 and ϖ = 1200.

Where subindex γ is the forecasting strategy used, and $V[.]$ are the portfolio realized variances on day t calculated as in (5). $V\left[r^{\varpi,u}_{\pi,t+h+\frac{k}{K}}\right]$, $V\left[r^{\varpi,n}_{\pi,t+h+\frac{k}{K}}\right]$ and $V\left[r^{\varpi}_{\pi,t+h+\frac{k}{K}}\right]$ are the return variances under the unhedged, *Naïve* and RMVHR strategies, respectively. *HE* is usually interpreted in percentage rather than proportion. The closer the *HE* is to 100%, the larger is the proportion of cash price risks offset by hedging and thus the more effective the hedging strategy. While a *HE* = 100% denotes a perfect hedge, where the hedged portfolio experiences zero return variance, *HE* = 0% implies that the hedged portfolio has equal amount of risk as the cash position itself. A negative *HE* indicates that the hedged position is riskier than the unhedged one.

A key question lies on whether *HE* differences across strategies are statistically significant. To derive statistical inference on *HE*, we follow Wang et al. (2015) and conduct the Diebold and Mariano (Diebold & Mariano, 1995) test on portfolio squared returns. Details are offered in Appendix C. The DM is a pairwise test that informs us if there is difference between the strategies compared. However, its pairwise nature does not allow identifying which among all the strategies considered allows for better hedging performance. Therefore, we also conduct the model confidence set (MCS) test to further compare among strategies (Hansen et al., 2003; Hansen et al., 2011). Intuitively, the MCS selects the set of 'best' models (M^*) given a collection of candidate forecast models (M_0), where 'best' is defined relative to a loss differential. Details are offered in Appendix D.

3. Data and sampling

To produce (co)variance forecasts we use futures tick price data retrieved from CME Time and Sales dataset for the period from 04/11/2014 to 06/17/2022 (01/02/2009 to 06/17/2022) for the crude oil crack (soybean crush) spread. The soybean crush cash prices are provided by CBOT and the crude oil crack cash prices are provided by NYMEX and we retrieve them from Barchart. For the soybean crush spread we focus on day trading hours (detailed in Appendix E table E.1) which concentrate most of the market activity. For the crude oil crack spread, we take the entire day *t* trading session from 5:00 p.m. on day *t*-1 to 4:00 p.m. (4:15 p.m. prior to 2016, see Appendix E table E.2) central time on day *t*. The September soybean (meal/oil) contracts are excluded from the dataset as they are lightly traded (Smith, 2005; Wang et al., 2014).

To derive intraday cash prices, we use the nearby futures contract price defined as the closest-to-expiration contract with the largest trading volume. We rollover to the next-to-expire contract when it experiences higher trading volume than the nearby. For each day, we calculate the difference between the nearby futures closing price and the spot price, to obtain an estimate of the cost of carry for that day. By assuming constancy of the cost of carry within a day, we derive intraday spot prices by subtracting the cost of carry from the nearby futures price. Different industries may use different contracts to hedge, from the nearby to further-out contracts. We choose an intermediate between contract liquidity, which is typically larger in closer-to-expire contracts, and expiration date and hedge cash positions in the first-deferred contract. Similar to the nearby contract, we build a first-deferred continuous series by rolling on the same day that the nearby is rolled.

We sample data following the method proposed by Liu et al. (2015), Asai & McAleer (2015), Anatolyev & Kobotaev (2018), and Zhang et al. (2019). The method consists of building 5 subgrids of 5-minute spaced intervals and is discussed in Appendix F. This method has the merit of cleaning microstructure noise, synchronizing price data for (co)variance calculation, and better exploiting the information in the high-frequency data, by effectively using use 1-minute sampled data.

Table 1 presents summary statistics for sample realized (co)variances for futures and spot prices in the soybean crush and crude oil crack commodity markets. Figure G.1. in Appendix G depicts these (co)variances over time. (Co)variances are directly comparable across markets as we work with log price returns, which are equivalent to percent price returns and thus independent from the units of measurement. Energy commodities experience larger realized variance than agricultural commodities. Average realized variances fluctuate from 4.99 to 24.20 for the oil crack commodities and from 1.79 to 3.37 for the soybean crush (table 1). Realized spot-futures covariances are also larger for the oil crack (4.27 to 7.89) than the soybean crush (1.62 to 1.98). Crack commodity realized (co)variances also show more extreme values than the crush commodities. For example, the crude oil spot price realized variance goes as high as 2,487.05, while the realized covariance between the gasoline spot and futures prices goes as low as -513.54. The soybean meal spot price experiences a maximum realized variance of 399.03 and a minimum spot-futures covariance of -32.46. Consistent with the extreme values, standard deviations suggest larger dispersion in day-to-day (co)variances in the oil crack than in the soybean crush. Higher variability in (co)variances may pose challenges for forecasting accuracy and thus for hedging effectiveness. Skewness and kurtosis measures suggest (co)variances are right skewed and leptokurtic in each market. Figure G.1. in Appendix G shows that the crack market became extremely volatile during 2015-16 coinciding with a price plunge, and the first half of 2020 during the covid-19 pandemic. While the crack market experiences extreme spikes, the crush market is relatively less spiky.

4. Results

We investigate whether there is value in using intraday data to hedge the crack and crush spread margins by considering different forecasting approaches summarized in table 2. First, we forecast realized variances and covariances in expression (7) separately using the HAR-RV and the HAR-RC, respectively. We substitute each of these forecasts into (7) to derive the three hedging ratios for each portfolio. We denote this approach as *Matrix* in the following paragraphs. The combined forecast errors due to the multiple forecasts required in *Matrix* can be large, which may lead to low hedging performance. Thus, we also consider single-commodity portfolios. The *Univariate* method uses HAR-RC and HAR-RV to independently forecast the spot-futures realized covariance and the futures realized variance in expression (8). We then use (8) to generate the hedging ratios for each commodity. The *Vector* and *Cholesky* approaches use VHAR-Log and VHAR-Chol, to forecast the spot-futures covariance matrix for each commodity. We then substitute values in expression (8) to generate the optimal hedging ratio. The *Beta* approach relies on a HAR-Beta to directly forecast the hedging ratio for each commodity (i.e., the left-hand-side of equation (8) directly). In summary, *Matrix* minimizes the overall portfolio variance and thus takes inter-commodity (co)variances into account. Instead, the *Univariate, Vector, Cholesky,* and *Beta* approaches derive hedging ratios for each commodity individually. While the latter models allow deriving single-commodity portfolio hedging ratios and *HE*, we also consider their ability to hedge the spreads. Specifically, we derive the β vector in (7) by setting the non-diagonal values of the (3×3) right-hand side matrices to 0, and diagonal values to the results derived from each of our candidate measures (*Univariate, Vector, Cholesky,* and *Beta*).

4.1 Forecasting hedging ratios

Table 3 presents the mean and standard deviation of the out-of-sample forecast hedging ratios for each commodity across rolling windows. *Matrix* yields highly unstable hedging ratios, specially in the soybean crush market. For example, the average forecast hedging ratio for soybean is 0.486 with a standard deviation of 28.33. Given the large standard deviations, mean values are not highly representative. *Univariate* and *Beta* produce hedging ratios that are generally between 0.74 and below the *Naïve* hedging ratio of 1. *Cholesky* and *Vector* yield forecast hedging ratios in the range between 0.92 and 1.15 and usually display the minimum distance to 1.

Figures H.1 to H.6 in Appendix H depict the evolution of forecast hedging ratios over time for crack and crush spread commodities. To compare across graphs we only present values in the range between -0.5 to 2.5, where most of the results lie. We observe that crack market *Matrix* hedging ratios experience large volatility throughout the out-of-sample period, which is consistent with the information from table 1. Meanwhile, the other four methods produce relatively stable hedging ratios over time. Similarly, *Matrix*-generated hedging ratios show more volatility compared to other methods in the crush spread over the whole sample. To evaluate whether variability in the hedging ratio pays off in terms of improving hedging performance, in the following subsection we present *HE* results.

4.2 Hedging effectiveness and model selection

We now focus on measuring the *HE* to identify whether the use of intraday data yield superior results relative to the *Naïve* hedging strategy. As discussed, *HE* measures the proportional reduction in the hedged portfolio's return variance, relative to the return variance of the unhedged portfolio. Table 4 presents the average *HE* for both the crack and crush spread portfolios (second column). The table also presents the hedging results when each commodity is hedged individually, rather than as part of a spread portfolio (columns 3 to 7). Comparison across portfolios suggests gasoline as the commodity that is more challenging to hedge. Gasoline *HE* ratios register the smallest values, independently on the hedging strategy used. This may be related to gasoline experiencing the largest spot price realized variance (table 1) over the period studied. We will shed further light on this issue when discussing the futures-cash price realized correlation. *Beta* emerges as the best strategy in hedging both the crack and crush portfolios. Specifically, *Beta* reduces the crack (crush) portfolio return variance by 70% (61.7%) on average. *Beta* is also the best strategy in 4 of the 6 single-commodity hedging portfolios. It achieves reductions in the portfolio return's variance above 60%, except for gasoline (43.3%) and soybean meal (59%). The *Naïve* (*Univariate*) strategy slightly surpasses *Beta* in crude oil (soybean oil) hedging with a *HE* = 90.5% (69.2%), while *Univariate* performs identically to *Beta* in soybean.

Matrix is the worst strategy to hedge the crack portfolio, with a *HE* of 24.8%, clearly below the other strategies' *HE* close to 70%. *Matrix* performs even worse when hedging the crush portfolio, with a negative *HE*. This suggests that combined forecast errors associated to *Matrix* worsen hedging performance. In contrast, the other strategies are capable of reducing the portfolio's variance by about 60-70%. Overall, *Beta* achieves either the best or second-best performance over the other strategies in terms of *HE.* Consistent with the literature, this suggests that both simplicity and intraday information are relevant for hedging purposes. Simplicity because *Beta* forecasts the hedging ratio directly rather than its components, and intraday information because *Beta* relies on the RMVHR based on intraday price data.

While our HE results rely on a rolling window size of 800 days (section 2.4), we assess the robustness of our findings to the change of rolling window size by considering a shorter (400 observations) and a longer (1,200 observations) horizon. Overall, the results (presented in Appendix I) are consistent under different estimation horizons, with *Beta* being the best model overall.

Figures 1 and 2 present the differences between *Beta*'s and *Naïve*'s *HE* over time measured in the left vertical axis and depicted in black dots for the crack and crush spreads, respectively. While positive values indicate *Beta* is superior to *Naïve*, negative values suggest the opposite. The right vertical axis measures the realized correlation coefficient between cash and futures depicted in grey lines. The realized correlation ρ_t is calculated from the realized variances and covariances defined in section 2.2, i.e., $\rho_t = RC_{sf,t}/\sqrt{RV_{f,t}RV_{s,t}}$. Since the *Naïve* strategy relies on the hypothesis that $\rho_t = 1$, we expect the difference between the two strategies' *HE* to experience large departures from zero during those periods where ρ_t is lower. Consistent with expectations, figure 2 suggests that *Beta* provides on average better results than *Naïve* in those periods where ρ_t is the lowest in the soybean industry. While both positive and negative *HE* differences arise during these periods, the latter are less frequent and smaller than positive differences. This is especially true for the most recent period, comprising the increased relevance of renewable diesel, which is characterized by large drops in ρ_t for soybean meal and oil. Notice however, that better on average performance comes at the cost of larger variance in performance differences. This pattern is also imprinted in figure 1, which sheds light on a couple of interesting patterns. First, gasoline is the market with the largest ρ_t volatility, which helps understanding the challenges associated to hedging gasoline.[5] Second, *HE* differences grow with volatility in ρ_t. As an example, compare *HE* differences in the crude oil market with the most stable ρ_t, with the heating oil and gasoline markets with intermediate and large volatility in ρ_t, respectively. In short, stable cash-futures realized correlations that are close to 1 do not provide much advantage to *Beta* over *Naïve*. It is when these correlations depart from 1 and are highly unstable that a flexible hedging strategy based on intraday data such as *Beta* can have an edge over *Naïve*.

Next, we apply the DM test to statistically compare each hedging strategy against the benchmark *Naïve* hedge. Results are reported in Appendix J and suggest that there is statistically significant value in using intraday data to hedge commodities in the agriculture space. However, for crack commodities advantages of intraday data are less significant. DM tests suggest that the superior performance of *Beta* over *Naïve* for the crack and crush spread margins is only statistically significant for the crush spread. When it comes to hedging individual commodities, there is always at least one intraday data-based strategy whose *HE* is statistically superior to *Naïve*'s *HE*, except for crude oil, the commodity whose ρ_t experiences less departures from 1 and gasoline, the commodity whose ρ_t experiences the largest departures from 1 (figure 1).

In addition to the DM test, we perform the MCS test to select a superior set of hedging strategies for each portfolio. Results along with further details are presented in Appendix J. We find that *Beta* is the only strategy that survives in all the 8 portfolios, with a p-value of 1.000 in 4 of the 8 portfolios and p-values larger than 0.58 in the other sets. *Naïve*, in contrast, only survives in half of the portfolios considered; crack spread, crude oil, gasoline and soybean. Hence, MCS results are consistent with DM results in that intraday-based strategies are more attractive in the agriculture than the energy space.

In table 5 we offer an estimate of the economic value of using the *Beta* hedging strategy relative to the *Naïve* approach. We concentrate on *Beta* as this is overall the best intraday data-based hedging strategy and measure the economic value by assessing the reduction in portfolio standard deviation (expressed in USD) achieved through hedging. We calculate a hedging effectiveness (*HE'**($)) measure equivalent to (19) in terms of standard deviations, as opposed to

[5] We acknowledge that our results are influenced by our empirical design, i.e., we hedge using the second-to-expire contract and rely on the storage model to derive intraday spot prices. In any case, a low spot-futures price correlation in this context implies challenges in predicting the future gasoline price and thus in hedging the commodity.

variances. *HE'**($) informs on the decline in unhedged returns standard deviation achieved through hedging. *HE'**($) is expressed in USD per crude oil (soybean) contract equivalent for the crack (crush) commodities following the approach detailed in Appendix K. We compare the hedging effectiveness between *Naïve* and *Beta* as

$$HE_t^{\prime *Beta-Naive}(\$) = HE_t^{\prime *Beta}(\$) - HE_t^{\prime *Naive}(\$). \tag{20}$$

Results suggest that *Beta* is more effective at hedging the outputs of the crude oil crack and the soybean crush spread industries than the inputs and the whole spread, which is consistent with results in tables J.1 and J.2 in Appendix J. In the soybean crush, for example, $HE'^{*Beta-Naive}_t(\$)$ is $9.88 ($7.12) for soybean meal (soybean oil) and drops to $1.15 ($2.41) for soybean (soybean crush spread). In the crude oil crack, $HE_t^{*Beta-Naive}(\$)$ is $28.62 and $4.66 for gasoline and heating oil, while values for crude oil (crack spread) are -$2.13 ($1.72). Notice that these values are not net of transactions costs, which are arguably larger for the *Beta* than for the *Naïve* strategy.[6]

The amounts in table 5 will easily grow with the period during which the *Beta* hedging strategy is adopted, and the number of contracts held. For example, a firm hedging gasoline (heating oil) through the *Beta* strategy for a month, i.e. 20 trading days, may see a decline in the portfolio's standard deviation on the order of $572.40 ($93.20) per contract. In contrast, the bad performance of crude oil hedging would result in an increase in the portfolio's standard deviation of $42.60 per contract. Declines observed in soybean meal (soybean oil) are $197.60 ($142.40), while the decline in soybean is $23.00. On a yearly basis (250 trading days), values would jump to $7,155.00, $1,165.00, -$532.50, $2,470.00, $1,780.00 and $287.50 for gasoline, heating oil, crude oil, soybean meal, soybean oil and soybeans, respectively. Hedging the combined crack and crush portfolios with the *Beta* strategy results in smaller annual declines in portfolio's return standard deviation on the order of $430.00 and $602.50, respectively, relative to the *Naïve* strategy. These values are in terms of one contract but would be more substantial for large companies trading a large number of contracts on a daily basis. In summary, table 5 suggests gains from hedging individual commodities using intraday data, except for the crude oil, with these gains ranging from $287.50 to 7,155.00 per contract and year.

4.3 Robustness checks

To assess the robustness of the HAR-Beta model, we examine two alternative models: the ARMA-RMVHR model, which directly forecasts hedging ratios using an ARMA process, and the DCC-GARCH model (Engle, 2002), which forecasts the return covariance matrix using daily returns.

[6] While the *Naïve* strategy does not require adjusting the positions in the futures market over time, the *Beta* does, as forecast realized (co)variances change daily and with them the RMVHR. For example, suppose the forecast RMVHR for day *t* and day *t*+1 are 1 and 1.1, respectively. On day *t*+1 the hedger will have to take an additional 0.1 futures contract relative to day *t*. Information on transactions costs is, however, not publicly available.

The latter allows us to compare our results with conventional models that rely on daily data. Both models and results are presented and discussed in Appendix L to preserve space. Overall, the exceptional performance of HAR-Beta persists in the face of alternative model structures, reaffirming its robustness. Furthermore, results suggest that models relying on intraday data perform better than those using daily returns.

5. Conclusion

Energy and food price volatility has been increasingly relevant during the last decades, due to recurring global supply and demand imbalances and disruptive events such as the 2008 financial crisis, the covid-19 pandemic, and the ongoing Russian invasion of Ukraine. Increased price volatility has induced greater marketing and operation risks in the energy and food supply chains and increased the interest in derivative markets as hedging instruments. The momentum gained by the realized (co)variance forecasting models based on intraday data, has recently led to the proposal of realized minimum-variance hedging ratios (RMVHR) as an alternative to the widely used MVHR based on low frequency data. In this paper we extend this literature by considering, for the first time, the value of intraday data for hedging commodities. By doing so, we show how high-frequency market data can be useful to perform low frequency tasks. We concentrate on the crude oil crack and the soybean crush spread industries. We assess whether complexity pays off by considering multiple-commodity portfolios to hedge the crack and crush spreads and single-commodity portfolios to hedge the commodities in these spreads individually. This results in a total of 8 portfolios.

We make three contributions to the literature. First, we propose a strategy to overcome the lack of intraday spot commodity prices that can be paired with the intraday futures prices based on the cost of carry model for storable commodities. Second, we consider multi-asset hedging portfolios which have not yet been studied using intraday data. This adds substantial complexity to hedging by requiring forecasts of covariances across different commodity prices. Third, we study a wide array of minimum variance hedging strategies based on intraday data and on heterogeneous autoregressive (HAR)-type forecast models to produce the required (co)variances. We compare results against the *Naïve* hedging strategy which has been proven hard to beat (Wu et al. 2015). We use the latter as a benchmark as it does not rely on data to forecast future price (co)variance. Instead, it assumes that futures and spot prices are perfectly correlated over the holding period and thus adopts a hedging ratio equal to 1.

We use the HAR structure as the forecasting tool given its consistent performance in different empirical settings. The hedging strategies considered range from the simplest *Beta* to the most complex *Matrix* approach. *Beta* directly forecasts the daily RMVHR for each commodity independently of the rest. The *Matrix* approach is targeted at a multi-commodity portfolio and requires considering the inter- and intra-commodity futures and futures-spot covariances to forecast the RMVHRs. Intermediate strategies *Vector* and *Cholesky* forecast the spot-futures realized covariance matrix for a single-commodity allowing for spillovers across spot and futures markets. *Univariate* forecasts the components of the spot-futures realized covariance matrix separately. The performance of these strategies is evaluated through the hedging effectiveness (*HE*) ratio and differences in performance across strategies are tested statistically using the Diebold-Mariano (DM) and the Model Confidence Set (MCS) tests. Our results show, for the first time, how intraday data can be useful to commodity hedgers by assuming they rebalance their portfolios daily, a practice that may arguably be more appealing to large than small firms in the commodity business.

We show that intraday data help producing better hedging results relative to the *Naïve* strategy in several instances. Our results further imply that simple is better. In this regard, the crack and crush spread portfolios are better hedged through strategies that ignore the price covariance across different commodities in the portfolio. In other words, obtaining the optimal hedging ratio for each commodity in the portfolio independently of the rest, results in better *HE* than determining hedging ratios simultaneously by allowing for cross-commodity covariances. *HE* results show there is always at least one intraday data-based strategy that outperforms the *Naïve* except for crude oil. We conclude that the simplest intraday data-based strategy, *Beta* is the best strategy overall. Superior performance of intraday data-based strategies is usually statistically significant for agricultural commodities, but less so for energy commodities. Our findings align with previous research in the stock and exchange rates markets and suggest that intraday data can have a value in hedging commodities, particularly in the agricultural space.

While the use of intraday high-frequency data has been associated to sophisticated high-frequency traders, our analysis shows that commercial strategies can also benefit from these data. By using intraday futures transactions price data and daily cash prices, commodity hedgers may achieve better performance than using a *Naïve* hedging approach. Our estimates place the advantage of using intraday data between $7,155.00 and $287.50 per contract and year on average, with these values representing the decline in the portfolio's standard deviation achieved through hedging.

These findings carry significant implications for commercial traders engaged in commodity futures markets. In practical terms, these traders can enhance risk management by leveraging intraday data and making marginal adjustments to their portfolios in alignment with daily forecasted hedging ratios. Additionally, improving hedging outcomes can also enhance welfare by increasing individual firms' utility, improve allocation of commodities over time, as well as investment decisions. The availability of high-frequency data enables the recalibration of hedging portfolios beyond daily frequencies, potentially leading to more precise and effective hedging strategies. It is important to note that while intraday adjustments may incur higher costs compared to low-frequency dynamic hedging, the assessment of whether the benefits outweigh these costs remains an open question. Future research focused on refining hedging activities could explore this avenue, offering valuable insights for commercial traders and high-frequency traders undertaking hedging responsibilities for their clients. Another promising avenue for research in the field of hedging involves enhancing forecasting models based on high-frequency data.

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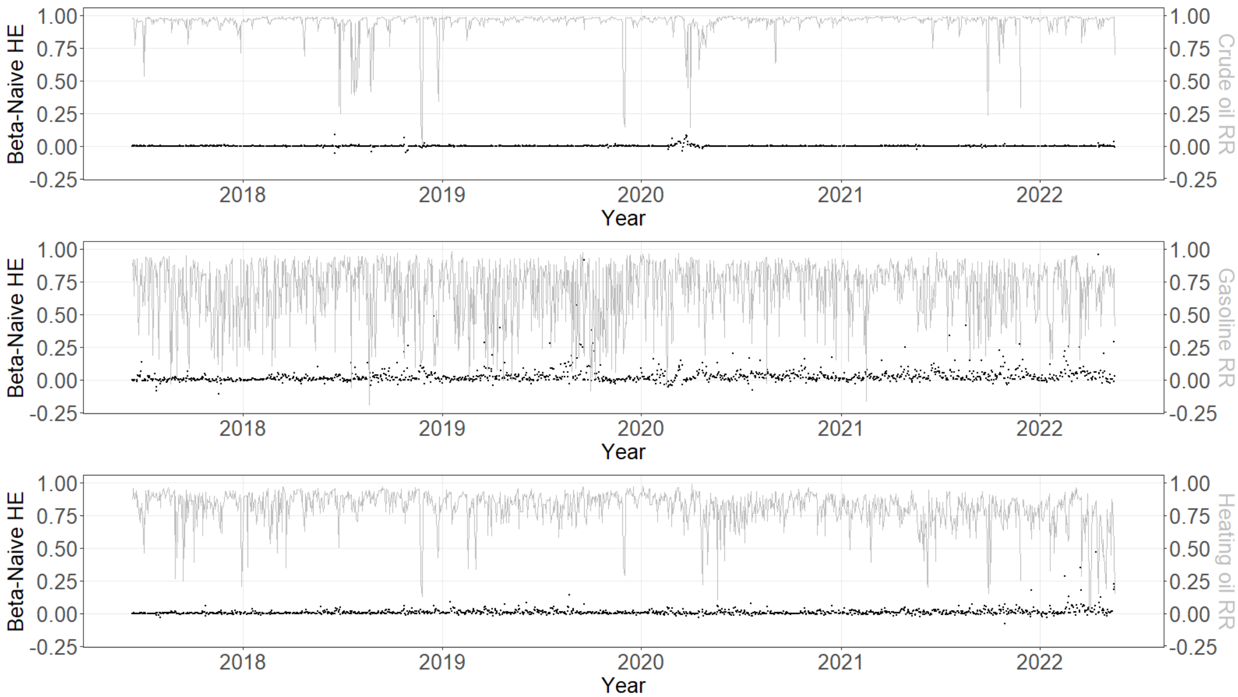
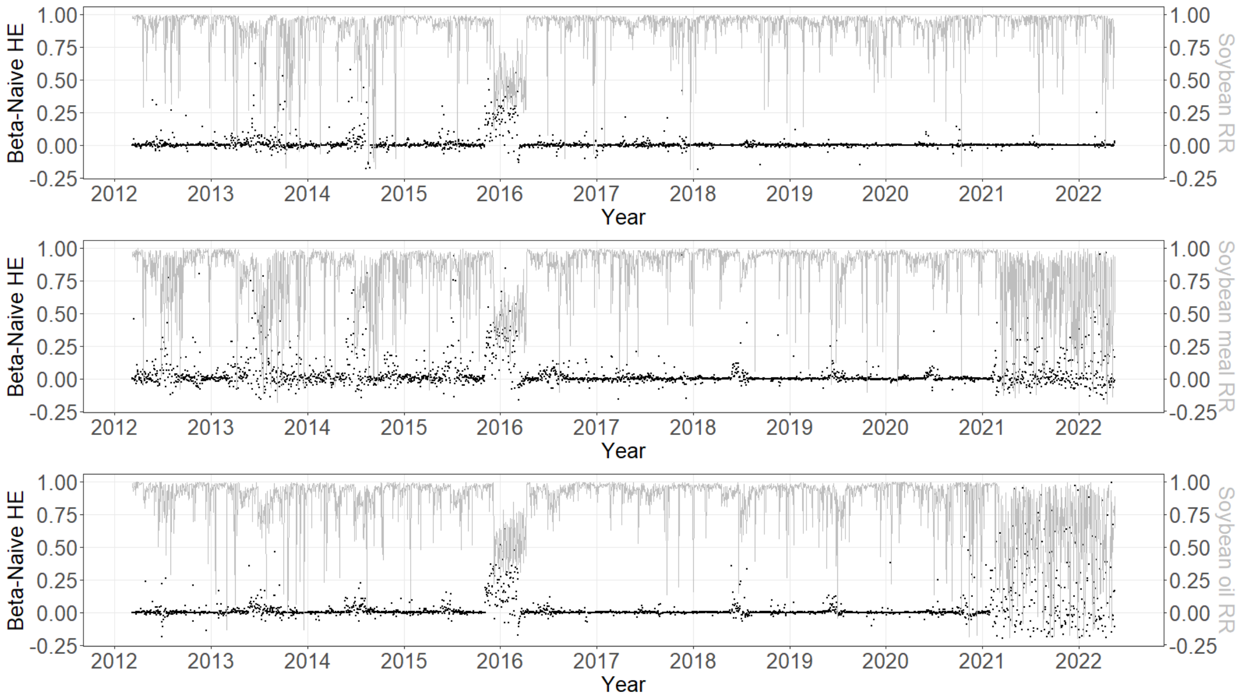
Figures

Figure 1. Differences in hedging effectiveness between *Beta* and *Naïve* and realized correlation coefficient between cash and futures price returns for the crude oil crack spread commodities



Note: Hedging effectiveness differences between *Beta* and *Naïve* in black dots are measured in the left-vertical axis. Realized correlation in grey lines is measured in the right vertical axis.

Figure 2. Differences in hedging effectiveness between *Beta* and *Naïve* and realized correlation coefficient between cash and futures price returns for the soybean crush spread commodities



Note: Hedging effectiveness differences between *Beta* and *Naïve* in black dots are measured in the left-vertical axis. Realized correlation in grey lines is measured in the right vertical axis.

Tables

Table 1. Descriptive statistics of sampled realized (co)variances for futures and spot prices in the crude oil crack (top panel) and soybean crush (bottom panel) commodity markets

	$RV_{s,t}$	$RV_{f,t}$	$RC_{sf,t}$	$RV_{s,t}$	$RV_{f,t}$	$RC_{sf,t}$	$RV_{s,t}$	$RV_{f,t}$	$RC_{sf,t}$
	Crude oil			Gasoline			Heating oil		
Mean	11.94	8.41	7.89	24.20	7.16	5.49	7.85	4.99	4.27
Max	2487.05	1928.58	2013.52	1946.64	556.27	420.12	720.53	362.08	319.78
Min	0.34	0.24	-16.07	0.42	0.25	-513.54	0.32	0.23	-14.87
St.Dev	74.25	49.83	50.98	105.08	22.83	23.18	23.29	13.27	10.87
Skew.	23.05	31.07	32.75	12.85	13.63	2.95	17.39	17.70	16.90
Kurtosis	666.02	1119.59	1213.86	203.39	254.55	236.56	456.89	411.62	405.13
	Soybean			Soybean meal			Soybean oil		
Mean	2.11	1.79	1.62	3.37	2.36	1.98	2.84	2.06	1.76
Max	187.18	116.59	45.42	399.03	102.76	132.58	256.82	53.76	48.46
Min	0.12	0.08	-9.03	0.10	0.08	-32.46	0.20	0.13	-17.68
St.Dev	5.17	3.92	2.38	11.15	4.05	3.86	8.29	2.82	2.69
Skew.	19.56	18.64	6.70	19.96	13.51	15.33	16.24	7.19	6.48
Kurtosis	576.72	474.48	81.64	564.77	277.20	440.88	371.87	87.07	80.60

Note: Table 1 presents summary statistics of realized (co)variances for spot and futures prices for each commodity for the sample period. The sample period starts on 04/11/2014 (01/02/2009) and ends on 06/17/2022 (06/17/2022) for the crude oil crack (soybean crush) spread commodities.

Table 2. Hedging strategies

	Forecasting model	RMVHR
Matrix	HAR-RV & HAR-RC	$\beta\ = -V\left[r_{f,t}\right]^{-1} Cov\left[r_{f,t}, r_{s,t}\right] 1_m$
Univariate	HAR-RV & HAR-RC	$\beta^i = -\frac{RC_{sf,t}^i}{RV_{f,t}^i}$
Vector	HAR-Log	$\beta^i = -\frac{RC_{sf,t}^i}{RV_{f,t}^i}$
Cholesky	HAR-Chol	$\beta^i = -\frac{RC_{sf,t}^i}{RV_{f,t}^i}$
Beta	HAR-Beta	$\beta^i = -\frac{RC_{sf,t}^i}{RV_{f,t}^i}$
Naïve	No	No

Note: β is a 3x1 matrix of three hedging ratios, while β^i is the hedging ratio for commodity *i*

Table 3. Descriptive statistics of forecast hedging ratios

	Matrix	Cholesky	Vector	Univariate	Beta
Crude oil	1.277	1.047	1.003	0.936	0.987
	(2.062)	(0.132)	(0.049)	(0.116)	(0.051)
Gasoline	0.735	1.146	0.954	0.748	0.853
	(3.330)	(0.277)	(0.120)	(0.233)	(0.077)
Heating oil	0.878	0.989	0.942	0.911	0.913
	(1.544)	(0.074)	(0.047)	(0.090)	(0.045)
Soybean	0.486	1.034	0.987	0.904	0.964
	(28.33)	(0.118)	(0.087)	(0.188)	(0.091)
Soybean meal	1.17	1.006	0.928	0.854	0.884
	(16.70)	(0.181)	(0.125)	(0.162)	(0.111)
Soybean oil	1.14	0.983	0.937	0.891	0.896
	(10.51)	(0.135)	(0.120)	(0.169)	(0.146)

Note: Table 3 shows the mean forecast hedging ratios for each commodity based on different hedging strategies. The values reported in parenthesis are standard deviations. The number of observations is 1267 (2566) for the crack (crush) complex.

Table 4. Hedging effectiveness

	Matrix	Cholesky	Vector	Univariate	Beta	Naïve
Crack	0.248	0.678	0.698	0.692	**0.700**	0.696
Crush	-23.78	0.589	0.612	0.605	**0.617**	0.586
Crude oil	-	0.888	0.903	0.890	0.903	**0.905**
Gasoline	-	0.256	0.399	0.419	**0.433**	0.382
Heating oil	-	0.657	0.665	0.664	**0.667**	0.657
Soybean	-	0.622	0.665	0.671	**0.671**	0.644
Soybean meal	-	0.532	0.578	0.550	**0.590**	0.527
Soybean oil	-	0.648	0.682	**0.692**	0.690	0.627

Note: Table 4 presents the average hedging effectiveness for the crack and crush hedging portfolio and for each commodity individually. *HE* measures the proportional reduction in the hedged portfolio's return variance, relative to the return variance of the unhedged portfolio. Bold values indicate the hedging strategy with the largest *HE* for each hedging scenario.

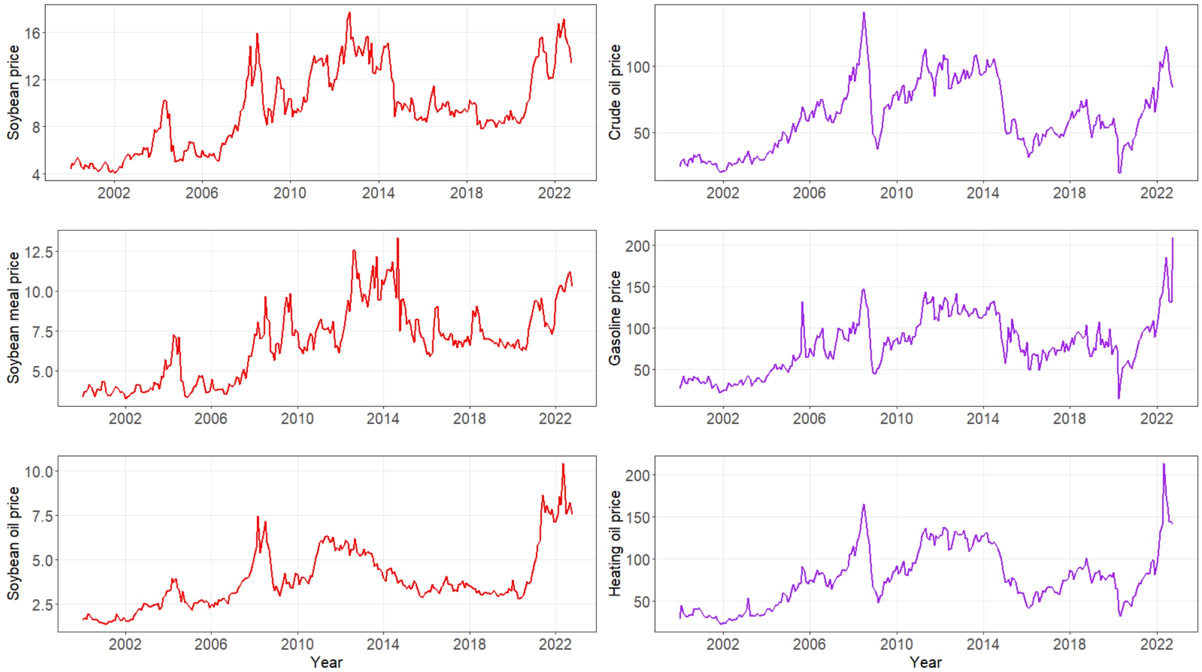
Table 5. Economic value of switching from Naïve to Beta

	Crude oil	Gasoline	Heating oil	Crack
$HE'^{*Beta-Naive}_t$ ($)/day	-2.13	28.62	4.66	1.72
$HE'^{*Beta-Naive}_t$ ($)/month	-42.60	572.40	93.20	34.40
$HE'^{*Beta-Naive}_t$ ($)/year	-532.50	7155.00	1165.00	430.00
	Soybean	Soybean meal	Soybean oil	Crush
$HE'^{*Beta-Naive}_t$ ($)/day	1.15	9.88	7.12	2.41
$HE'^{*Beta-Naive}_t$ ($)/month	23.00	197.60	142.40	48.20
$HE'^{*Beta-Naive}_t$ ($)/year	287.50	2470.00	1780.00	602.50

Note: The values are expressed in USD/contract/day (month/year), assuming 20 trading days in a month and 250 trading days in a year. They represent the cash price return standard deviation that is reduced through hedging. For the crack (crush) commodities we express the economic values per contract of crude oil (soybeans) equivalent. Hence, all prices are expressed in 1,000 barrels of crude oil (5,000 bushels of soybeans) equivalent.

Appendix A – Historical prices

Figure A.1. Historical prices of crude oil crack and soybean crush complex



Note: the figure shows monthly nominal prices of the crude oil crack (right panel) and soybean crush (left panel) complex from January 2000 to October 2022

Appendix B – Cholesky decomposition

We decompose the covariance matrix into the product of H_t and its conjugate transpose $H_t H_t'$ and the half vectorization of H_t gives:

$$Y_t = vech(H_t) = (Y_{s,t}, Y_{sf,t}, Y_{f,t}), \tag{B.1}$$

where $Y_{s,t}, Y_{sf,t}, Y_{f,t}$ represent the elements of the Cholesky-transformed realized (co)variance. Similar to the VHAR-Log model, the VHAR-Chol model is specified as:

$$\begin{bmatrix} Y_{s,t+h} \\ Y_{sf,t+h} \\ Y_{f,t+h} \end{bmatrix} = \begin{bmatrix} \varphi_0 \\ \eta_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} \varphi_{d,s} & \varphi_{d,sf} & \varphi_{d,f} \\ \eta_{d,s} & \eta_{d,sf} & \eta_{d,f} \\ \theta_{d,s} & \theta_{d,sf} & \theta_{d,f} \end{bmatrix} \begin{bmatrix} Y_{s,t} \\ Y_{sf,t} \\ Y_{f,t} \end{bmatrix}$$

$$+ \begin{bmatrix} \varphi_{w,s} & \varphi_{w,sf} & \varphi_{w,f} \\ \eta_{w,s} & \eta_{w,sf} & \eta_{w,f} \\ \theta_{w,s} & \theta_{w,sf} & \theta_{w,f} \end{bmatrix} \begin{bmatrix} Y^w_{s,t} \\ Y^w_{sf,t} \\ Y^w_{f,t} \end{bmatrix} + \begin{bmatrix} \varphi_{m,s} & \varphi_{m,sf} & \varphi_{m,f} \\ \eta_{m,s} & \eta_{m,sf} & \eta_{m,f} \\ \theta_{m,s} & \theta_{m,sf} & \theta_{m,f} \end{bmatrix} \begin{bmatrix} Y^m_{s,t} \\ Y^m_{sf,t} \\ Y^m_{f,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{sf,t} \\ \varepsilon_{f,t} \end{bmatrix}, \tag{B.2}$$

where superindices *w* and *m* represent the average Y_t over the past 5 days and 22 days, respectively. The forecast covariance matrix is obtained by taking the *invvech* of the forecast Y_{t+h}:

$$H_{t+h} = invvech(Y_{t+h}), \tag{B.3}$$

And multiplying by its conjugate transpose:

$$\begin{bmatrix} RV_{s,t+h} & RC_{sf,t+h} \\ RC_{sf,t+h} & RV_{f,t+h} \end{bmatrix} = H_{t+h} H_{t+h}' \tag{B.4}$$

Appendix C – Diebold and Mariano test

The minimum-variance hedging framework aims to minimize the realized variance of the hedging portfolio, and a perfect hedge implies that the intraday return variance of the portfolio is zero. Given expression (6) in the paper, a perfect forecast of the hedging ratio leads to the perfect hedge. Thus, non-zero return variance of the hedged portfolio can be treated as forecasting errors. We define the loss differential as the difference between the realized variances of the *Naïve* hedging strategy $g^{\varpi}_{1,t+h} = \sum\left(r^{\varpi,n}_{\pi,t+h+\frac{k}{K}}\right)^2$ and the RMVHR strategy $g^{\varpi}_{\beta,t+h} = \sum\left(r^{\varpi}_{\pi,t+h+\frac{k}{K}}\right)^2$, which yields $D^{\varpi}_{1\beta,t+h} = g^{\varpi}_{1,t+h} - g^{\varpi}_{\beta,t+h}$. We generate as many $D^{\varpi}_{1\beta,t+h}$ observations as total number of rolling windows. Next, we conduct the Diebold-Mariano (DM) test on the loss differentials. The null hypothesis is $H_0\colon D^{\varpi}_{1\beta,t+h} = 0$, indicating no statistically significant difference between the two hedging strategies. The original DM test is not robust to serial correlation which we create by using the rolling window approach. To correct for this issue, we conduct the DM test following the approach of Giacomini & White (2006). We run a regression of $D^{\varpi}_{1\beta,t+h}$ on a constant and use Newey-West standard errors (Newey and West 1987). If the intercept is significantly different from 0 then the two strategies have significantly different performances.

Appendix D – MCS test

Intuitively, the MCS selects the set of 'best' models (M^*) given a collection of *n* candidate forecast models (M_0), where 'best' is defined relative to a loss differential. The procedure sequentially identifies $M^* \subset M_0$ given a confidence interval α. The equivalence test is applied to the collection of models in M_0. Rejection of the null hypothesis implies that the objects in M_0 are not equally "good." Then, an elimination rule is used to remove from M_0 the model with the poorest sample performance. This procedure is repeated until the null is accepted and the MCS is defined by a set of "surviving" models (M^*). M_0 contains all models derived from each forecasting strategy and we sequentially test the null hypothesis of equal prediction accuracy, i.e.

$$H_0: E\left(D^{\varpi}_{\beta\beta',t+h}\right) = 0 \; \forall \beta, \beta' \in M_0, \tag{D.1}$$

where β and β' include all different forecasts to produce the RMVHR based on the different econometric specifications described earlier, including the *Naïve* approach. Following Hansen et al. (2011), two types of statistics are used to test the null hypothesis, the range statistics T_R, and *max* statistics T_{max}.

According to Hansen et al. (2011), let

$$D^{\varpi}_{\beta\beta',t+h} = g^{\varpi}_{\beta,t+h} - g^{\varpi}_{\beta',t+h}, \; \beta, \beta' = 1, \dots, n, \; t = 1, \dots, T, \tag{D.2}$$

be the loss differential between model β and model β' in the model set M_0, where $g^{\varpi}_{\beta,t+h}$ and $g^{\varpi}_{\beta',t+h}$ are the loss functions used in the DM test (Appendix C). Then, we define the sample loss of model β relative to the average losses across *n* models in the set M_0 as:

$$\overline{D}^{\varpi}_{\beta} = n^{-1} \sum_{\beta' \in M_0} \overline{D}^{\varpi}_{\beta\beta'}, \tag{D.3}$$

where $\overline{D}^{\varpi}_{\beta\beta'} = (T)^{-1} \sum_{t=1}^{T} D^{\varpi}_{\beta\beta',t+h}$, is the relative sample loss between model β and model β'. The T_R and T_{max} statistics are associated to the null hypotheses (D.4) and (D.5), respectively.

$$H_{0,\beta\beta'}: E\left(D^{\varpi}_{\beta\beta',t+h}\right) = 0, \forall \beta, \beta' \in M_0, \tag{D.4}$$

and $H_{0,\beta}: E\left(\overline{D}^{\varpi}_{\beta}\right) = 0, \forall \beta \in M_0$ (D.5) which form the basis to test (D.1). For these two null hypotheses we construct two *t*-statistics:

$$t_{\beta\beta'} = \frac{\bar{D}^{\varpi}_{\beta\beta'}}{\sqrt{\widehat{var}(\bar{D}^{\varpi}_{\beta\beta'})}}, \tag{D.6}$$

and $t_{\beta} = \frac{\bar{D}^{\varpi}_{\beta}}{\sqrt{\widehat{var}(\bar{D}^{\varpi}_{\beta})}}$, (D.7)

respectively, where $\widehat{var}(\bar{D}^{\varpi}_{\beta\beta'})$ and $\widehat{var}(\bar{D}^{\varpi}_{\beta})$ represent the estimates of $var(\bar{D}^{\varpi}_{\beta\beta'})$ and $var(\bar{D}^{\varpi}_{\beta})$. Next, we use the two *t*-statistics to form the test statistics for the null hypotheses:

$T_{max,M_0} = \arg\max_{\beta \in M_0} t_{\beta}$, and (D.8)

$$T_{R,M_0} = \arg\max_{\beta\beta' \in M_0} |t_{\beta\beta'}|. \tag{D.9}$$

As stated in Hansen et al. (2011), the test statistics T_{max,M_0} and T_{R,M_0} have nonstandard asymptotic distributions under the null hypothesis, thus we employ a bootstrap procedure. When the null hypothesis is rejected at the significance level α%, the model with worst performance is removed from the set M_0 with elimination rules associated with T_{max,M_0} and T_{R,M_0}, which are defined as:

$e_{max,M_0} = \arg\max_{\beta \in M_0} t_{\beta}$, and (D.10)

$$e_{R,M_0} = \arg\max_{\beta \in M_0} \sup_{\beta\prime \in M_0} t_{\beta\beta\prime} \tag{D.11}$$

respectively. The elimination process is repeated until the null hypothesis is not rejected and we construct the superior model set M^* with a (1- α) % confidence band. In our application we conduct the bootstrap procedure with 10,000 iterations with block length 2 to correct for possible serial correlation in the data.

Appendix E – Trading hours

Table E.1. Trading hours of CME soybean/meal/oil futures, 01/02/2009 - 06/17/2022

Date	Opening time	Closing time
01/02/2009 - 04/05-2013	9:30 a.m.	13:15 p.m.
04/08/2013 - 07/01/2015	8:30 a.m.	13:15 p.m.
07/02/2015 - 06/17/2022	8:30 a.m.	13:20 p.m.

Table E.2. Trading hours of CME crude oil/gasoline/heating oil futures, 04/11/2014 - 06/17/2022

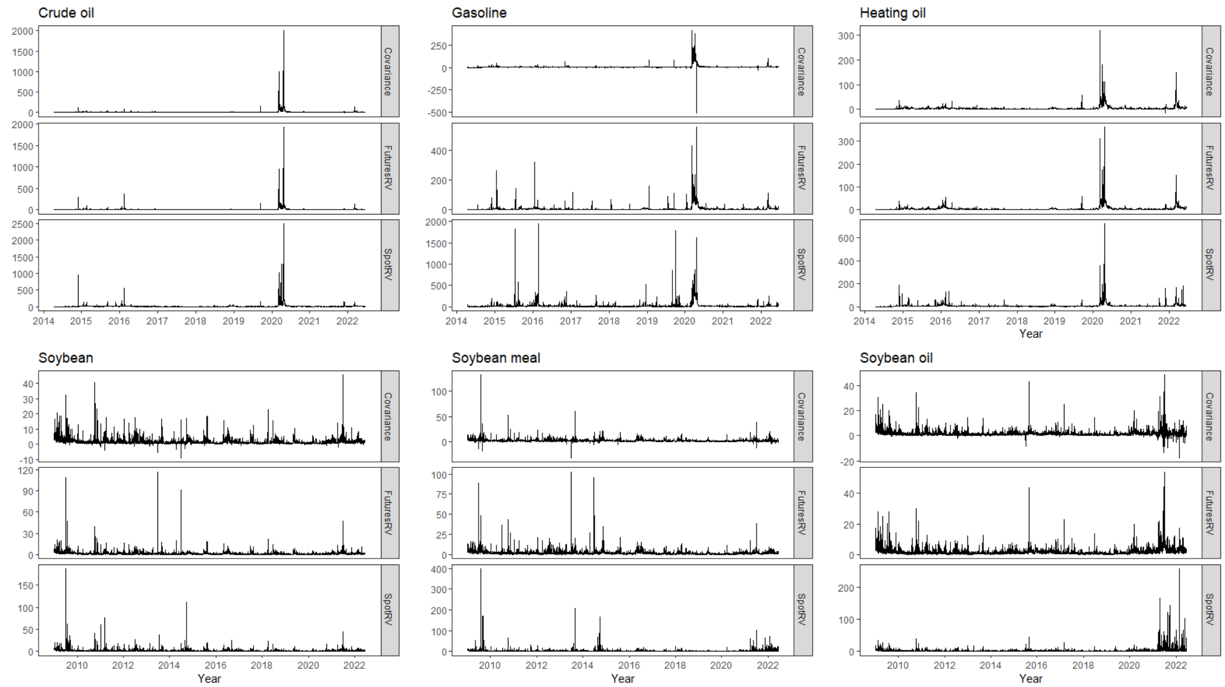
Date	Opening time	Closing time
04/11/2014 - 04/08-2016	17:00 a.m.	16:15 p.m.
04/11/2016 - 06/17/2022	17:00 a.m.	16:00 p.m.

Appendix F – Sampling

We sample prices every 5 minutes to filter the microstructure noise from the price data. The 5-minute sampling results in a loss of an important amount of data. Following Asai & McAleer (2015) Anatolyev & Kobotaev (2018) and Zhang et al. (2019) we use an averaging method to compensate for this loss. The method consists of building 5 subgrids of 5-minute spaced intervals starting at minute 0, 1, 2, 3, and 4 of each day trading session, respectively. For each subgrid, we sample prices every 5-minutes. Last day's closing price is added prior to the first 5-minute interval of each subgrid as the overnight price. Whenever a transaction cannot be identified at exactly 5 minutes after the previous transaction, the last transaction available from the data is taken. Then, 5-minute intraday returns for each subgrid are calculated with the sampled prices. On each day, 5 realized (co)variances are produced, one for each subgrid, which we average to form a single realized measure.

Appendix G – Sample (co)variances over time

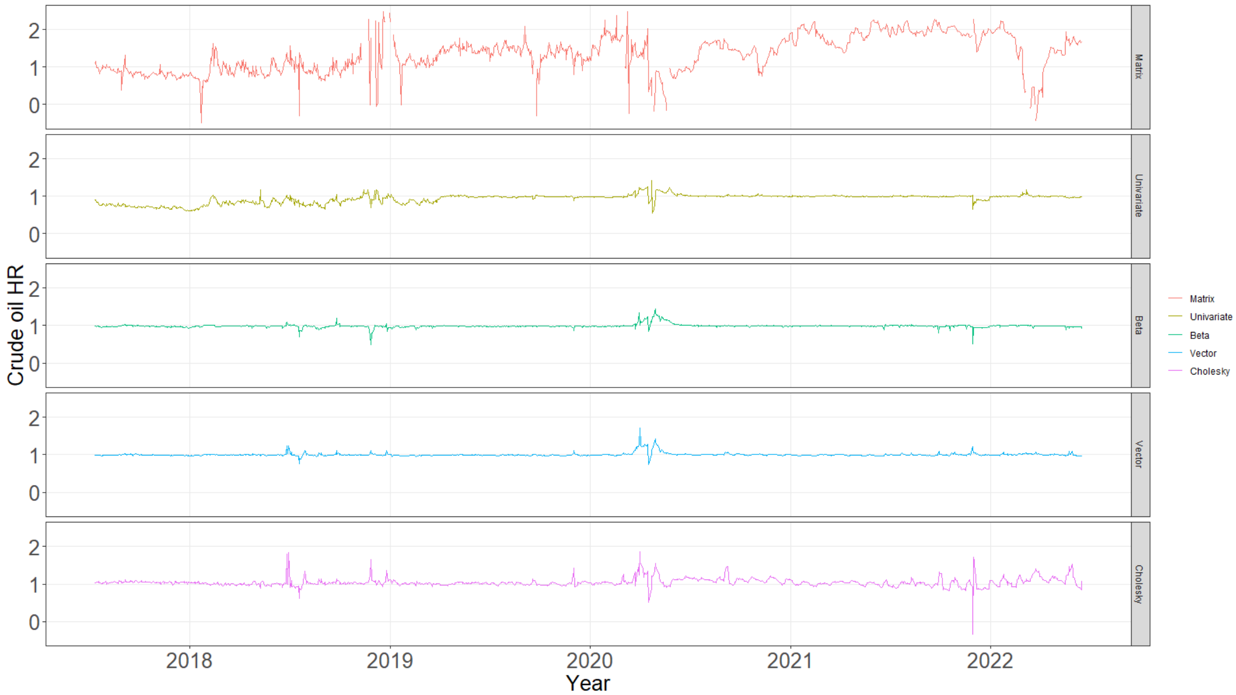
Figure G.1. Sample period realized (co)variances for the crude oil crack (top three panels) and the soybean crush spread (top bottom panels) commodity markets



Note: For each commodity, the (co)variances are arranged from top to bottom as: realized cash-futures price covariance, realized variance in futures position, and realized variance in spot position. We removed the negative price that crude oil registered in April 2020 due to the need to take logarithms on the prices.

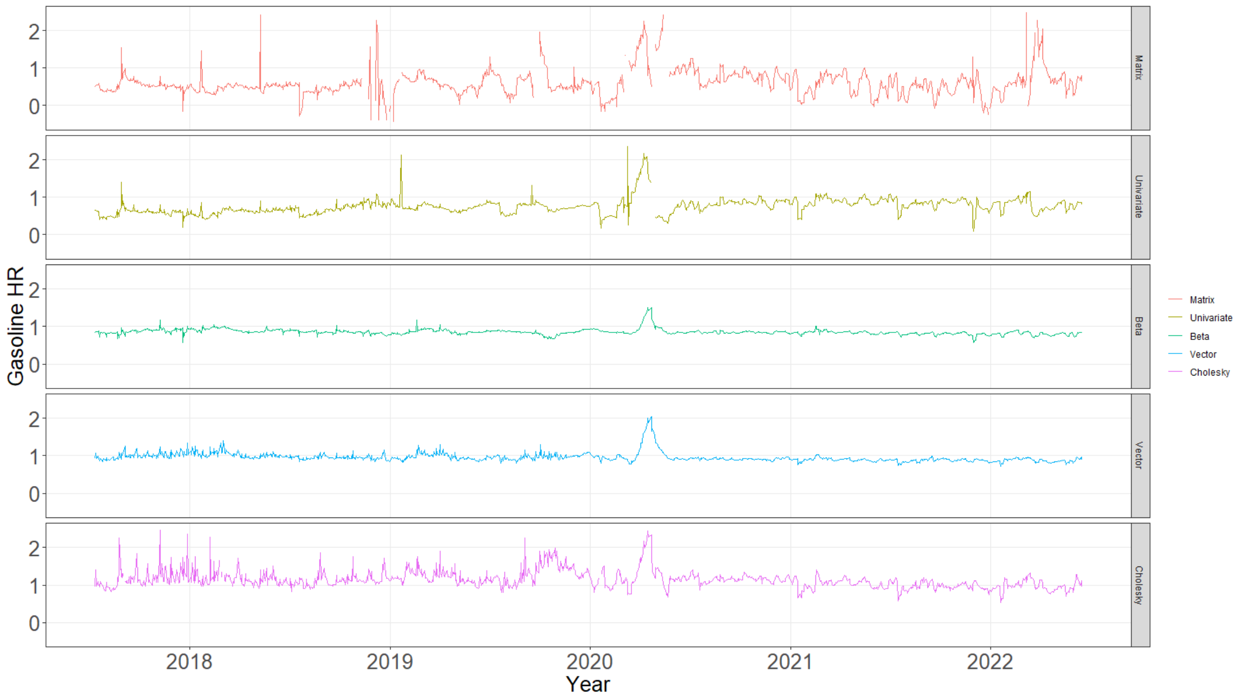
Appendix H – Forecast hedging ratios for the crack and crush spread outputs

Figure H.1. Forecast hedging ratio for crude oil



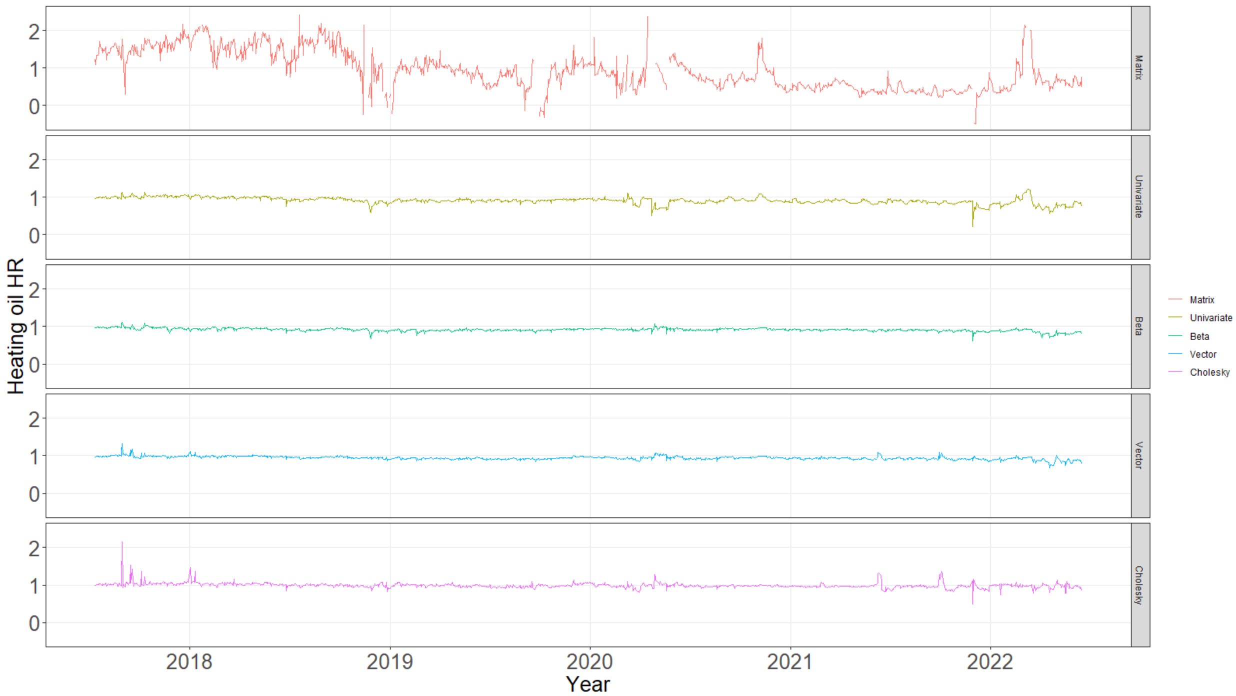
Note: The figure shows forecast hedging ratios for crude oil derived from different hedging strategies. The vertical axis is truncated to be in the range [-0.5, 2.5].

Figure H.2. Forecast hedging ratio for gasoline



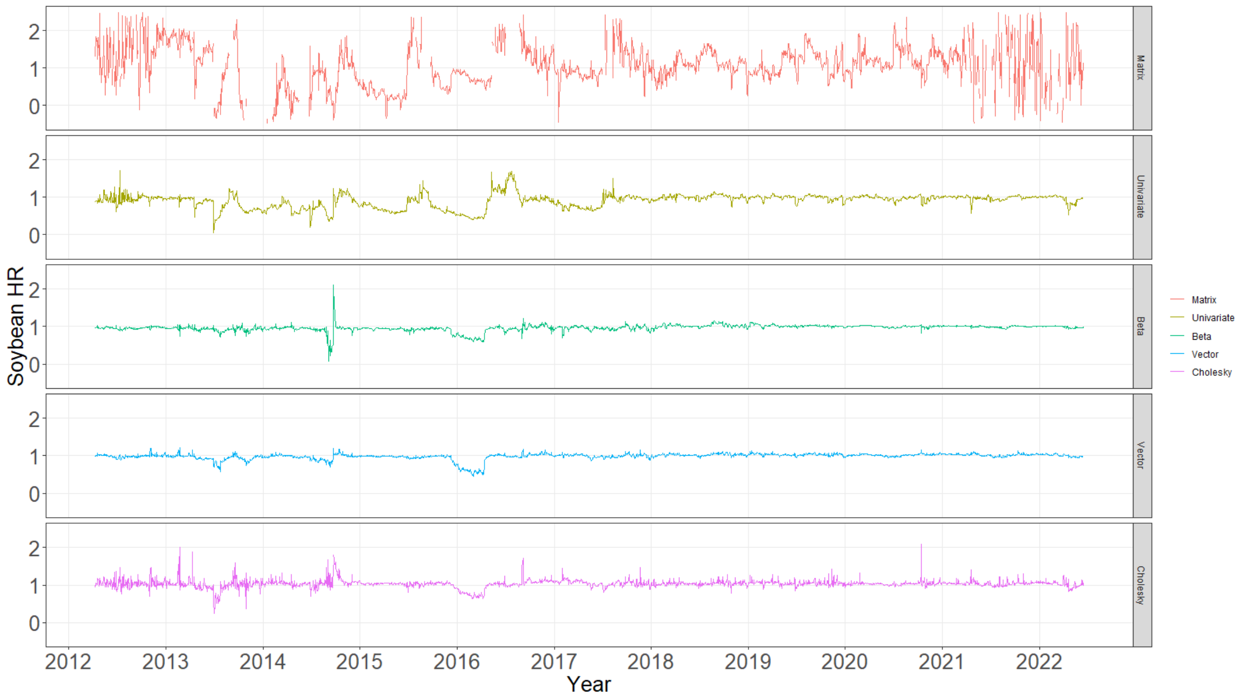
Note: The figure shows forecast hedging ratios for gasoline derived from different hedging strategies. The vertical axis is truncated to be in the range [-0.5, 2.5].

Figure H.3. Forecast hedging ratio for heating oil



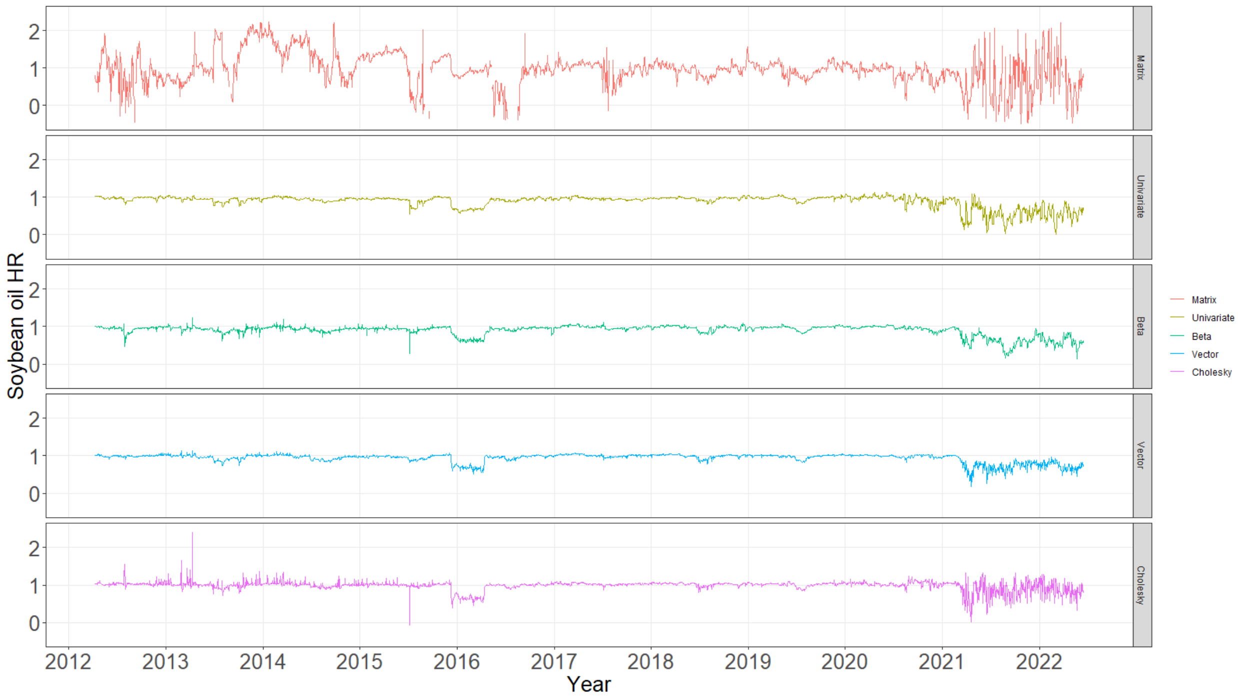
Note: The figure shows forecast hedging ratios for heating oil derived from different hedging strategies. The vertical axis is truncated to be in the range [-0.5, 2.5].

Figure H.4. Forecast hedging ratio for soybean



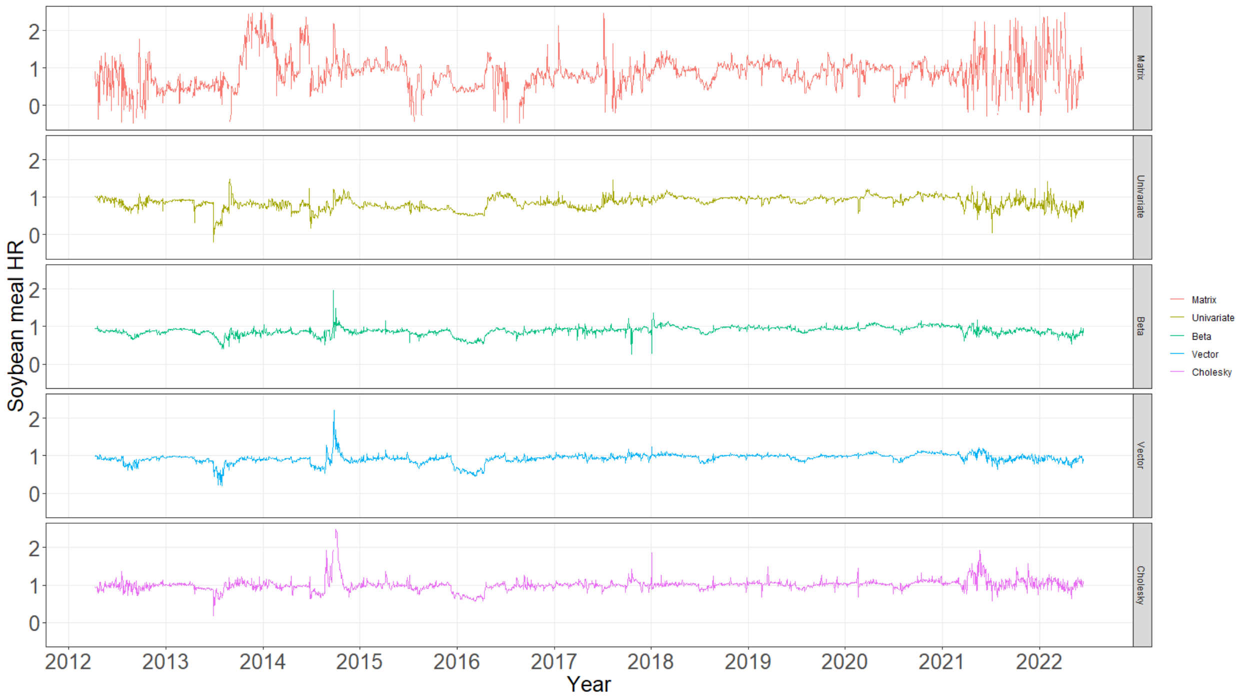
Note: The figure shows forecast hedging ratios for soybean derived from different hedging strategies. The vertical axis is truncated to be in the range [-0.5, 2.5].

Figure H.5. Forecast hedging ratio for soybean oil



Note: The figure shows forecast hedging ratios for soybean oil derived from different hedging strategies. The vertical axis is truncated to be in the range [-0.5, 2.5].

Figure H.6. Forecast hedging ratio for soybean meal



Note: The figure shows forecast hedging ratios for soybean meal derived from different hedging strategies. The vertical axis is truncated to be in the range [-0.5, 2.5].

Appendix I – Hedging effectiveness robustness check

Table I.1. Hedging effectiveness with ϖ = 400

	Matrix	Cholesky	Vector	Univariate	Beta	Naïve
Crack	0.444	0.651	0.669	0.661	**0.670**	0.665
Crush	-1.036	0.617	0.639	0.632	**0.643**	0.615
Crude oil	-	0.847	0.860	0.851	**0.861**	0.844
Gasoline	-	0.266	0.384	0.394	**0.416**	0.366
Heating oil	-	0.631	0.639	0.636	**0.640**	0.626
Soybean	-	0.651	0.689	**0.701**	0.696	0.676
Soybean meal	-	0.566	0.604	0.577	**0.610**	0.553
Soybean oil	-	0.664	0.703	0.707	**0.710**	0.654

Note: Table I.1 presents the average hedging effectiveness for the crack and crush hedging portfolio and for each commodity individually, with the rolling window size equal to 400 observations. *HE* measures the proportional reduction in the hedged portfolio's return variance, relative to the return variance of the unhedged portfolio. Bold values indicate the hedging strategy with the largest *HE* for each hedging scenario.

Table I.2. Hedging effectiveness with ϖ = 1200

	Matrix	Cholesky	Vector	Univariate	Beta	Naïve
Crack	0.611	0.689	0.712	0.699	**0.712**	0.709
Crush	-1.114	0.597	0.618	0.612	**0.622**	0.593
Crude oil	-	0.874	0.910	0.889	0.910	**0.913**
Gasoline	-	0.250	0.415	0.412	**0.443**	0.388
Heating oil	-	0.629	0.637	0.635	**0.638**	0.626
Soybean	-	0.701	0.726	0.726	**0.729**	0.714
Soybean meal	-	0.545	0.587	0.574	**0.601**	0.546
Soybean oil	-	0.642	0.676	**0.686**	0.682	0.610

Note: Table J.3 presents the average hedging effectiveness for the crack and crush hedging portfolio and for each commodity individually, with the rolling window size equal to 1200 observations. *HE* measures the proportional reduction in the hedged portfolio's return variance, relative to the return variance of the unhedged portfolio. Bold values indicate the hedging strategy with the largest *HE* for each hedging scenario.

We find that the 1,200-observation rolling window yields results virtually identical to the 800-observation window. *Beta* outperforms in 6 out of 8 portfolios, except for crude oil (*Naïve*) and soybean oil (*Univariate*). When we use the 400-observation window, the relative performance of *Beta* improves, as it is only beaten in soybean (by the *Univariate* strategy). All three window sizes' *HE* values are very close to each.

Appendix J – Diebold-Mariano and MCS test results

Table J.1 reports both the DM test statistics and p-values. The DM test null hypothesis states that there is no difference between the performance of the *Naïve* and the alternative intraday data-based hedging strategy. Consistent with results in table 4 in the main paper, DM tests show the poor performance of *Matrix* against the *Naïve* through very large test statistics. While p-values reject the null hypothesis for crack but not crush, after removing 3 significant outliers from crush *Matrix* forecasts, *Matrix*'s *HE* remains lower than *Naïve*'s *HE* and the null hypothesis is rejected.[7] This allows concluding that *Naïve* outperforms *Matrix*. A total of 4 out of 8 DM test statistics corresponding to the *Beta-Naïve* comparison show significance. These are the DM tests for the crush spread, heating oil, soybean meal and soybean oil. In all these cases, *Beta* has larger *HE* than *Naïve,* which allows concluding that *Beta* outperforms *Naïve* in half of the portfolios. In the remaining half, *Beta* performs equally well as *Naïve.* A total of 5 out of 8 DM statistics are significant when comparing *Vector* and *Naïve*, and all involve higher *HE* for *Vector*. Specifically, *Vector* outperforms *Naïve* in the crush spread and the three crush commodities, as well as heating oil. In the remaining portfolios, *Vector*'s performance is not statistically different from *Naïve*. The *Cholesky* approach does not have any significant result. The *Univariate* strategy outperforms *Naïve* in soybean oil, at the 10% significance level underperforms *Naïve* in the crude oil, and performs equally well in the rest. In summary, hedging strategies based on intraday data that forecast the spot-futures covariance matrix either explicitly (*Vector*) or implicitly (*Beta*) and ignore spillovers across different commodities are either superior or equal to the *Naïve* hedging ratio, with *Vector* and *Beta* beating *Naïve* when hedging the crush spread margins. When it comes to hedging individual commodities, there is always at least one intraday data-based strategy that outperforms the *Naïve*, except for crude oil, the commodity whose ρ_t experiences less departures from 1 and gasoline, the commodity whose ρ_t experiences the largest departures from 1 (figure 4). Soybean oil benefits the most from intraday data-based hedging methods with 3 out of the 4 intraday strategies outperforming the *Naïve* at the 1% significance level, followed by heating oil and soybean meal (2 out of 4) and then soybean (1 out of 4). This suggests that there is statistically significant value in using intraday data to hedge commodities, especially in the agriculture space. For crack commodities, however, advantages of intraday data are rarely significant.

In addition to the DM test, we perform the MCS test to select a superior set of hedging strategies for each portfolio. The MCS test is performed with 10,000 bootstrap iterations and a block length of 2. Table J.2 shows the p-values of the MCS test for the spread and individual commodity portfolios. The p-value shows the significance level at which the strategy is excluded from the superior set. We find that *Beta* is the only strategy that survives in all the 8 portfolios, with a p-value of 1.000 in 4 of the 8 portfolios and p-values larger than 0.586 in the other sets. *Vector* survives 6 out of 8 portfolios, excluding heating oil and soybean meal. *Univariate* and

[7] Results are available from the authors upon request.

Cholesky are excluded three times from the MCS. The *Naïve* is the worst performer and is excluded from four portfolios: crush, heating oil, soybean meal and soybean oil. *Matrix* is excluded from both spread portfolios.

The MCS test suggests that heating oil should be hedged using intraday data and the hedging ratio should be based on the *Beta* strategy. The crush spread can be hedged by either *Beta* or *Vector* and soybean meal with *Beta* or *Cholesky*. Soybean oil can be hedged through *Beta*, *Univariate* or *Vector*. The use of intraday data and the complexity associated to doing so does not seem to make a difference in the case of crude oil, gasoline, and soybean hedging according to the MCS test. Crude oil and gasoline are the most liquid futures markets of the crack spread, while soybean is the most liquid market of the crush spread. All strategies except *Matrix* survive the MCS in the crack spread, the most highly traded spread in the commodity space. Similar to the DM test, the MCS suggests agricultural commodities as the group possibly benefiting the most from intraday data-based hedging. Notice that *Naïve* is excluded more often from the crush than the crack compound.

Table J.1. DM test

	Matrix	Cholesky	Vector	Univariate	Beta
Crack	1.283e+2	3.030	2.237	2.558	1.027
	(4.838e-3)***	(0.244)	(0.563)	(0.621)	(0.484)
Crush	6.852e+4	3.137e-3	4.964e-3	8.378e-3	6.036e-3
	(0.290)	(0.710)	(2.371e-4)***	(0.171)	(5.333e-4)***
Crude oil	-	0.270	0.012	0.013	0.011
	-	(0.163)	(0.931)	(0.062)*	(0.952)
Gasoline	-	9.494	5.155	1.855	0.959
	-	(0.277)	(0.380)	(0.267)	(0.492)
Heating oil	-	3.013e-4	2.384e-4	9.780e-4	6.006e-4
	-	(0.156)	(4.087e-5)***	(0.904)	(1.663e-3)***
Soybean	-	6.638d-5	6.675e-5	4.548e-4	1.375e-4
	-	(0.100)	(4.600e-2) ***	(0.617)	(0.248)
Soybean meal	-	5.402e-4	4.914e-4	9.657e-4	5.031e-4
	-	(0.982)	(6.380e-3) ***	(0.694)	(5.570e-4) ***
Soybean oil	-	4.799e-4	1.168e-3	1.801e-3	1.304e-3
	-	(0.159)	(9.948e-6) ***	(2.477e-4) ***	(9.568e-7) ***

Note: Table J.1. reports the DM test statistics when comparing advanced hedging strategies to the Naïve strategy when hedging the crack and crush spread as well as hedging single commodities. The values presented in the parenthesis are *p*-values of the test. *, **, and *** indicate 10%, 5%, and 1% significance level respectively.

Table J.2. MCS test

	Matrix	Cholesky	Vector	Univariate	Beta	Naïve
Crack	0.022**	0.403	0.626	0.586	0.586	1.000
Crush	0.085*	0.027**	0.708	0.014**	1.000	0.085*
Crude oil	-	0.307	1.000	0.307	0.992	0.992
Gasoline	-	0.255	0.738	0.738	0.631	1.000
Heating oil	-	0.067*	0.065*	0.005***	1.000	0.060*
Soybean	-	0.415	1.000	0.415	0.590	0.382
Soybean meal	-	0.124	0.064*	0.042**	1.000	0.038**
Soybean oil	-	0.002***	0.380	0.972	1.000	0.001***

Note: Table J.2 shows the *p*-values obtained from the MCS test with 10,000 bootstraps and a block length of 2. *, **, and *** indicate that the model is excluded from the MCS at 10%, 5%, and 1% significance levels, respectively.

Appendix K – Economic Value

In table 5 in the article we offer an estimate of the economic value of using the *Beta* hedging strategy relative to the *Naïve* approach. We concentrate on *Beta* as this is overall the best intraday data-based hedging strategy and measure the economic value in USD. To obtain the dollar values, we calculate a hedging effectiveness (*HE**) measure equivalent to (19) in the main paper in terms of standard deviations, as opposed to variances. The latter informs on the percent by which the standard deviation of the unhedged portfolio returns $\left(std\left(r_{\pi,t}^{u}(\%)\right)\right)$ can be reduced through hedging as follows:

$$HE_t^*(\%) = \left(std\left(r_{\pi,t}^{u}(\%)\right) - std\left(r_{\pi,t}(\%)\right)\right)/std\left(r_{\pi,t}^{u}(\%)\right). \tag{K.1}$$

where $std\ ()$ is calculated by taking the square root of the realized portfolio variance (expression (5) in the main article).

Alternatively, (K.1) can be expressed as:

$$HE_t^*(\%)std\left(r_{\pi,t}^{u}(\%)\right) = \left(std\left(r_{\pi,t}^{u}(\%)\right) - std\left(r_{\pi,t}(\%)\right)\right). \tag{K.2}$$

Let's focus on the left-hand side of (K.2) which informs on the absolute decline in standard deviation as a result of hedging. However, since the standard deviation is expressed in percent, this measure is not directly useful for our purpose. We want to express the standard deviation in USD rather than in percent, which results in our economic measure. For simplicity, assume we have a single-commodity portfolio. The unhedged portfolio returns only depend on percent spot price returns, which we measure as $r_{\pi,t}^{u}(\%) = 100\log\frac{p_{s,t}^{i}}{p_{s,t-1}^{i}} \approx 100\frac{p_{s,t}^{i}-p_{s,t-1}^{i}}{p_{s,t-1}^{i}}$. As discussed in section 2.1. in the main paper, superindex $i = a, b, c$ denotes the input (a) or output (b, c). Since $p_{s,t-1}^{i}$ is known, the unhedged returns standard deviation can be expressed as $std\left(r_{\pi,t}^{u}(\%)\right) \approx \frac{100}{p_{s,t-1}^{i}}std(p_{s,t}^{i})$. We transform $std\left(r_{\pi,t}^{u}(\%)\right)$ into $std\left(r_{\pi,t}^{u\prime}(\$)\right)$ in USD per crude oil (soybean) futures contract equivalent for the crack (crush) commodities in expression (K.3)

$$std\left(r_{\pi,t}^{u\prime}(\$)\right) \approx\ std\left(r_{\pi,t}^{u}(\%)\right)\frac{p_{s,t-1}^{i}}{100}\frac{i}{a}q \approx std\left(\frac{i}{a}p_{s,t}^{i}\right)q, \tag{K.3}$$

where a, b, c are the technical coefficients representing the crude oil crack or the soybean crush production functions, and q measures the number of barrels (bushels) of crude oil (soybeans) per crude oil (soybeans) futures contract. By multiplying $p_{s,t}^{i}$ by $\frac{i}{a}$, we transform the initial price $p_{s,t-1}^{i}$

in crude oil / soybean equivalent price. Hence, we now can use the idea in (K.2) to we express the reduced variability of cash price returns as a result of hedging in USD as:

$$HE_t'^{*}(\$) = HE_t^{*}(\%)std\left(r_{\pi,t}^{u'}(\$)\right)q = HE_t^{*}(\%)std\left(\frac{i}{a}p_{s,t}^{i}\right)q. \tag{K.4}$$

For crude oil (gasoline) [heating oil] q=1,000 barrels of crude oil equivalent and *i/a* = 1 (2/3) and [1/3]. Similarly, for soybeans (soybean meal) [soybean oil] q=5,000 bushels of soybeans and *i/a* = 1 (11/10) [9/10]. We compare the hedging effectiveness between *Naïve* and *Beta* as

$$HE_t'^{*Beta-Naive}(\$) = HE_t'^{*Beta}(\$) - HE_t'^{*Naive}(\$). \tag{K.5}$$

For the spread portfolios we follow the same process based on the price $p_{s,t}$ calculated as $p_{s,t} = -p_{s,t}^{a} + \frac{b}{a}p_{s,t}^{b} + \frac{c}{a}p_{s,t}^{c}$.

Appendix L – Alternative models

We assess the time-series properties of the optimal hedging ratios derived using methods described in Table 2 and find they display significant autocorrelation and are stationary.[8] ARMA models are widely used in modeling serial correlation which we consider to assess the robustness of our results to other econometric specifications unrelated to HAR. Specifically, we use the following ARMA-RMVHR (p, q) model, where the realized hedging ratio is forecasted using the ARMA structure:

$$\beta_t = \varepsilon_t + \sum_{i=1}^{p} \varphi_i \, \beta_{t-i} + \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j} \tag{L.1}$$

For model parsimony, we define (p, q) = (1, 1).

Next, we explore a GARCH model that leverages daily frequency returns to forecast the daily conditional covariance matrix and calculate the forecasted hedging ratio. Specifically, we examine the bivariate DCC-GARCH model, a well-known GARCH variant that allows us estimating time-varying MVHRs. The conditional means for the two return series of the DCC-GARCH are represented as:

$$r_{i,t}^{*} = \mu_{i,t} + \varepsilon_{i,t} + \theta \, \varepsilon_{i,t-1}, \tag{L.2}$$

$$\varepsilon_{i,t} = h_{i,t}^{\frac{1}{2}} \eta_{i,t} \tag{L.3}$$

where, $r_{i,t}^{*}$ is the log daily price difference of the spot (*i*=*s*) or the futures (*i*=*f*) price at time *t*, $\eta_{i,t} \sim i.i.d.\,N(0,1)$, and $h_{i,t}$ is the conditional variance. The equation for the conditional variance of a DCC-GARCH (1,1) is as follows:

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t}^2 + \beta_i h_{i,t-1} \tag{L.4}$$

DCC-GARCH allows the conditional correlation matrix R_t between spot and futures returns to vary independently from their covariance matrix H_t:

$$H_t = D_t R_t D_t \tag{L.5}$$

where D_t is the diagonal matrix of the conditional standard deviations $\left(h_{i,t}^{\frac{1}{2}} \right)$ at time *t* and R_t follows the dynamic process specified in Engle (2002).

Table L. 1 compares the Hedging Effectiveness (*HE*s) of the ARMA and GARCH models with

[8] Results are available upon request.

HAR-Beta and the *Naïve* strategy. The *Beta* approach demonstrates superior performance in 7 out of 8 hedging scenarios, uncontested by the two alternative models except in the soybean market, with ARMA's *HE* slightly higher than that of *Beta*'s. When comparing with the *Naïve* strategy, the ARMA model demonstrates superior performance in 4 out of 8 cases, whereas the GARCH model outperforms in 6 out of 8 cases. Table L. 2 presents the results of the Model Confidence Set (MCS) test, including the two alternative models and the HAR-type models. The MCS test reveals that *Beta* remains effective in all hedging scenarios. Additionally, with the inclusion of the new models, the *Cholesky* approach is favored in one more case, and the *Univariate* and *Naïve* strategy are deemed optimal in two more scenarios, at a 10% confidence level. Both the ARMA and GARCH models are excluded from the set the largest number of times (three) at a 10% significance level and 1% significance level, respectively. Hence, none of these three models exhibit a superior performance advantage over the others significantly, which is consistent with Wang et al. (2015). While ARMA shows larger *HE* than *Beta* for soybean (table L.1), ARMA and *Beta* models exhibit identical MCS p-values when hedging soybean, implying their performances are not statistically different. Overall, the exceptional performance of HAR-Beta persists in the face of alternative model structures, reaffirming its robustness. Furthermore, our results suggest that models that rely on intraday high-frequency data generally outperform the model based on daily frequency data.

Table L. 1. Hedging effectiveness with alternative models

	Beta	Naïve	ARMA	GARCH
Crack	**0.700**	0.696	0.681	0.690
Crush	**0.617**	0.586	0.606	0.594
Crude oil	0.903	**0.905**	0.886	0.896
Gasoline	**0.433**	0.382	0.358	0.398
Heating oil	**0.667**	0.657	0.617	0.658
Soybean	0.671	0.644	**0.675**	0.664
Soybean meal	**0.590**	0.527	0.557	0.552
Soybean oil	**0.690**	0.627	0.658	0.632

Note: Table L.1 presents the average hedging effectiveness of the *Beta* approach, *Naïve* strategy, the ARMA-RMVHR model, and DCC-GARCH model for the crack and crush hedging portfolio and for each commodity individually.

Table L. 2. MCS test with alternative models

	Matrix	Chol.	Vector	Univ.	Beta	Naïve	ARMA	GARCH
Crack	0.026**	0.614	0.659	0.626	0.657	1.000	0.659	0.626
Crush	0.089*	0.225	0.712	0.013**	1.000	0.181	0.035**	0.225
Crude oil	-	0.353	1.000	0.353	0.991	0.991	0.218	0.225
Gasoline	-	0.418	0.690	0.690	0.642	0.642	1.000	0.418
Heating oil	-	0.113	0.060*	0.154	1.000	0.154	0.065*	0.000***
Soybean	-	0.434	1.000	0.434	0.912	0.475	0.912	0.862
Soybean meal	-	0.072*	0.062*	0.205	1.000	0.072*	0.205	0.006***
Soybean oil	-	0.027**	0.383	0.975	1.000	0.027**	0.038**	0.038**

Note: Table L.2 shows the *p*-values obtained from the extended MCS test when adding ARMA-RMVHR and DCC-GARCH with 10,000 bootstraps and a block length of 2. *, **, and *** indicate that the model is excluded from the MCS at 10%, 5%, and 1% significance levels, respectively.